Precision test of the weak interaction with slow muons

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Neutrinos and Muons

- Neutrinos are perhaps the most mysterious particles, due to their weak interactions with other SM particles
- Their small masses are confirmed by neutrino oscillation experiments (both solar and atmospheric), and direct measurements as well (KATRIN, Project8). Then we may ask the question: are neutrinos of Dirac or Majorana type?
- The smallness of neutrino masses can be explained by models such as the seesaw mechanism. On the other hand, V+A current can be restored in models such as the Left-Right Symmetric model. All these hint at BSM physics
- Muons are rich in neutrinos in their decays, and are relatively free of QCD effects. This makes them an ideal test bed for the BSM related to neutrinos

Lepton decay theory

- In a generic V$\pm$A model, the effective Hamiltonian for a lepton decay (vector currents) $l^- \rightarrow l'^- \bar{\nu}_{l'} \nu_l$ is

$$\mathcal{H} = 4\frac{G_F}{\sqrt{2}}\left(g_{LL} J^{\dagger}_{\ell' L\mu} J^{\mu}_{\ell L} + g_{RR} J^{\dagger}_{\ell' R\mu} J^{\mu}_{\ell R} + g_{LR} J^{\dagger}_{\ell' L\mu} J^{\mu}_{\ell R} + g_{RL} J^{\dagger}_{\ell' R\mu} J^{\mu}_{\ell L}\right)$$

where

$$J^{\mu\dagger}_{\ell L} = \sum_j \bar{\ell}\frac{1}{2}\gamma^{\mu}(1-\gamma^5)U_{\ell j} N_{jL},$$

$$J^{\mu\dagger}_{\ell R} = \sum_j \bar{\ell}\frac{1}{2}\gamma^{\mu}(1+\gamma^5)V_{\ell j} N_{jR},$$

N_j is the mass eigenstate spinor for the *j*'th neutrino, and U_{lj} (V_{lj}) is the left-handed (right-handed) lepton mixing matrix

- Second term is due to right-handed gauge boson W_R, and the last two terms come from the possible mixing between W_L and W_R

Muon decays

- Differential muon decay rate in its rest frame follows ($W_{e\mu} = (m_\mu^2 + m_e^2)/2m_\mu$, $x_0 = m_e/W_{e\mu}$, $x = E_e/W_{e\mu}$, θ is between electron momentum and muon polarization vector P_μ)

$$\frac{d\Gamma}{dx\,d\cos\theta} = \frac{m_\mu}{4\pi^3} W_{e\mu}^4 G_F^2 \sqrt{x^2 - x_0^2}\,\left[F_{IS}(x) - \mathcal{P}_\mu \cos\theta F_{AS}(x)\right]$$

  where $W_{e\mu}$ is the maximum electron energy, F_{IS} (F_{AS}) is the isotropic (anisotropic) part of the spectrum

- If the light neutrino mass can be neglected, and the muon beam is unpolarized (or muon polarization is not measured), the isotropic part reads

$$F_{IS}(x) = Ix(1-x) + \frac{2}{9}\rho\left(4x^2 - 3x - x_0^2\right) + \eta x_0(1-x)$$

  where ρ and η are the Michel parameters. In the SM, $I = |g_{LL}|^2 = 1$, $\rho = \frac{3}{4}|g_{LL}|^2 = \frac{3}{4}$, and $\eta = 0$

Muon decays

- In a general V $\pm$ A theory, the Michel parameters become

$$I = \sum_{j,k} \left[\left| f_{LL}^{jk} \right|^2 + \left| f_{RR}^{jk} \right|^2 + \left| f_{LR}^{jk} \right|^2 + \left| f_{RL}^{jk} \right|^2 + \epsilon \mathrm{Re} \left(f_{LR}^{jk} f_{LR}^{kj*} + f_{RL}^{jk} f_{RL}^{kj*} \right) \right]$$

$$\rho = \sum_{j,k} \frac{3}{4} \left(\left| f_{LL}^{jk} \right|^2 + \left| f_{RR}^{jk} \right|^2 \right)$$

$$\eta = \epsilon \sum_{j,k} \left[\mathrm{Re} \left(f_{LL}^{jk} f_{RR}^{kj*} \right) \right]$$

where summation is over neutrinos with mass light enough in the muon decay, ϵ=0 (1) for Dirac (Majorana) neutrinos, and

$$f_{LL}^{jk} = g_{LL} U_{ej} U_{\mu k}^*, \quad f_{RR}^{jk} = g_{RR} V_{ej} V_{\mu k}^*,$$
$$f_{LR}^{jk} = g_{LR} U_{ej} V_{\mu k}^*, \quad f_{RL}^{jk} = g_{RL} V_{ej} U_{\mu k}^*.$$

Neutrino	$U_{\ell j}$	$V_{\ell j}$
Light ($j \leq 3$)	Not suppressed	Suppressed
Heavy ($j \geq 4$)	Suppressed	Not suppressed

- We immediately see that when $x \rightarrow x_0$, the η-term has an non-zero contribution to the I-term if the neutrinos are of Majorana type

