Workshop on Muon Physics at the Intensity and Precision Frontiers (MIP 2024)

Finding axions at muon experiments

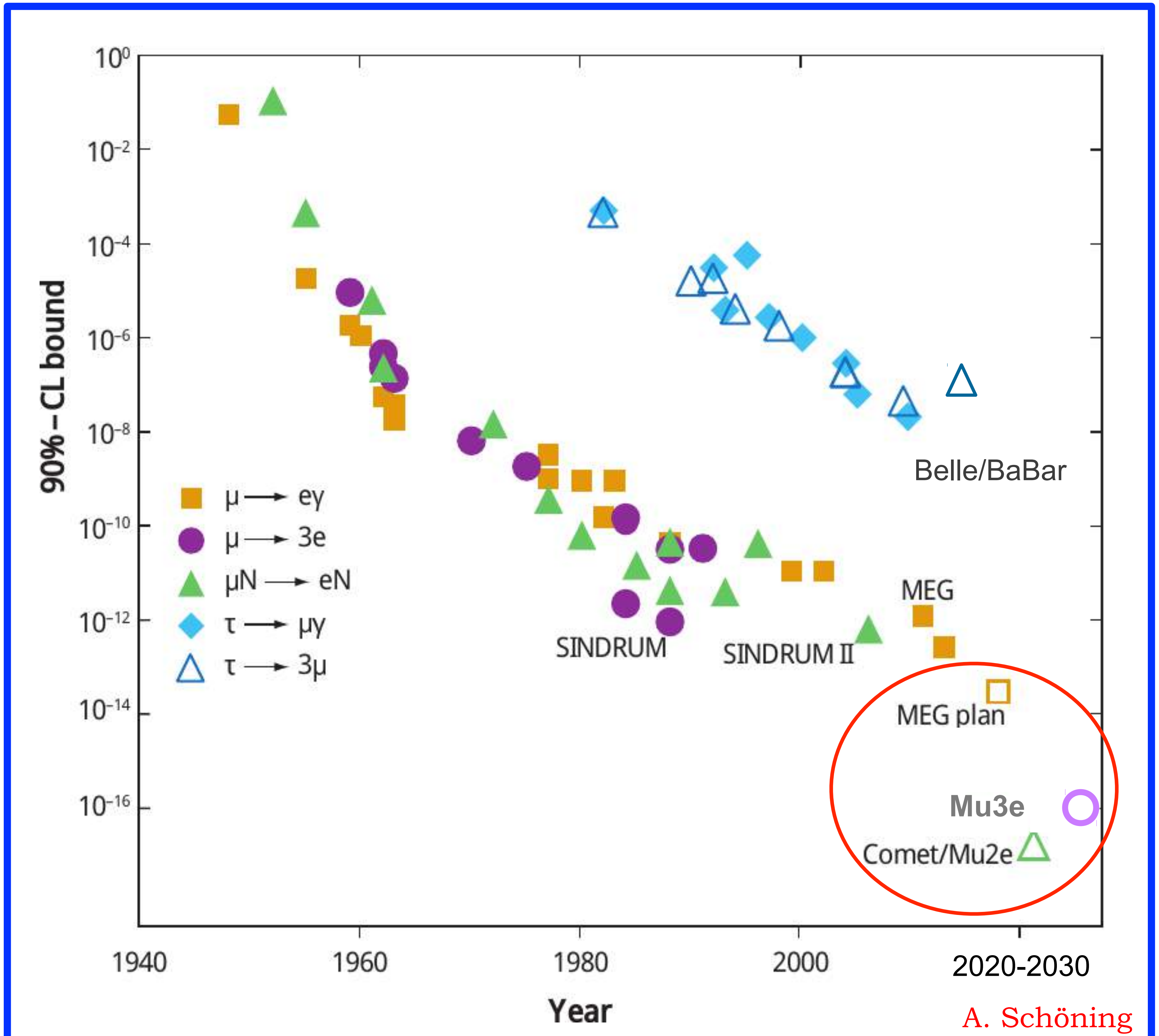
Lorenzo Calibbi

南開大學
Nankai University

PKU, Beijing, April 20th 2024

Motivation: we have high-intensity muon experiments!

Current landscape of searches for charged lepton flavour violation (CLFV):



Assume there is a *light*, *invisible*, new particle “*a*”
with *flavour-violating couplings* to leptons

Light:

$$m_a < m_\mu\,,\ m_\tau$$

Invisible:

- Neutral
- Feebly coupled (long-lived)

CLFV modes would then be $\mu \to e\,a,\ \ \tau \to \mu\,a,\ \ \mu \to e\,\gamma\,a,$ etc.

Interesting interplay with cosmology/astrophysics:

- Can a be Dark Matter? (yes, if long-lived enough)
- Bounds from star cooling/supernovae (if light and feeble enough)

Why should a be light and feebly-coupled?

Natural, if it is the (pseudo) Nambu-Goldstone boson (PNGB) of a spontaneously broken global U(1), *aka* an axion-like particle (ALP)

Examples of lepton-flavour-violating (LFV) ALPs:

	Broken global symmetry:	PNGB:	
			Wilczek '82
			Pilaftsis '93
• Neutrino masses →	Lepton Number	Majoron	Feng et al. '97
• Strong CP problem →	Peccei-Quinn	Axion	LC Goertz Redigolo Ziegler Zupan '16
• Flavour problem →	Flavour symmetry (Froggatt-Nielsen)	Familon	Di Luzio et al. '17, '19 LC Redigolo Ziegler Zupan '20
	…		…

Equivalent possibility: light Z' of a local U(1), e.g. L_i-L_j (with $g \ll 1$)

Heeck '16

General interactions to leptons (dimension 5 operators):

Shift symmetry (PNGB!) → mass arises m_a from (small) explicit U(1) breaking

$$\mathcal{L}_{a\ell\ell} = \frac{\partial^\mu a}{2 f_a} \left(C^V_{ij}\, \bar{\ell}_i \gamma_\mu \ell_j + C^A_{ij}\, \bar{\ell}_i \gamma_\mu \gamma_5 \ell_j \right)$$

U(1)-breaking scale → coupling suppression

Where does *lepton flavour violation* come from?

