Validation of the Geant4-based Calorimeter Acceptance Model in Fermilab Muon g-2 Experiment

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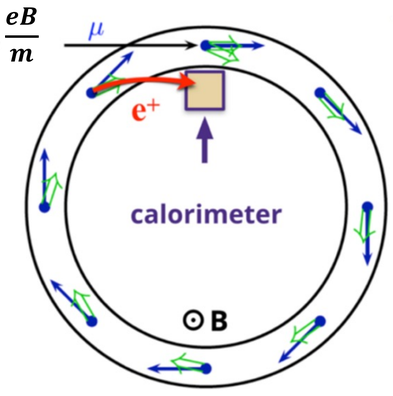
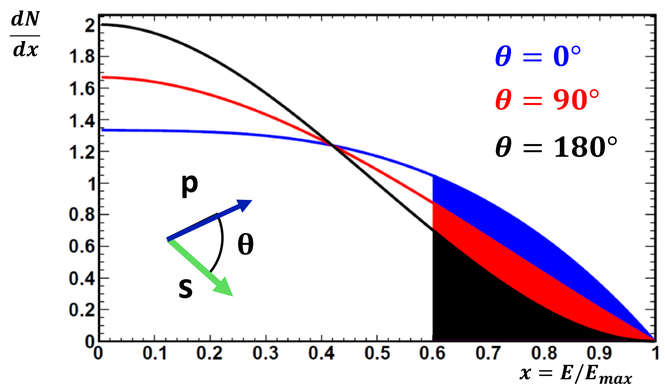
Muon's Magnetic Anomaly

Anomalous Spin Precession

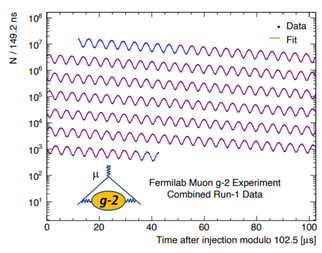
$$\omega_a = a_\mu \frac{eB}{m}$$



e^+ Spectrum Modulation

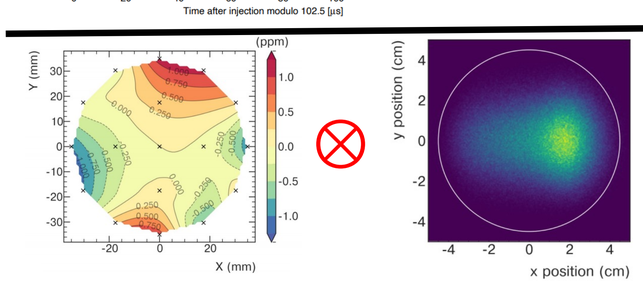


ω_a Analysis



Beam dynamic corrections

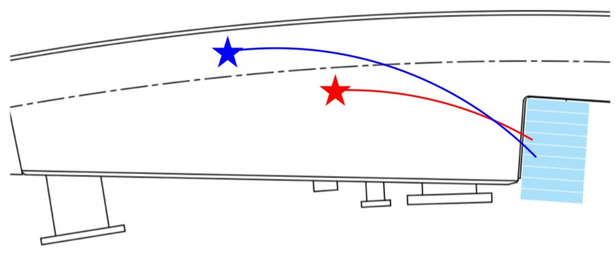
$$a_\mu \propto \frac{\omega_a \left(1 + C_e + C_p + C_{pa} + C_{ml} + C_{dd}\right)}{< \omega_p \times M > \left(1 + B_k + B_q\right)}$$



Muon- Weighted Magnetic field

Calorimeter Acceptance Maps

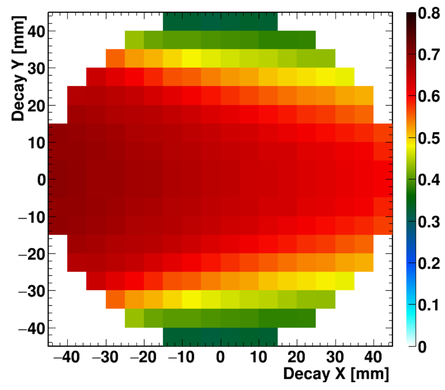
Modelling of Calorimeter response



- Sample decay time -> muon decay position
- Sample Michel spectrum -> e^+ initial momentum
- Parametrization of Edep in calo