Muon decays

- It is worth mentioning that if the heavy neutrino mass (m_j) is light enough to appear in the decay, then an extra contribution to the η-term reads

$$\frac{m_j}{2m_\mu}\left[x_0(1-x)+x_0\sqrt{1-x_0^2}\right]\sum_k \mathrm{Re}\left[f_{LL}^{jk}f_{RL}^{jk*}+f_{RR}^{jk}f_{LR}^{jk*}+\epsilon\left(f_{LL}^{jk}f_{RL}^{kj*}+f_{RR}^{jk}f_{LR}^{kj*}\right)\right]$$

and the phase space factor should be changed to

$$\sqrt{x^2-x_0^2}\left(1-\frac{m_j^2}{\left(m_\mu^2+m_e^2\right)(1-x)}\right)$$

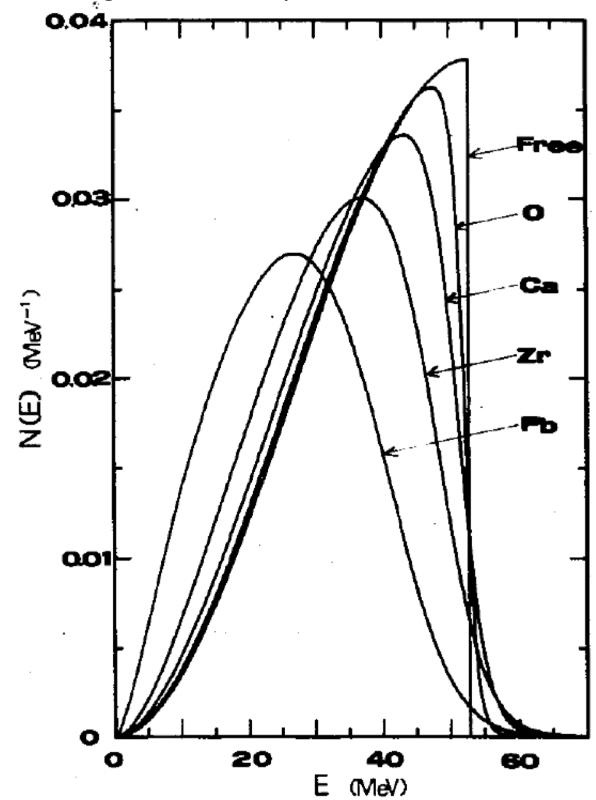
- Finally, if there are scalar and tensor currents, the η-term becomes [JHEP11 (2022) 117]

$$\begin{aligned}(\eta)_{jk}=\frac{1}{2}\mathrm{Re}[&(f_{LL}^V)_{jk}(f_{RR}^S)_{jk}^*+(f_{RR}^V)_{jk}(f_{LL}^S)_{jk}^*+(f_{LR}^V)_{jk}((f_{RL}^S)_{jk}^*+6(f_{RL}^T)_{jk}^*)+(f_{RL}^V)_{jk}((f_{LR}^S)_{jk}^*\\&+6(f_{LR}^T)_{jk}^*)]+\frac{\epsilon}{8}\mathrm{Re}\Big[4(f_{LR}^S)_{jk}(f_{RL}^V)_{kj}^*+24(f_{LR}^T)_{jk}(f_{RL}^V)_{kj}^*+(f_{LL}^S)_{jk}(f_{RR}^S)_{kj}^*+4(f_{LL}^V)_{jk}\\&(f_{RR}^V)_{kj}^*+(L\leftrightarrow R)\Big],\end{aligned}$$

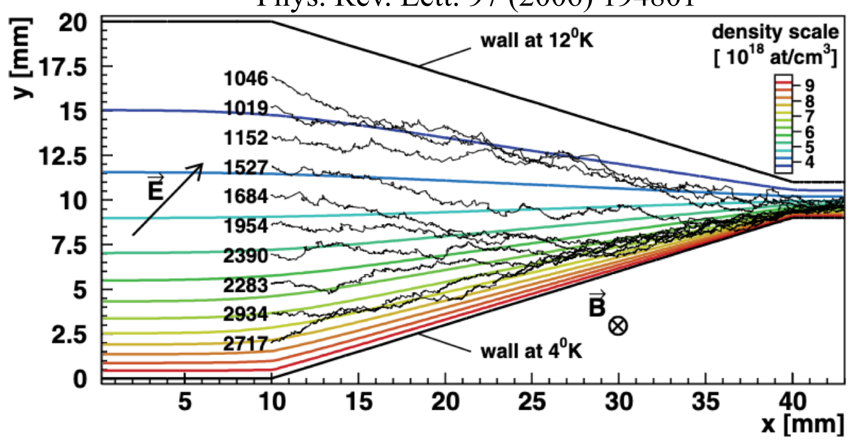
Muons in material

- A μ^- can be captured by a nucleus and the electron energy spectrum can be changed due to muon's atomic motion, making the end-point change significantly with different nuclei
- On the other hand, μ^+ can be produced as surface muons whose energy can be as low as a few eV via ionization energy loss in material. The PSI method can be used to compress muons to <1 mm