- If lepton U(1) charges are flavour non-universal ⇨ naturally flavour-violating couplings
- Alternatively, loop-induced flavour-violating couplings

Explicit examples at the end…

General interactions to leptons (dimension 5 operators):

Shift symmetry (PNGB!) → mass arises m_a from (small) explicit U(1) breaking

$$\mathcal{L}_{a\ell\ell} = \frac{\partial^\mu a}{2 f_a} \left(C^V_{ij}\ \bar{\ell}_i \gamma_\mu \ell_j + C^A_{ij}\ \bar{\ell}_i \gamma_\mu \gamma_5 \ell_j \right)$$

U(1)-breaking scale → coupling suppression

Where does *lepton flavour violation* come from?

This generic Lagrangian induces 2-body LFV decays such as:

$$\Gamma(\ell_i \to \ell_j\, a) = \frac{1}{16\pi} \frac{m^3_{\ell_i}}{F^2_{ij}} \left(1 - \frac{m_a^2}{m^2_{\ell_i}} \right)^2 \qquad F_{ij} \equiv \frac{2 f_a}{\sqrt{|C^V_{ij}|^2 + |C^A_{ij}|^2}}$$

Feng et al. '97

$\mu \to e\, a$: signal and background

Signal: monochromatic positron with $p_e = \sqrt{\left(\frac{m_\mu^2 - m_a^2 + m_e^2}{2m_\mu}\right)^2 - m_e^2}$

Differential decay rate: $$\frac{\mathrm{d}\Gamma(\ell_i \to \ell_j\, a)}{\mathrm{d}\cos\theta} = \frac{m_{\ell_i}^3}{32\pi F_{\ell_i\ell_j}^2}\left(1 - \frac{m_a^2}{m_{\ell_i}^2}\right)^2 \left[1 + 2P_{\ell_i}\cos\theta \frac{C^V_{\ell_i\ell_j} C^A_{\ell_i\ell_j}}{(C^V_{\ell_i\ell_j})^2 + (C^A_{\ell_i\ell_j})^2}\right]$$

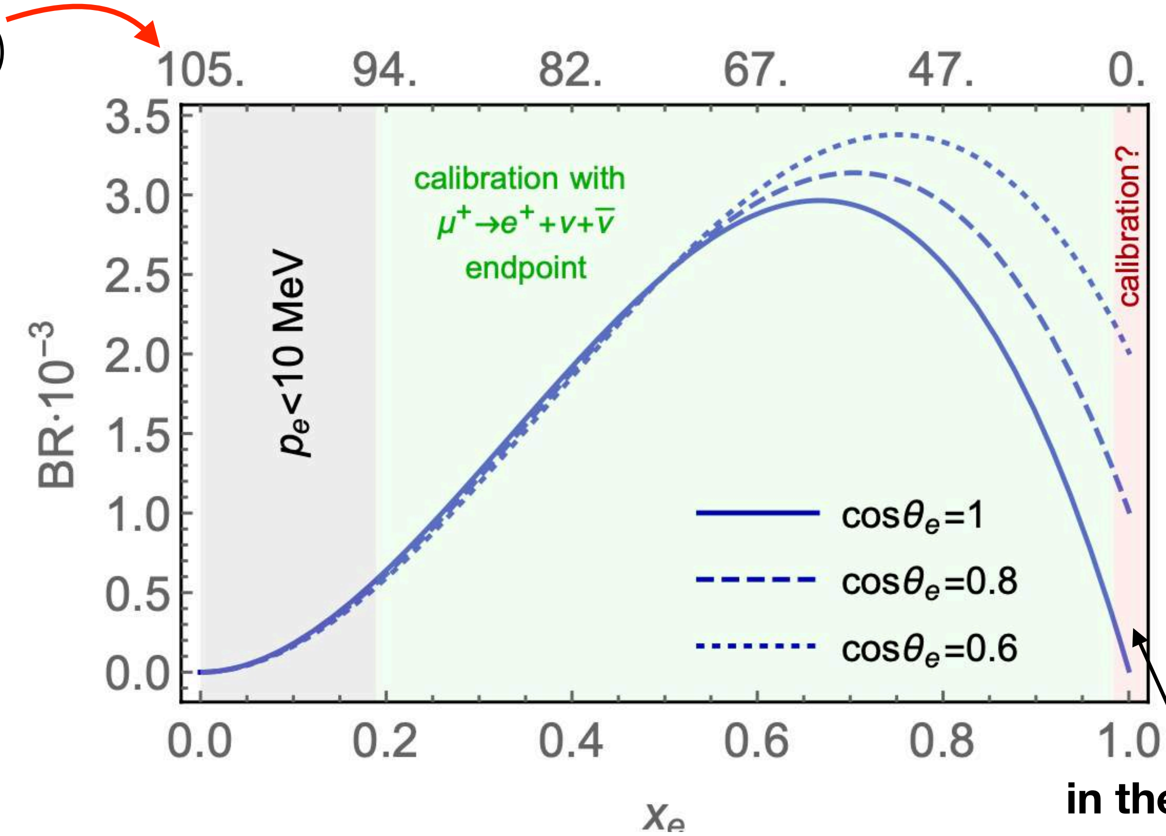
signal depends on the chirality of the couplings

Michel spectrum: $$\frac{\mathrm{d}^2\Gamma(\mu^+ \to e^+\, \nu_e\, \bar{\nu}_\mu)}{\mathrm{d}x_e\, \mathrm{d}\cos\theta} \simeq \Gamma_\mu\left((3 - 2x_e) - P_\mu(2x_e - 1)\cos\theta\right) x_e^2$$ $x_e = \frac{2p_e}{m_\mu}$

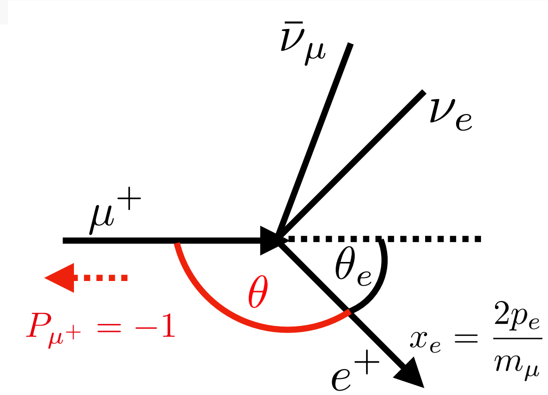
μ polarization

And "surface" muons are highly polarized (produced by pion decays at rest on the surface of the production target) → the SM background can be suppressed

m_a (MeV)