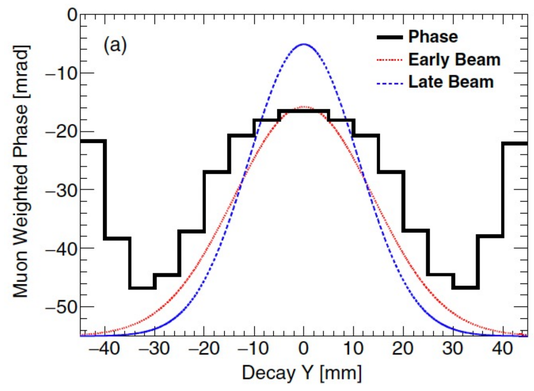
Acceptance = detected/generated

- Depends on decay location and material near the calo



Application: C_{pa} correction to ω_a

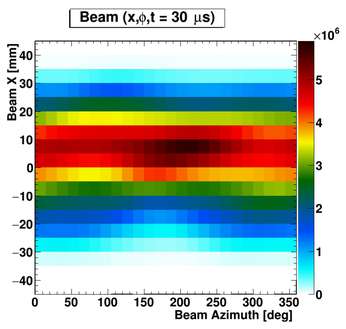
- **How well the maps agree with data (systematics) ?**



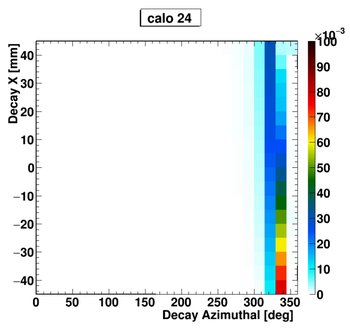
Validation of The Maps

Simulation

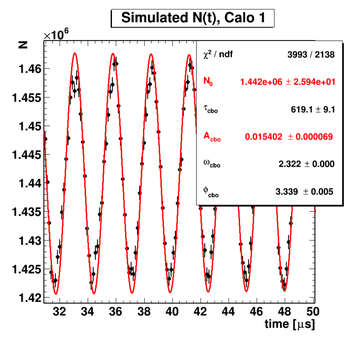
1. Muon Beam Sim.