Prog. of Theo. Phys., Vol. 78, No.1, 1987

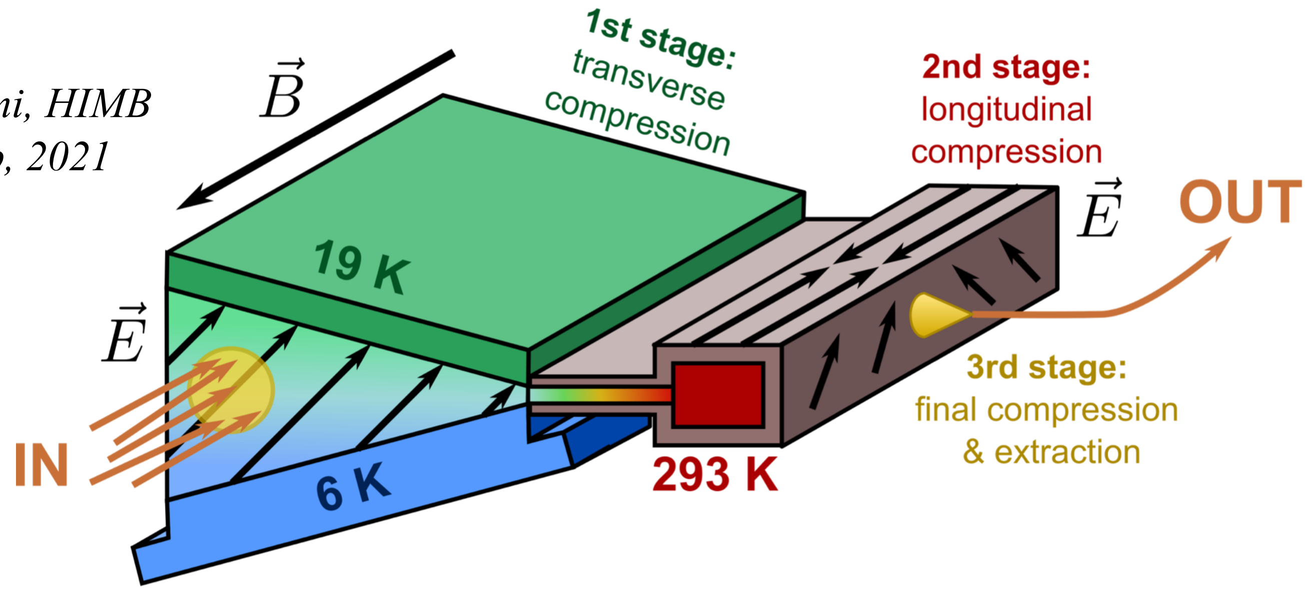


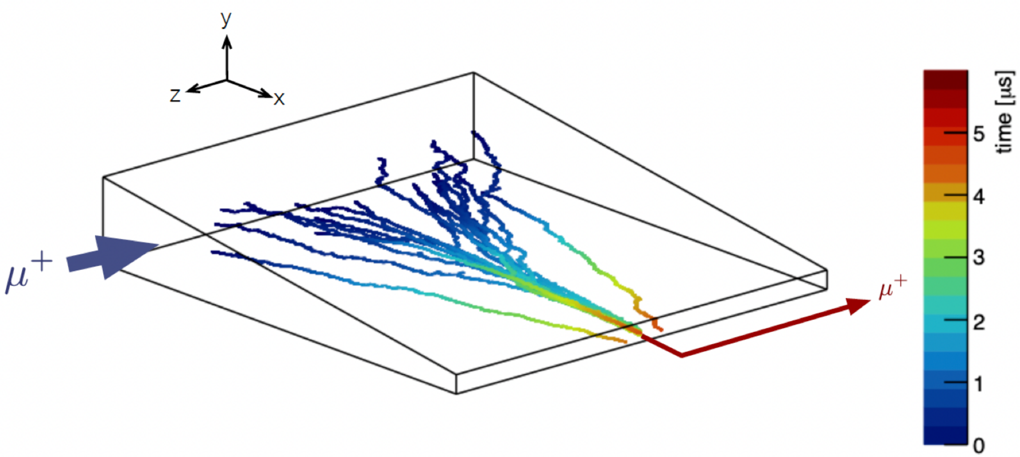
Phys. Rev. Lett. 97 (2006) 194801



Muon compression at PSI

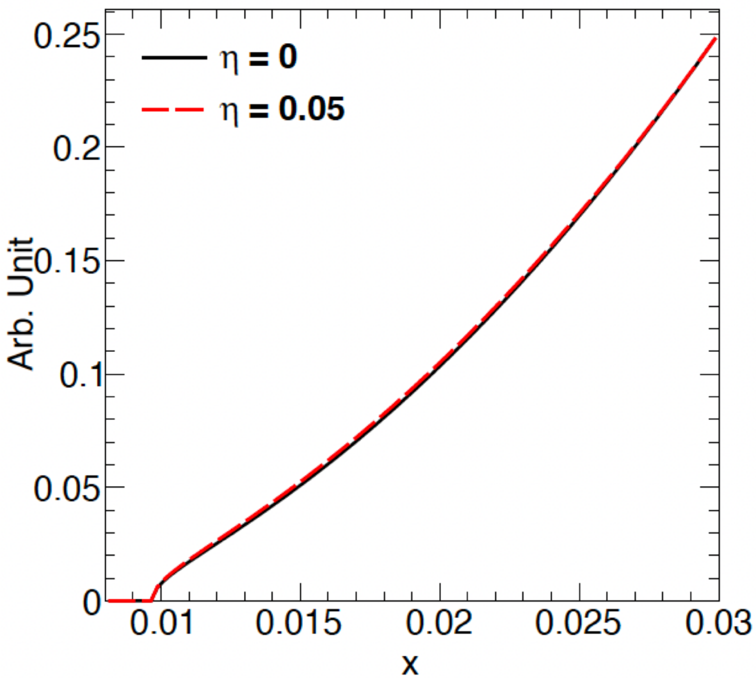
A. Antognini, HIMB workshop, 2021

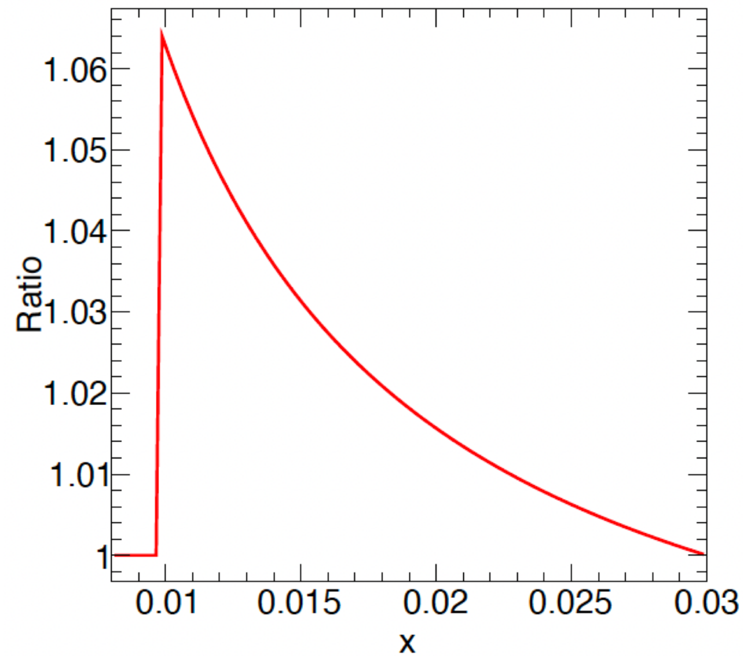




- Input muon energy 4 ±0.8 MeV
- Input diameter 7cm
- Input divergence 200 mrad
- Output energy 10-100 eV
- Output diameter 1mm

The η-term





- The η-term is most pronounced in the region where $x \rightarrow x_0$ (see demonstration of x distributions for SM and η=0.05)
- The current best measurement for η is -0.0027 ± 0.0050 (TWIST). It can be further improved with (1) more muon decay statistics, and (2) more precise resolution of electron energy measurement, as resolutions a a few percent with smear out the distribution