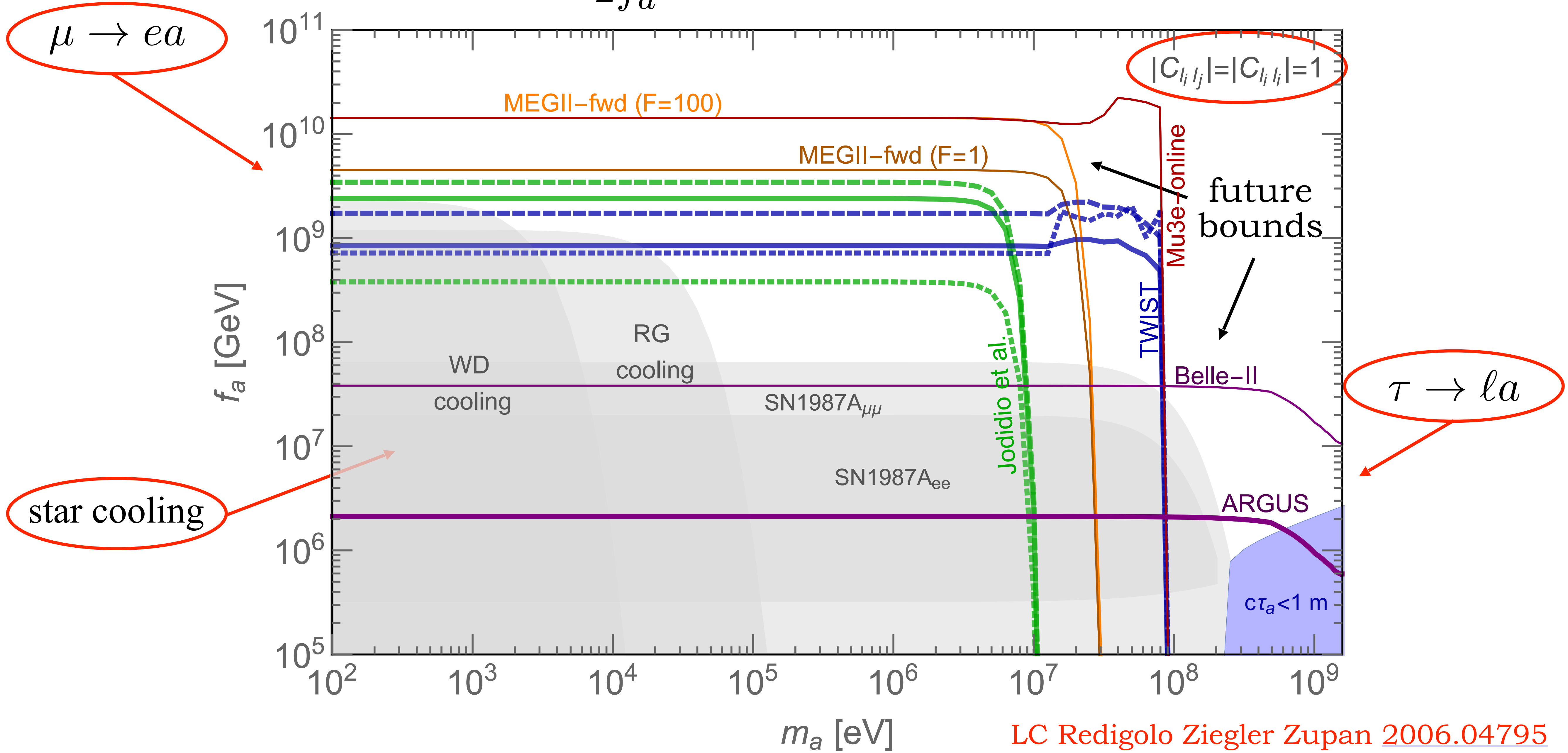


the bkd goes to zero in the "forward" direction (the direction opposite to the muon polarization)



Lepton-flavour-violating invisible ALPs

$$\mathcal{L}_{aff} = \frac{\partial_\mu a}{2 f_a} \bar{f}_i \gamma^\mu (C^V_{f_i f_j} + C^A_{f_i f_j} \gamma_5) f_j$$



LC Redigolo Ziegler Zupan 2006.04795

Decays mediated by dim-5 operators: much larger NP scales can be reached than $\mu \to e\gamma$, $\mu \to 3e$, $\mu \to e$ conv. (from dim-6 ops, NP scale reach ~10^7−10^8GeV)