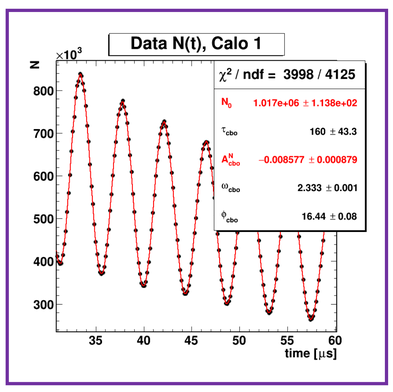
Acceptance maps



2. Predict N(t) using Maps

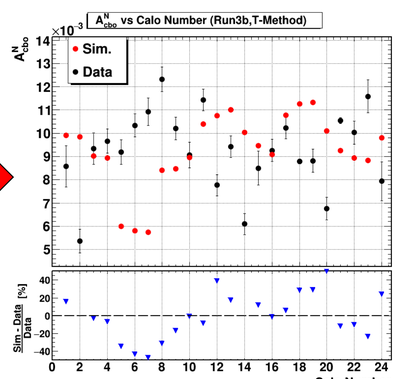


Data



fit

3. Compare Beam Dynamic Parameters

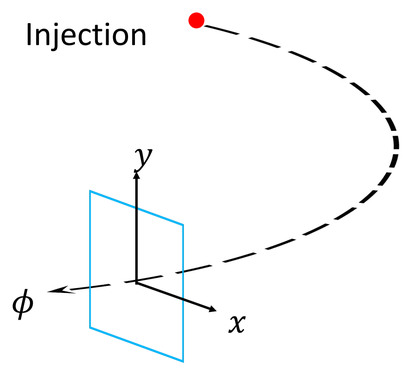


fit

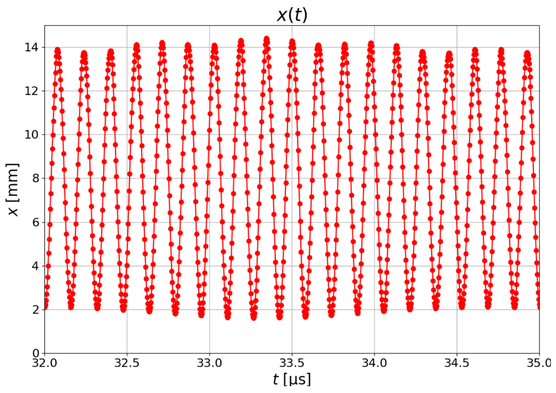
4. MC-Data Agreement = systematics

Muon Beam Simulation

G4 Tracking Planes

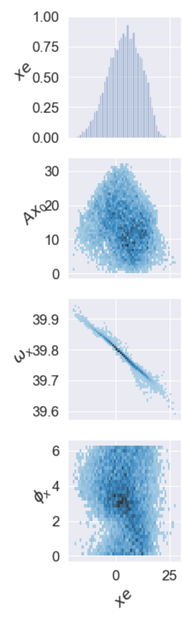


$$x(t) = x_e + A_x \cos(\omega_x + \phi_x)$$



Limitations

- Computationally expensive tracking $\mathcal{O}(min)$ / muon
- Not possible to record infinitesimal steps



- Fit with simplified beam dynamic functions

Fast Simulation of the Beam Distribution

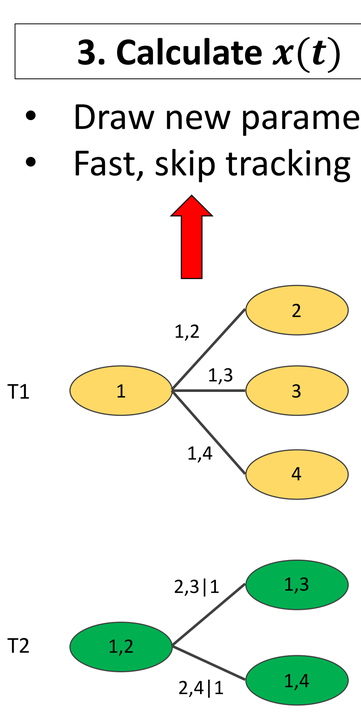
1. Input Distribution

Extract [3]:

- Marginals
- Dependence between parameters

2. Estimated Distribution

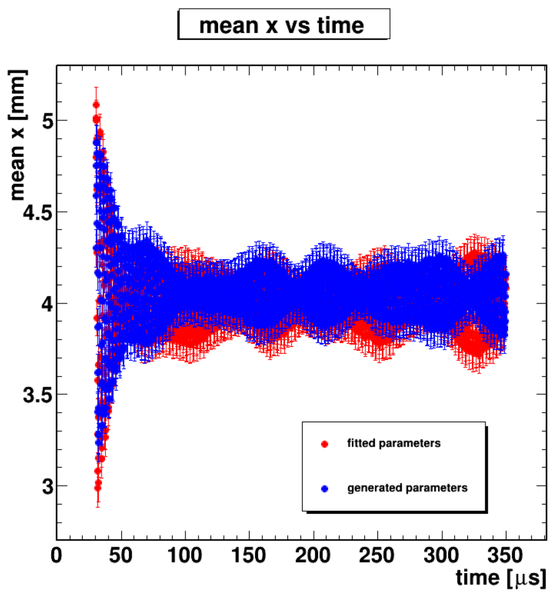
- **Vine Copula** [4]
- Multivariate distribution -> bivariate copulas
- Interpretability (vs. GAN)
- Efficient computation [1]



3. Calculate $x(t)$

- Draw new parameters
- Fast, skip tracking

4. Beam Distribution



Conclusion

- The calorimeter acceptance maps have been generated using geant4 simulation
- The maps are widely used in systematics studies of ω_a Analysis
- We predict the time spectrum of the detected positrons at the calorimeter using a simulated beam distribution and the acceptance maps
- The computational expensive beam simulation was parametrized by a copula-based model
- The fast simulation correctly reproduce the beam distribution

References

[1] T. Nagler and T. Vatter. Pyvinecopulib v0.6.4 (2023)

[2] Muon g-2 Colloboration. Phys. Rev. Lett., 131: 161802 (2023)

[3] K. Aas et al. Insurance: Mathematics and Economics, 44(2): 182-198 (2009)

[4] T. Bedford T & R. Cooke. Annals of Mathematics and Artificial Intelligence, 32(1): 245-268 (2001)

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