Cyclotron radiation of decay positron

- After beam focusing, slow positive muons are injected into a chamber of uniform magnetic field B, where the decay positrons will make helical motion and emit cyclotron radiation, with frequency

$$f = \frac{eB}{2\pi m_e \gamma} = \frac{f_0}{\gamma}$$

where f_0=55.984982 GHz for B=2T. The cyclotron radius is

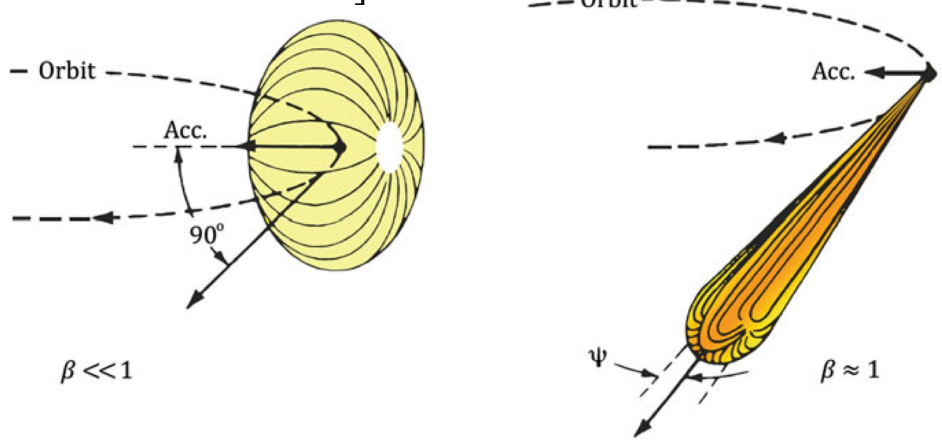
$$R = \frac{m_e c}{eB}\sqrt{\gamma^2 - 1}\sin\theta$$

where θ is the pitch angle (the angle between the positron momentum and the direction of the magnetic field)

- In a magnetic field of 2T, for a positron with $\theta=\pi/2$ and E_e=2 MeV, R=3.23 mm. Therefore, the inner diameter of the superconducting chamber can be about 1.29 cm if we are only interested in low energy positrons

Cyclotron and synchrotron radiation

[A. Balerna and S. Mobilio]