Past searches: $\mu \to e\, a$

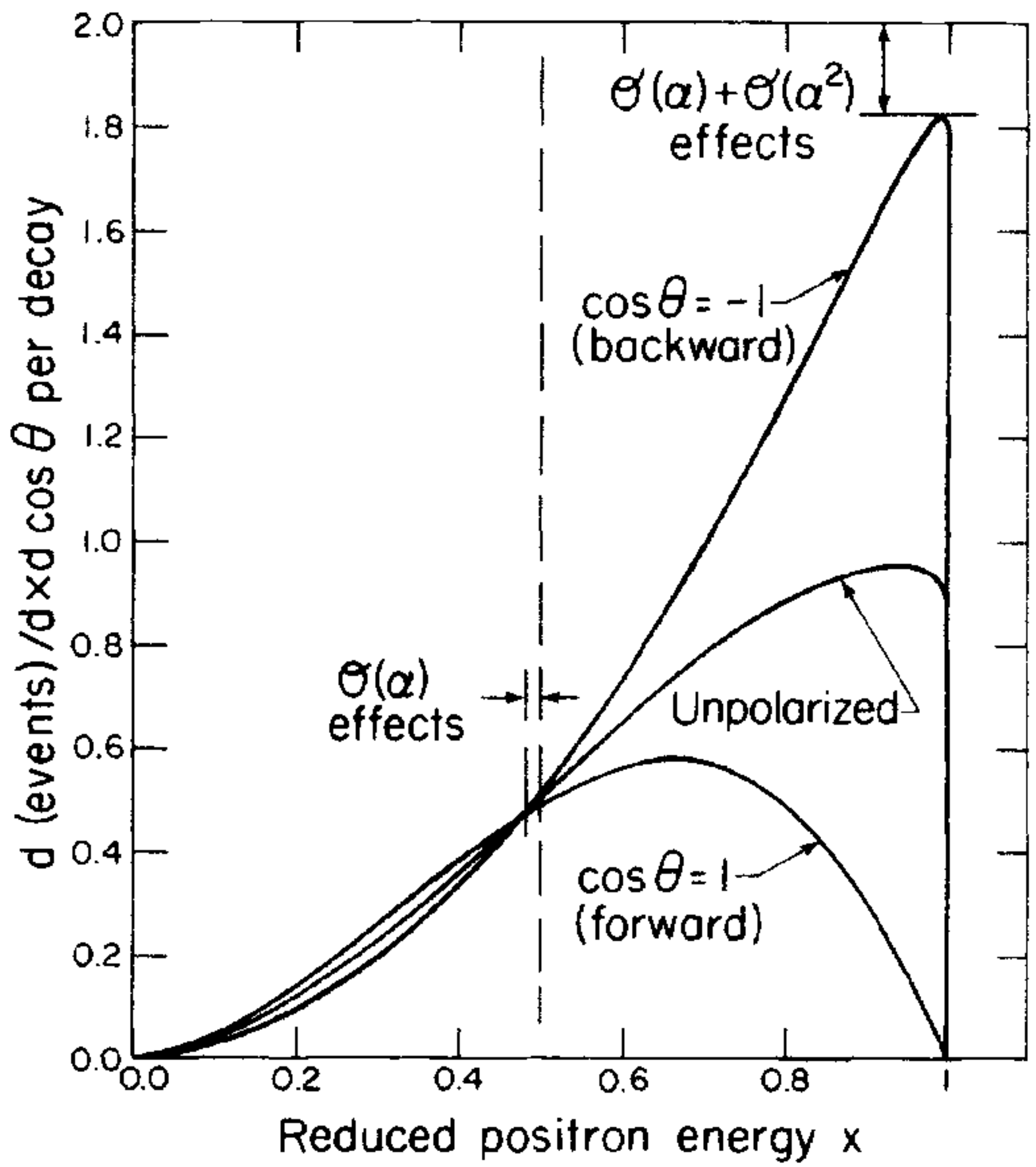
- Jodidio et al. (TRIUMF) 1986

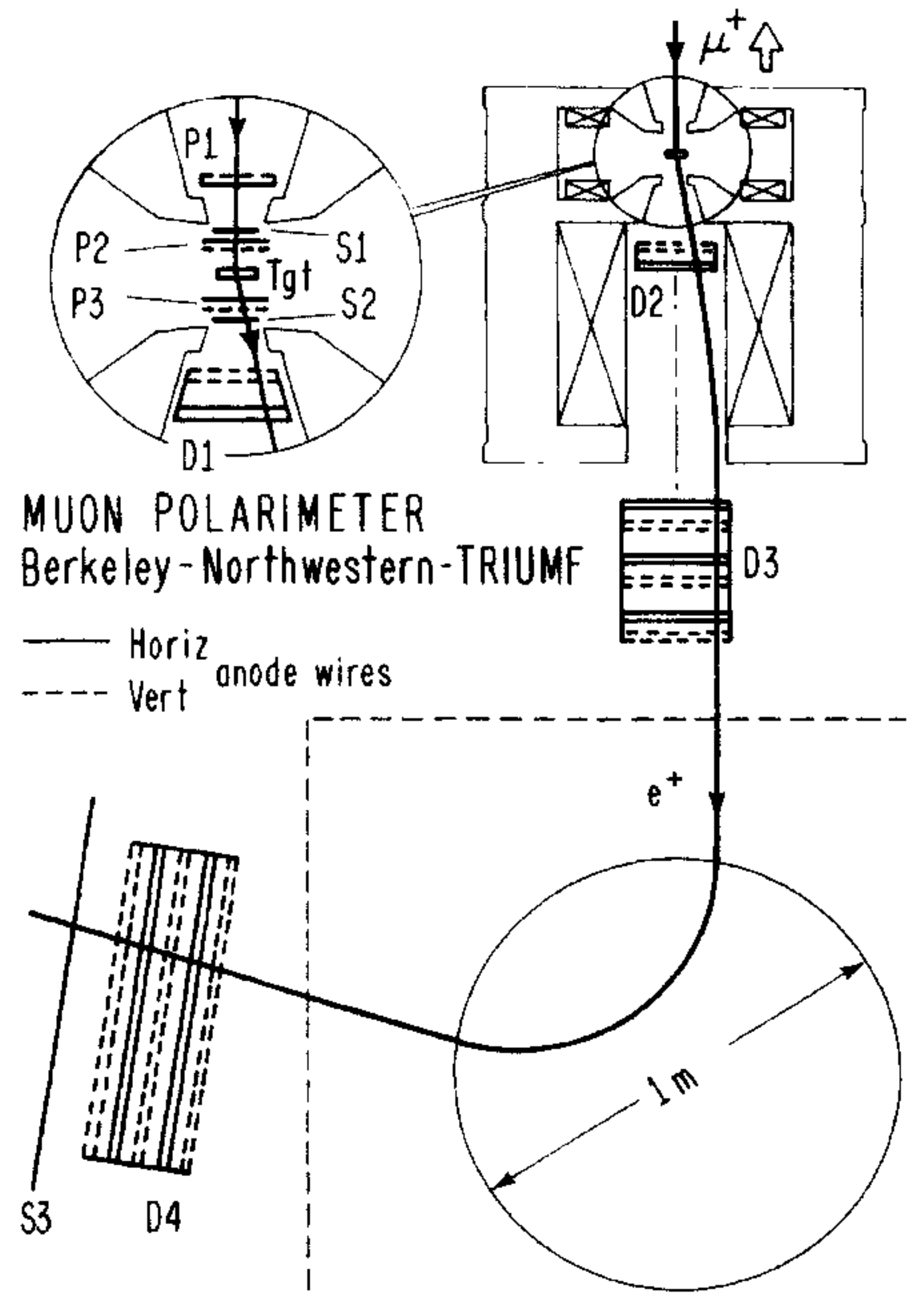
Search for RH currents with 1.8×10^7 polarized μ^+

Ordinary $\mu \to e\bar{\nu}\nu$

$$\frac{d^2\Gamma}{dx\, d\cos\theta} = \Gamma_\mu \left((3-2x) - P(2x-1)\cos\theta\right) x^2$$

$$x = 2E_e/m_\mu$$





Very good e+ momentum resolution
(~70 KeV at the e.p.)

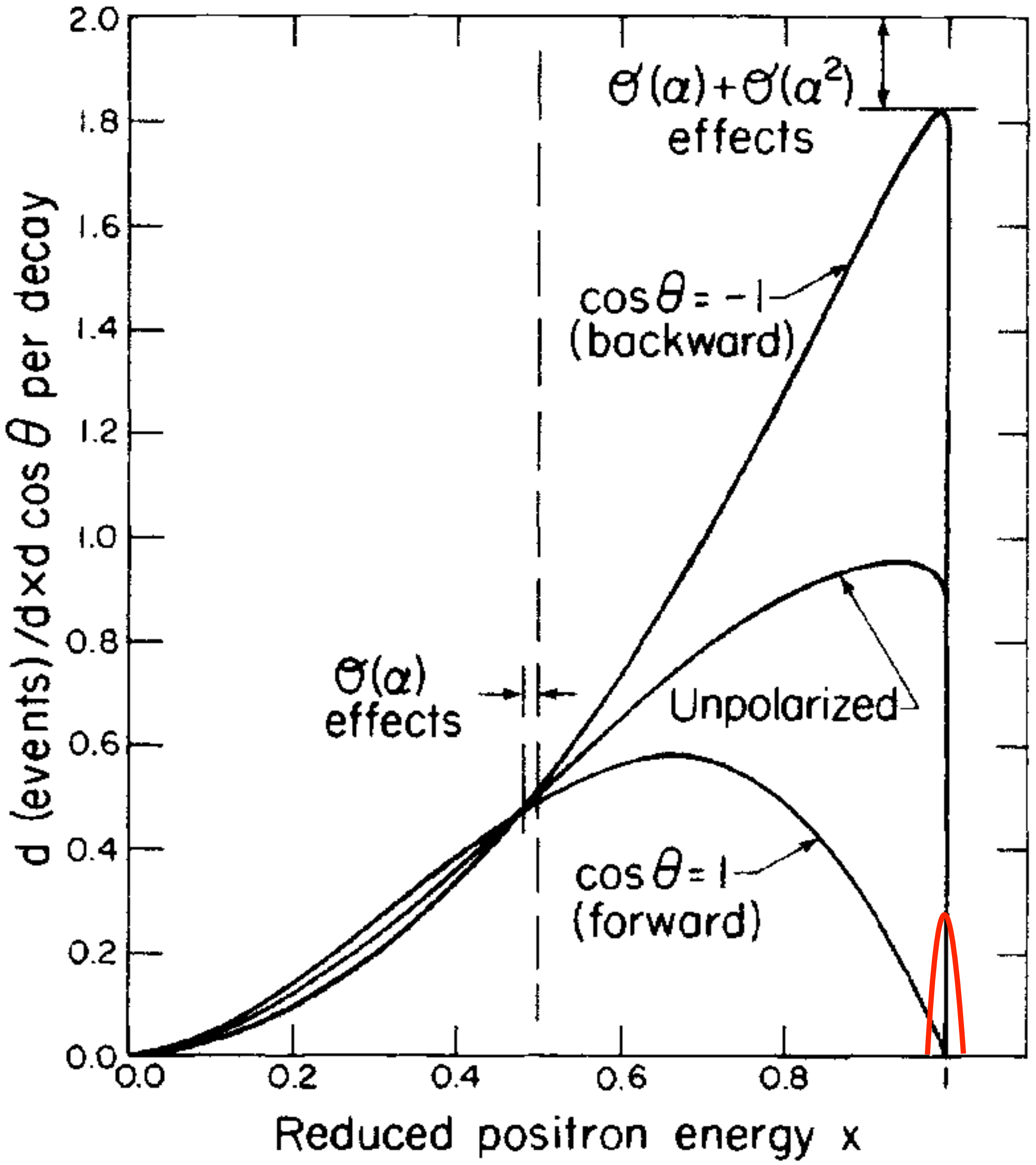
- Jodidio et al. (TRIUMF) 1986

Search for RH currents with 1.8×10^7 polarized μ^+ interpreted in terms of $\mu \to ea$ too

Ordinary $\mu \to e\bar{\nu}\nu$

$$\frac{d^2\Gamma}{dx\, d\cos\theta} = \Gamma_\mu \left((3-2x) - P(2x-1)\cos\theta\right) x^2$$

$$x = 2E_e/m_\mu$$



$\mu \to e\, a$ signal for $m_a \approx 0$: monochromatic e^+ at $m_\mu/2$

Unless it couples (V-A) like in the SM:

$$\frac{d\Gamma(\mu^+ \to e^+ a)}{d\cos\theta} = \frac{\Gamma_{\mu\to e\, a}}{2}\left[1 + 2P\cos\theta\frac{C^V_{e\mu}C^A_{e\mu}}{(C^V_{e\mu})^2 + (C^A_{e\mu})^2}\right]$$