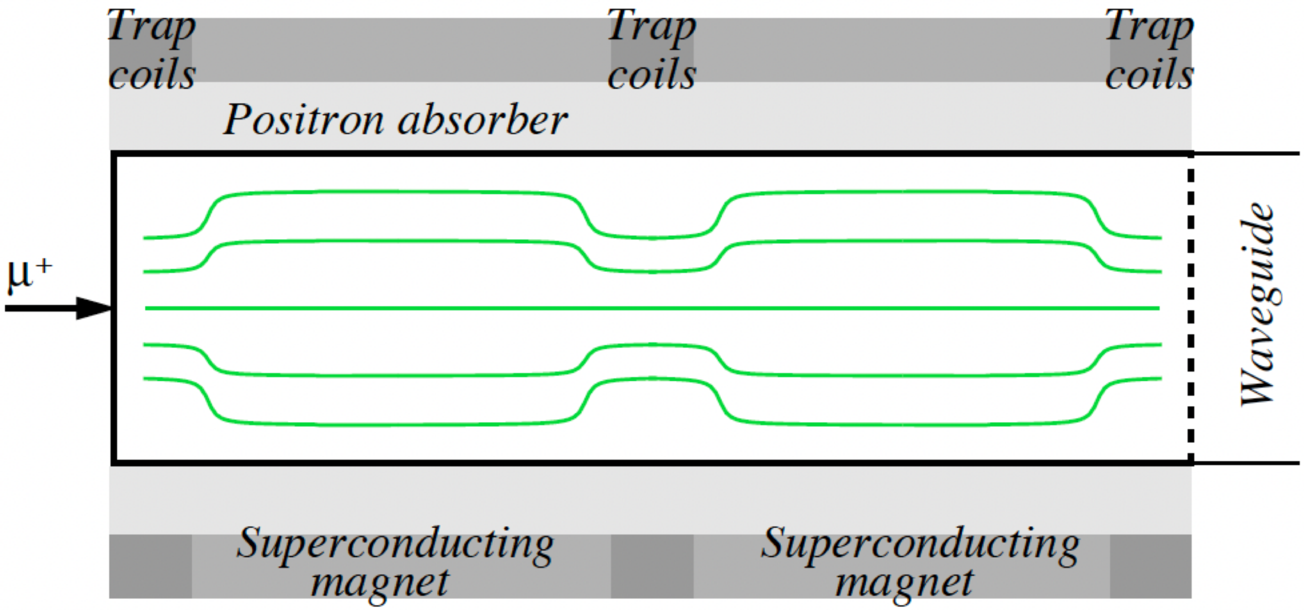
- The total cyclotron power radiated by each positron follows the Lienard extension of the Larmor formula (maximum intensity in the direction perpendicular to the acceleration)

$$P = \frac{e^4 B^2}{6\pi\epsilon_0 m_e^2 c}(\gamma^2 - 1)\sin^2\theta$$

- With $\beta \rightarrow 1$, the power is shifted to the synchrotron radiation with higher frequencies, and the power left for the n'th harmonic is

$$P_n = \frac{3^{1/6}\Gamma\left(\frac{2}{3}\right)}{4\pi^2} \cdot \frac{e^4 B^2}{\epsilon_0 m_e^2 c\gamma\sqrt{\gamma^2 - 1}\sin\theta} n^{1/3}$$

The apparatus

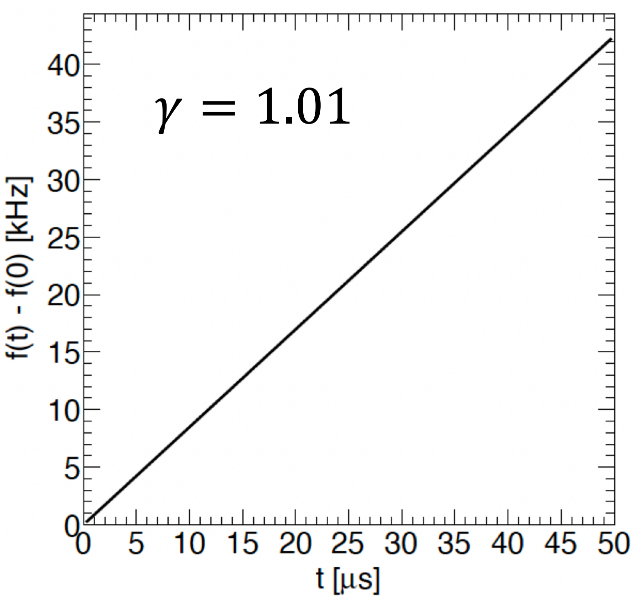


- Positron absorber: any positron with $\sqrt{\gamma^2 - 1}\sin\theta > 3.78$ will hit the wall and be absorbed
- Waveguide: the chamber inner wall is also a waveguide to transport the radiation in the TE_{11} mode
- Trap coils: periodically placed to make a parabolic field strength with $B = B_0 + \beta(z - z_1)^2 + \beta(z - z_2)^2 + \cdots$

Cyclotron radiation evolution

- The cyclotron frequency of a positron evolving with time can be approximated by

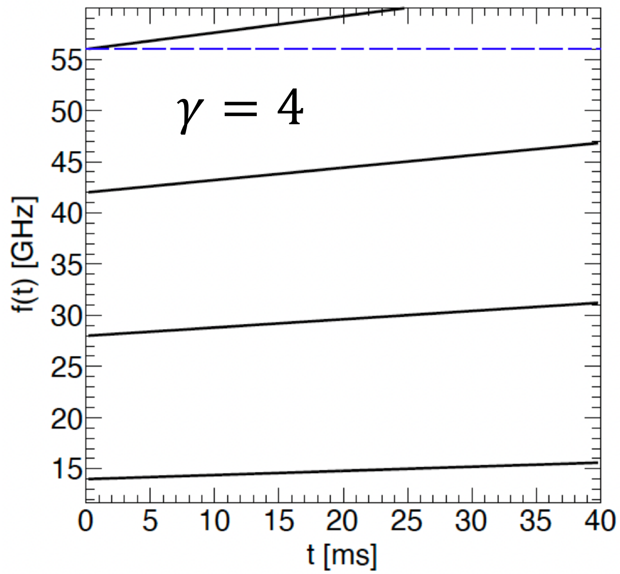
$$f(t) = f(0)\left[1 + \frac{\alpha(\gamma^2 - 1)}{\gamma}t\right], \text{ with } \alpha = \frac{e^4 B^2 \sin^2\theta}{6\pi\epsilon_0 m_e^3 c^3}$$



- For γ=1.01, the first harmonic (fundamental) frequency contains almost all the power
- For $\theta=\pi/2$ and γ=1.01, the radiated power is 1.27×10^{-15} W, which is similar to Project 8 and significant above noise
- The slope of the frequency yields the information about θ