for the *isotropic* case, they set the limit

$$\Rightarrow \quad BR(\mu^+ \to e^+ a) < 2.6\times10^{-6}$$

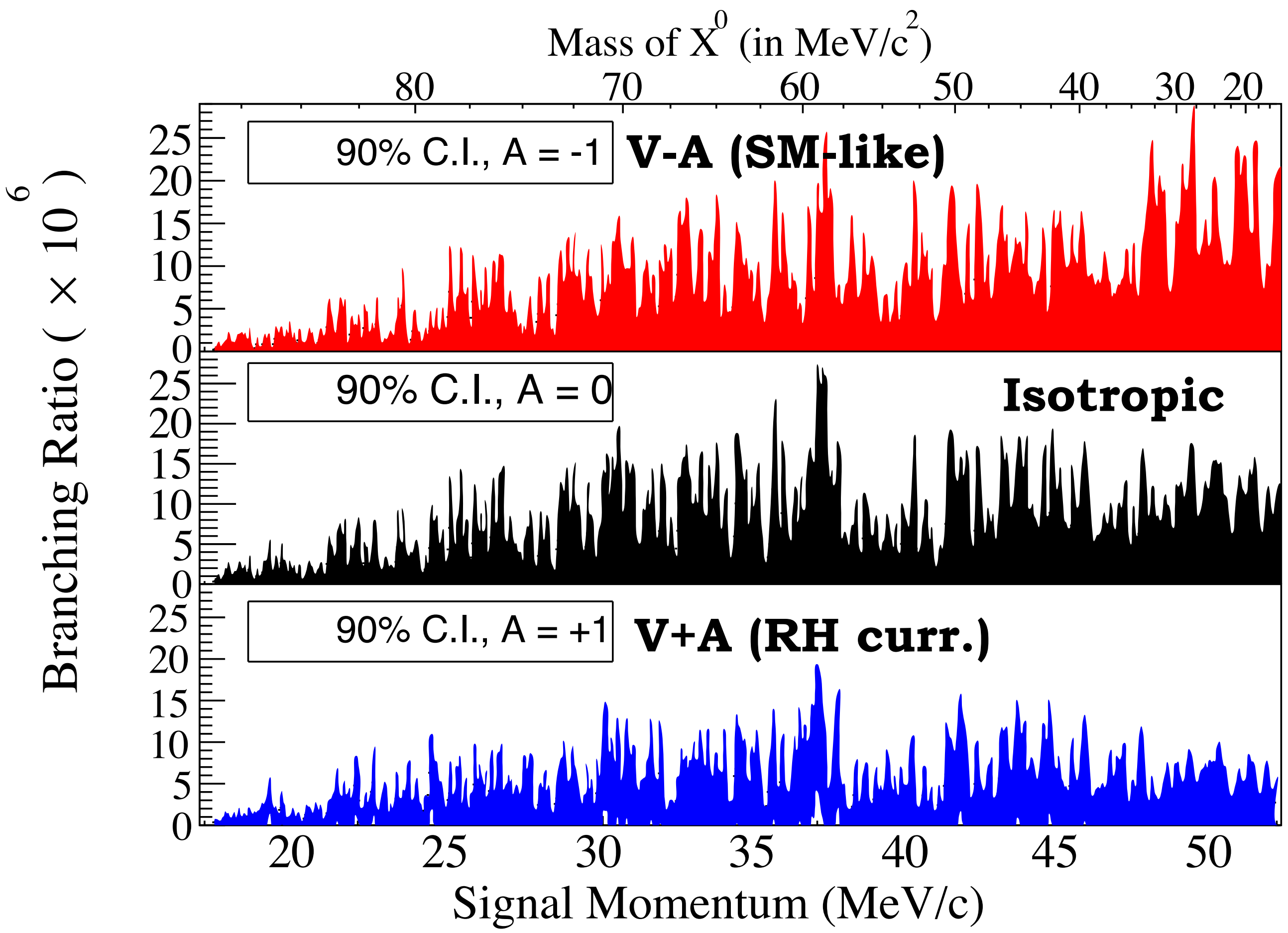
thus one gets

$$\Rightarrow \quad F_{\mu e} > 4.8\times10^9 \text{ GeV}$$

Past searches: $\mu \to e\, a$

- <u>TWIST 2014</u> Precise measurement of Michel parameters plus dedicated search for $\mu \to ea$ in the whole m_a range considering anisotropy of the signal

Limits (with $5.8\times10^8\ \mu^+$):



Decay Signal		90% C.L. (in ppm)	p-value
$A = 0$	Average	9	
	$p = 37.03$ MeV/c	26	0.66
	Endpoint	21	0.81
$A = -1$ SM-like	Average	10	
	$p = 37.28$ MeV/c	26	0.60
	Endpoint	58	0.80
$A = +1$	Average	6	
	$p = 19.13$ MeV/c	6	0.59
	Endpoint	10	0.90

For V-A coupl. and $m_a \approx 0$: $\text{BR}(\mu \to ea) < 5.8 \times 10^{-5}$

$$\Rightarrow \quad F_{\mu e} > 1.0 \times 10^9\ \text{GeV}$$

Present bounds based on old experiments and/or moderate luminosities (<10^9 total muon decays)

Modern facilities, e.g. $\pi E5$ beamline at PSI (where MEGII and Mu3e are located), can deliver >10^8 muons *per second*: next generation experiments must do better!