Cyclotron radiation evolution

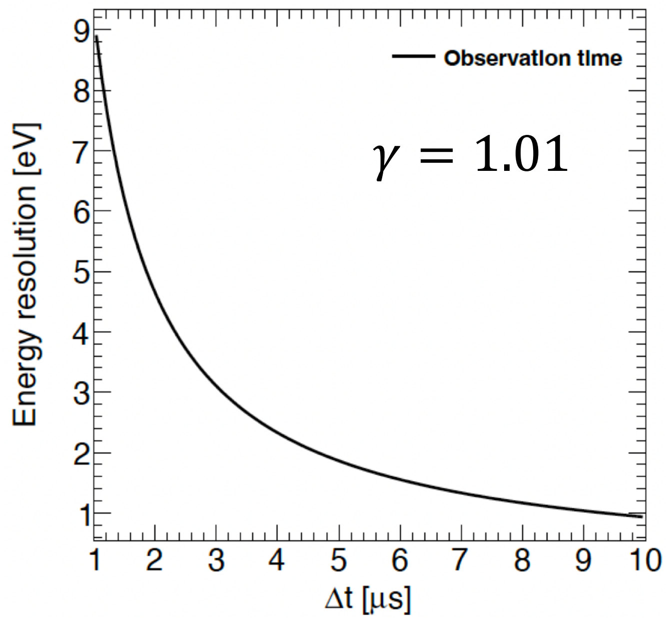
- For positrons with γ=4, most power is shifted to high frequency synchrotron radiations. However, significant power is still left for the first few harmonics of the radiation



- We should have a filter at the receiver to cut off frequencies higher than f_0, and we are left with 3 harmonics for γ=4. For $\theta=\pi/2$ and γ=4, the radiated power in the first harmonic is about 3.16×10^{-15} W, still large enough to be detected

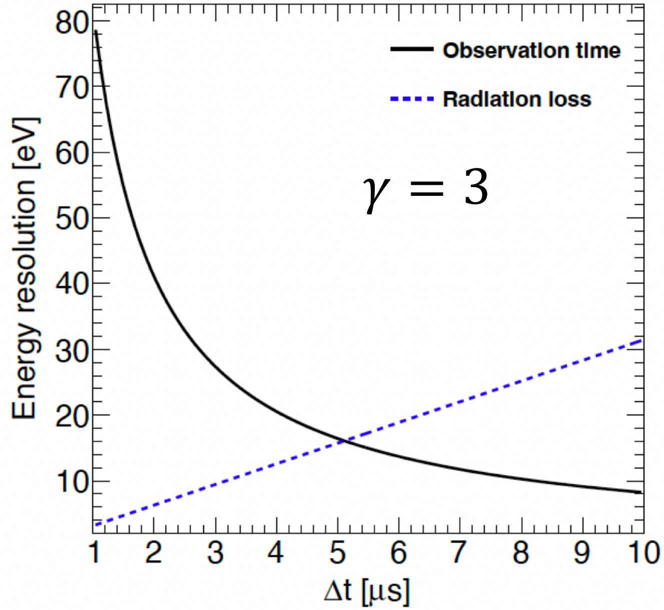
Frequency and resolution

- Cyclotron frequency lower than 14.3 GHz (for E_e>2 MeV) will be rejected by the receiver. The TE_{11} mode of the cylindrical waveguide also imposes a low cut-off at $1.8412c/(\pi d)$=13.6 GHz



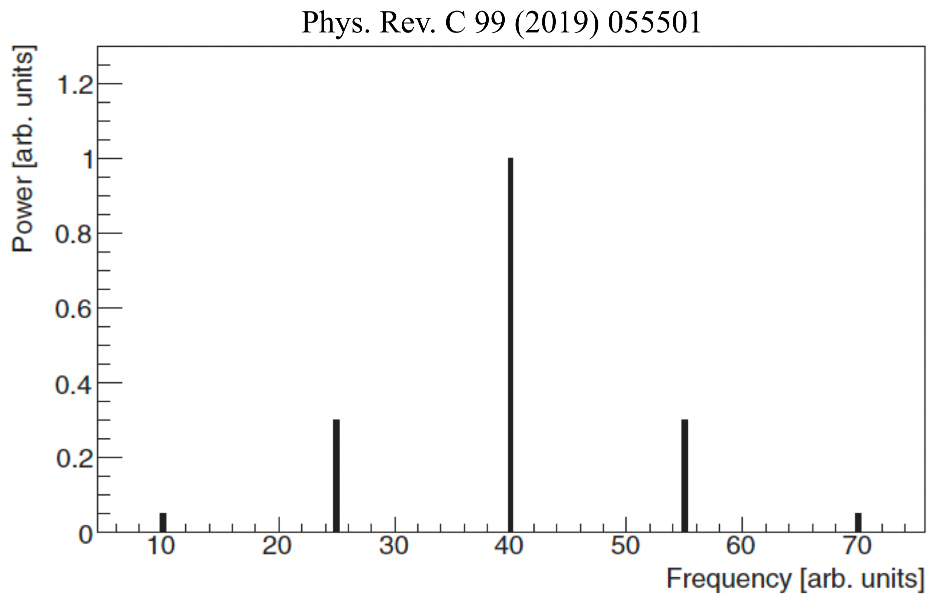
- The positron energy resolution depends on the observation time interval Δt which defined an "event". $\Delta E_e/E_e = \Delta f/f$, and $\Delta f \sim 1/\Delta t$. For $\gamma \sim 1$ and $\Delta t = 5\mu s$, the resolution can reach below 2 eV