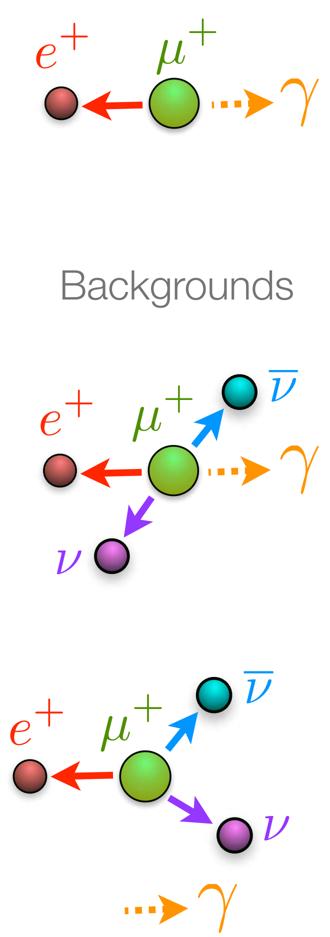
slide borrowed from A. Papa

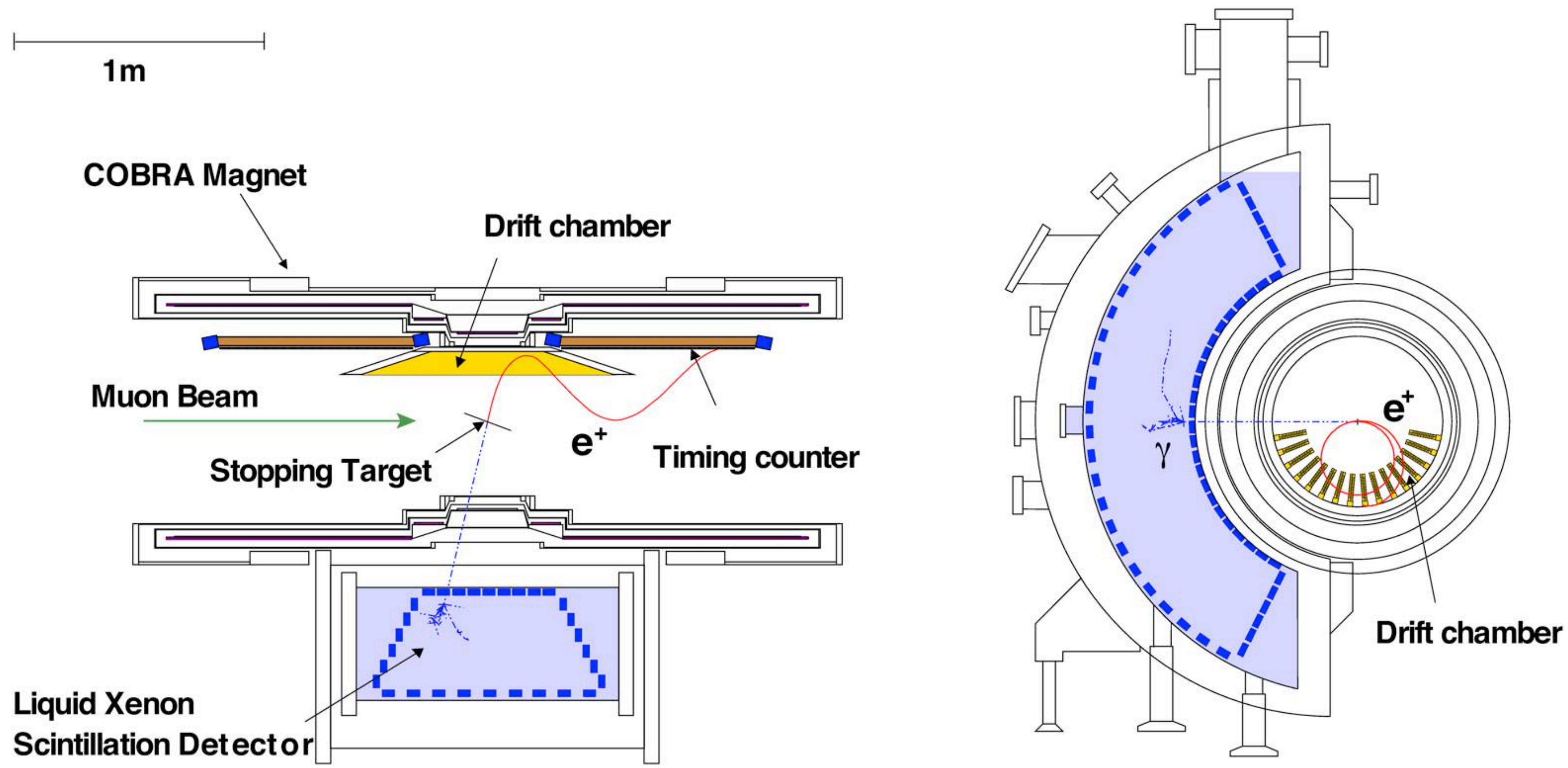
MEG: Signature and experimental setup

- The MEG experiment aims to search for $\mu^+ \to e^+ \gamma$ with a sensitivity of ~10^{-13} (previous upper limit $BR(\mu^+ \to e^+ \gamma) \leq 1.2 \times 10^{-11}$ @90 C.L. by MEGA experiment)
- Five observables (E_g, E_e, t_{eg}, ϑ_{eg}, ϕ_{eg}) to characterize $\mu \to e\gamma$ events

Signature

Backgrounds





53

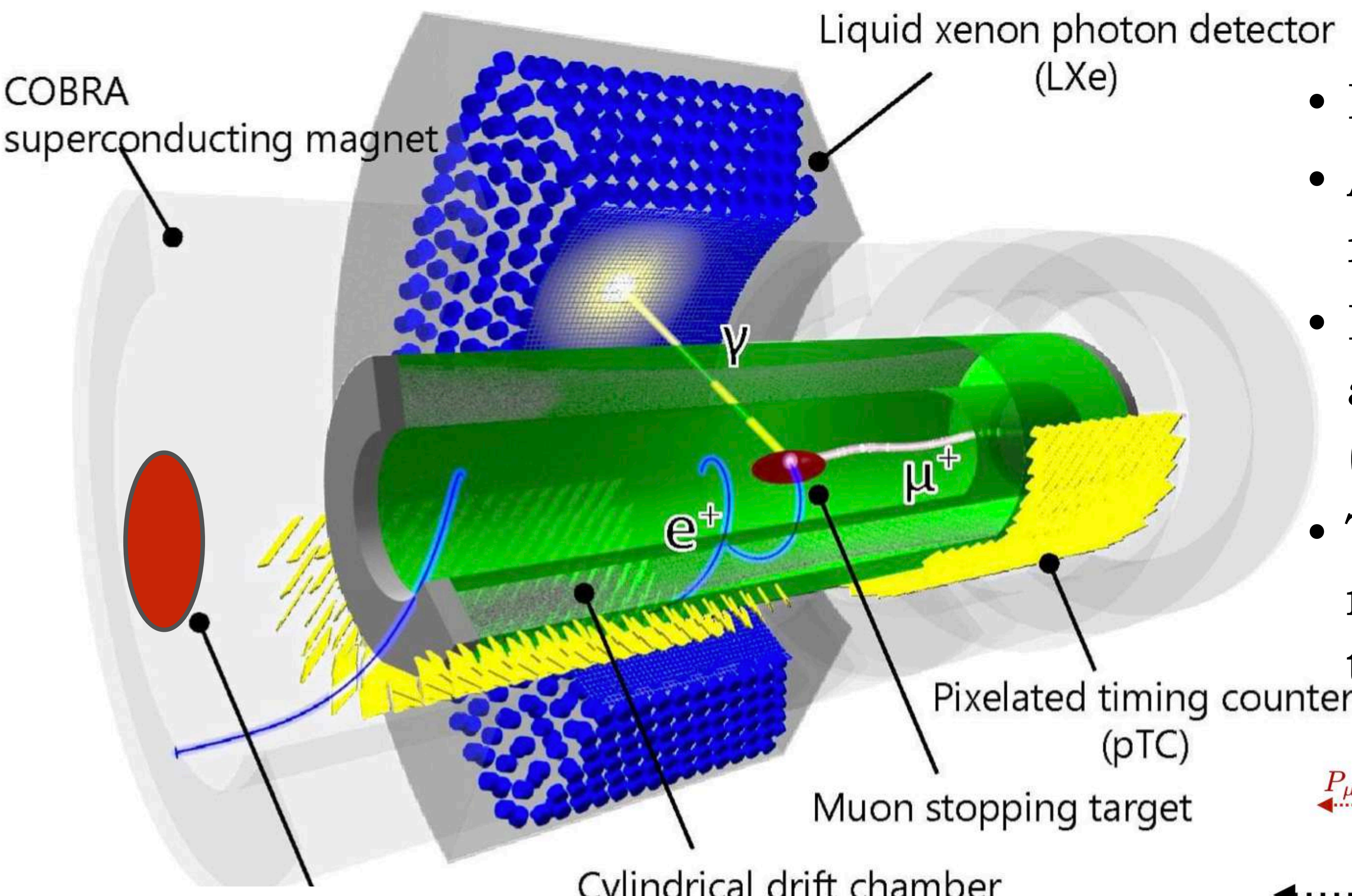
Final result (with 7.5×10^{14} μ^+ on target): $BR(\mu \to e\gamma) < 4.2 \times 10^{-13}$ (90% CL)