Frequency and resolution



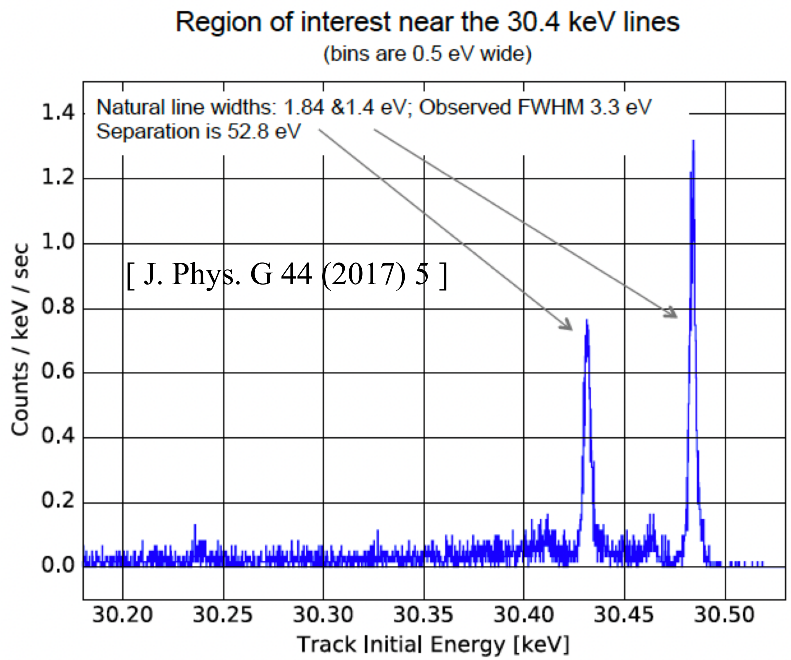
- Δt can not be too small since the thermal noise power is proportional to $kT\Delta f$
- For positrons with higher energy, Δt can neither be very large, because frequency will change dramatically within Δt due to radiation loss
- For γ=4 and Δt=5μs, the resolution can reach ~42 eV, which is still good enough for a positron with E_e~2 MeV

Doppler effect



- Due to the Doppler effect, the frequency spectrum follows a comb-like spectrum, and the power is shifted from the central line to sidebands
- The relative magnitude of the sidebands is characterized by the modulation index $\Delta\omega/\omega_a$, where $\Delta\omega$ is the frequency change due to the Doppler shift, and ω_a is the axial frequency
- To mitigate the Doppler effect, we can increase the number of trap coils so that $\Delta\omega/\omega_a$ remains small (e.g. <0.5) and power shunting is reduced

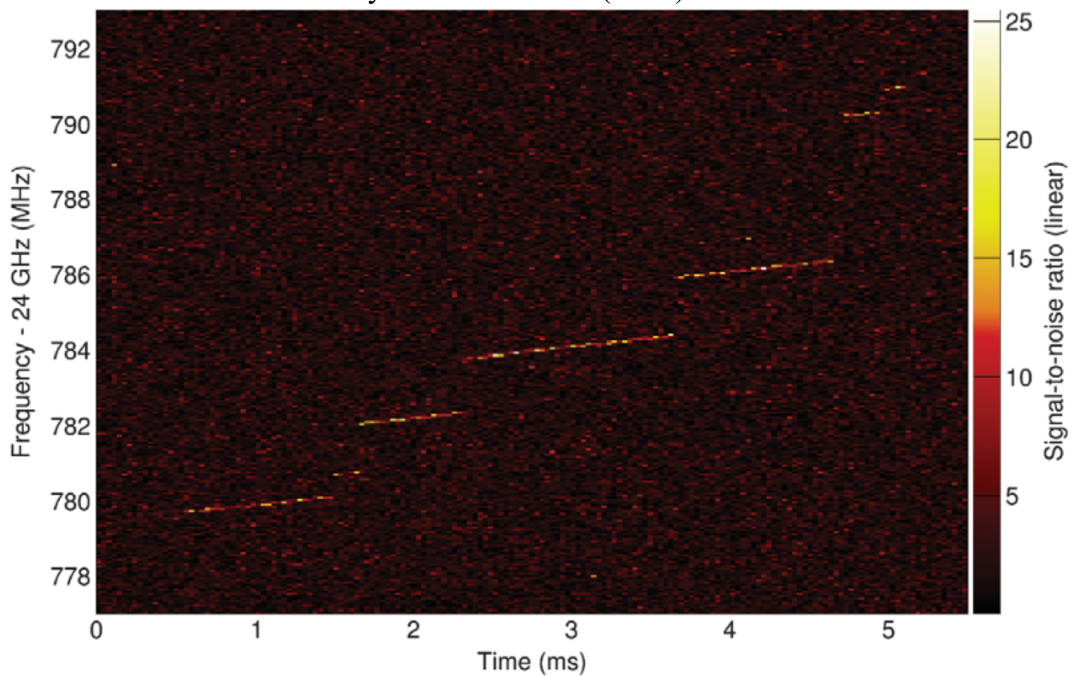
Signal calibration



- Calibration sources such as ^{83m}Kr can be used to generate electrons with known energy, and the cyclotron radiation frequency can be calibrated with it
- Artificial waves of different power can be injected into the chamber to calibration the efficiency at the receiver
- Positron pitch angle can be extracted from the energy loss slope

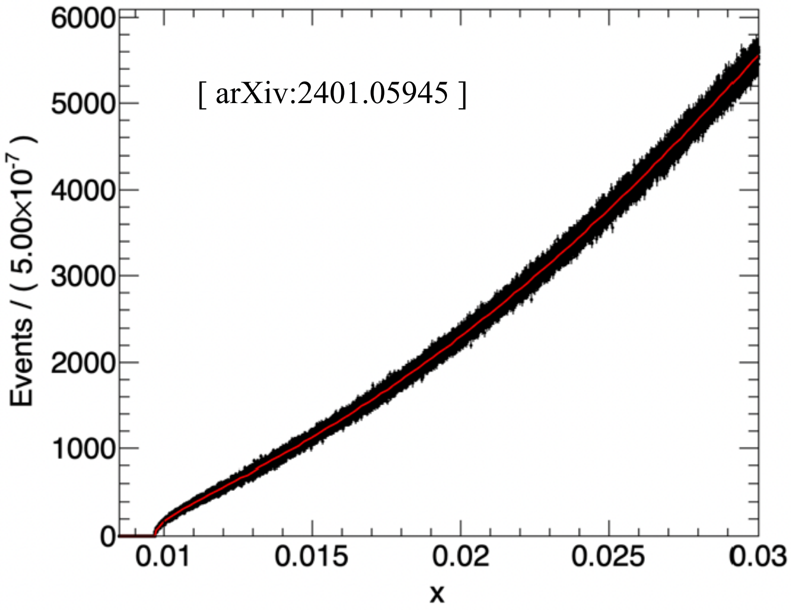
Test result from Project 8

Phys. Rev. Lett. 114 (2015) 162501



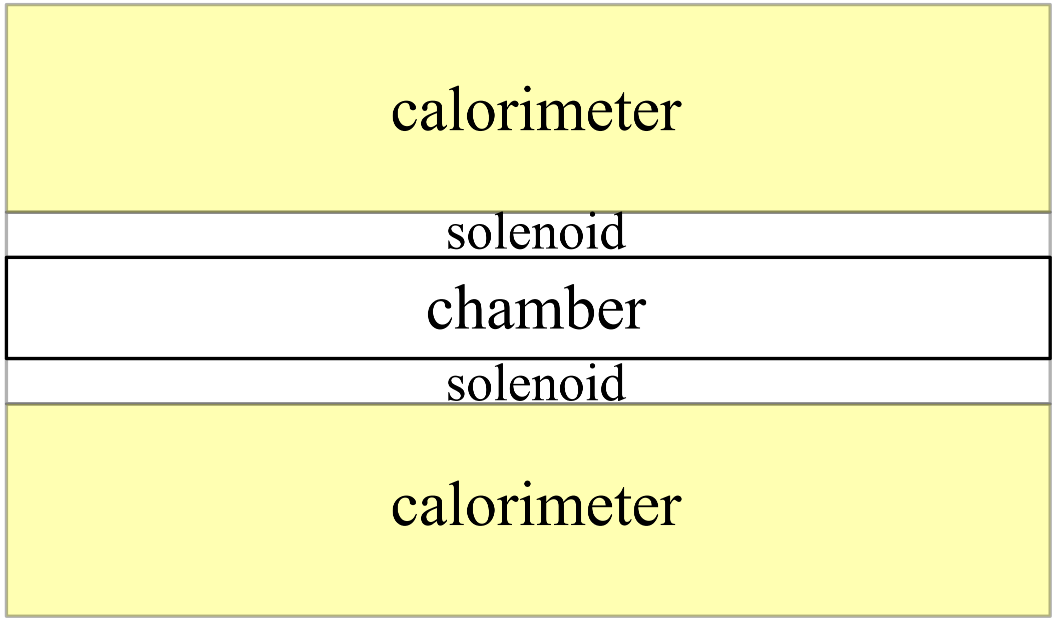
- Projected measured the cyclotron radiation frequency of 30 keV electrons from ^{83m}Kr decay as a function of time
- The frequency increases as the electron loses energy through radiation
- The frequency jumps result from the energy loss and pitch angle changes caused by electron collisions with the residual gas in the chamber

Pseudo-data fit



- With 10^8 decay positrons in the tail (x<0.03), the fit uncertainty of η is 4.3×10^{-4}, which is about an order of magnitude improvement over past experiment
- If the muon injection rate is 10^5/s, one expects about about 4.5/s positrons end up in the region of x<0.03, which does not cause a serious event pileup

A more complete design



- In a more sophisticated design, a calorimeter can be placed surrounding the solenoid, so positrons with high energy can be also measured
- High resolution calorimeter is preferred for an accurate measurement of the decay positron in high-x region in order to extract the Michel parameters. Some space position measurement + good timing can also probe eγ events

Summary

- Muons have a relatively long lifetime, can be easily produced at fixed targets and free of QCD effects. They can be used for precision test of the weak interaction, and potentially reveal the nature of neutrinos
- A new method is proposed to precisely measure the low energy decay positron spectrum using cyclotron radiations. The energy resolution is below ~50 eV for a 2 MeV positron
- Combined with slow muon sources (<100 eV muons), precision muon decay parameters can be measured and improved over past experiments
- If the magnetic field is 2 T, the detector chamber is a few cm’s in diameter, compact enough to fit into a calorimeter system, which are used to precisely measure the energy of high momentum positrons to 1-2% level