LC Redigolo Ziegler Zupan '20

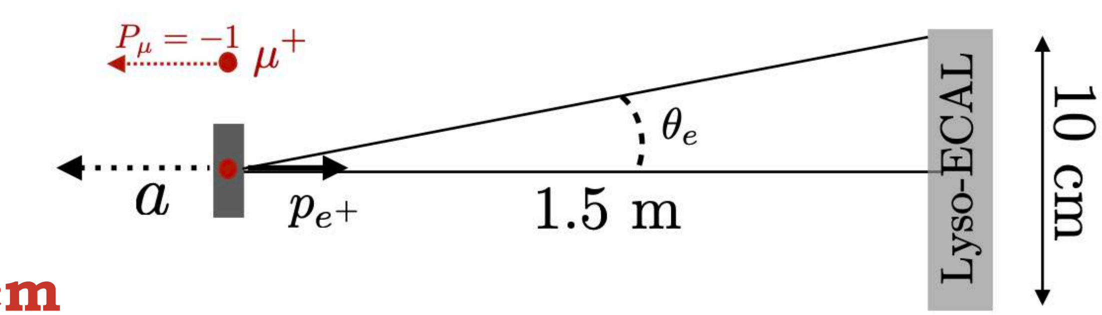
- Prospect at MEG II for $\mu \to e\, a$

What about a Jodidio-like search at MEG II for $m_a \approx 0$ with a *forward calorimeter*?
We propose a modified setup of MEG II (“MEGII-fwd”) and ~2 weeks dedicated run

idea from discussions with A. Papa and G. Signorelli, thanks!



- Muon beam already polarized
- A suitable magnetic field can reduce depolarization effects
- Reconfiguring the field we can also increase the acceptance (“magnetic focusing” up to F~100)
- Two weeks of run after MEG II main run are enough to improve the bound (even with F=0)



~1.5 m from the target, diameter ~ 10 cm

- Prospect at MEG II for $\mu \to e\, a$

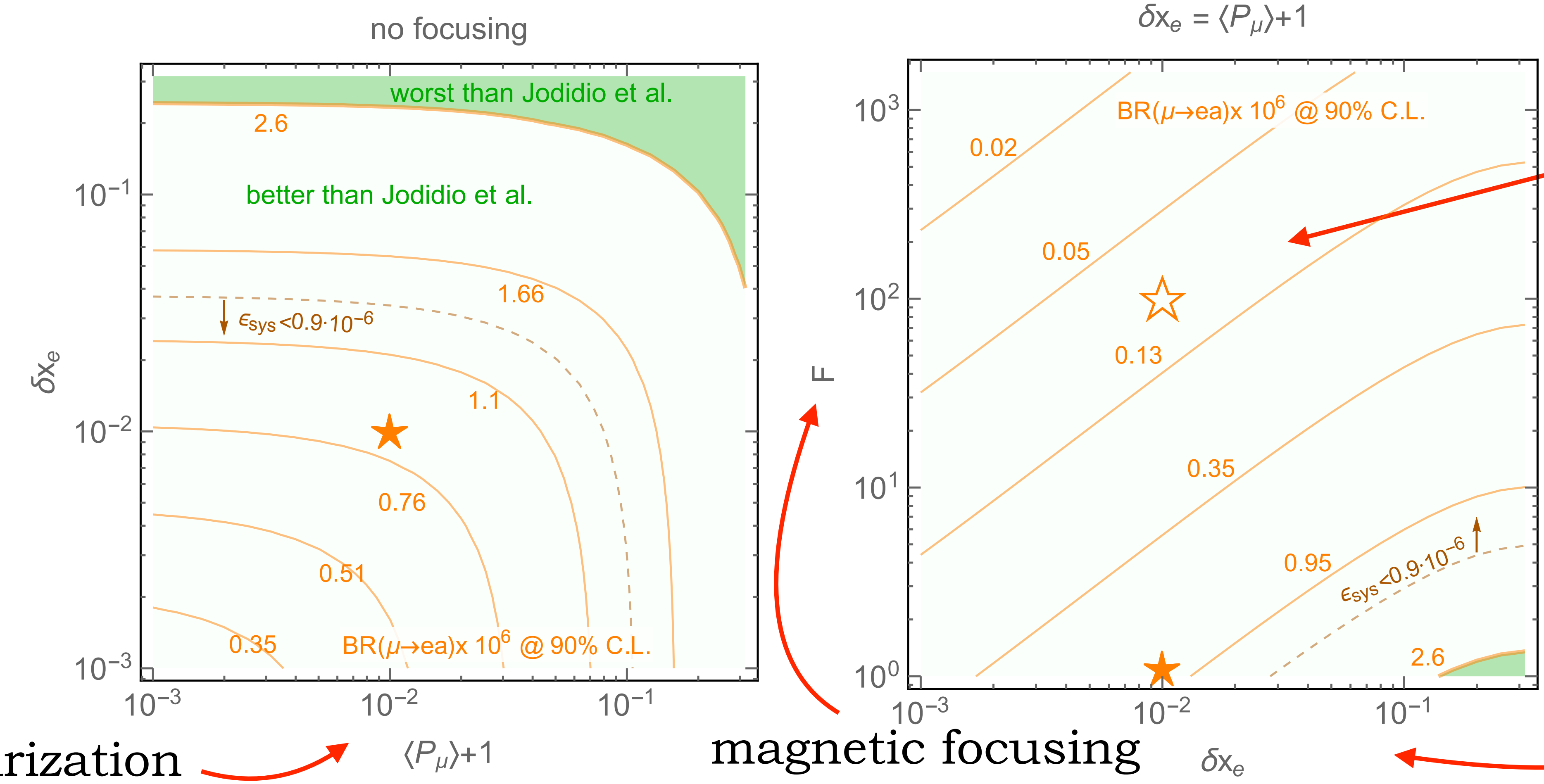
LC Redigolo Ziegler Zupan '20

What about a Jodidio-like search at MEG II for $m_a \approx 0$ with a *forward calorimeter*?
We propose a modified setup of MEG II ("MEGII-fwd") and ~2 weeks dedicated run

idea from discussions with A. Papa and G. Signorelli, thanks!

Our estimate of the sensitivity of a dedicate run (2 weeks with $10^8\ \mu^+/\mathrm{s}$):

$$F = 0 \qquad \text{MEGII-fwd}: \ 10^{14} \mu^+$$



$F = 100$
$\mathrm{BR} \sim 10^{-7}$

Positron momentum resolution

$$x_e = \frac{2p_e}{m_\mu}$$

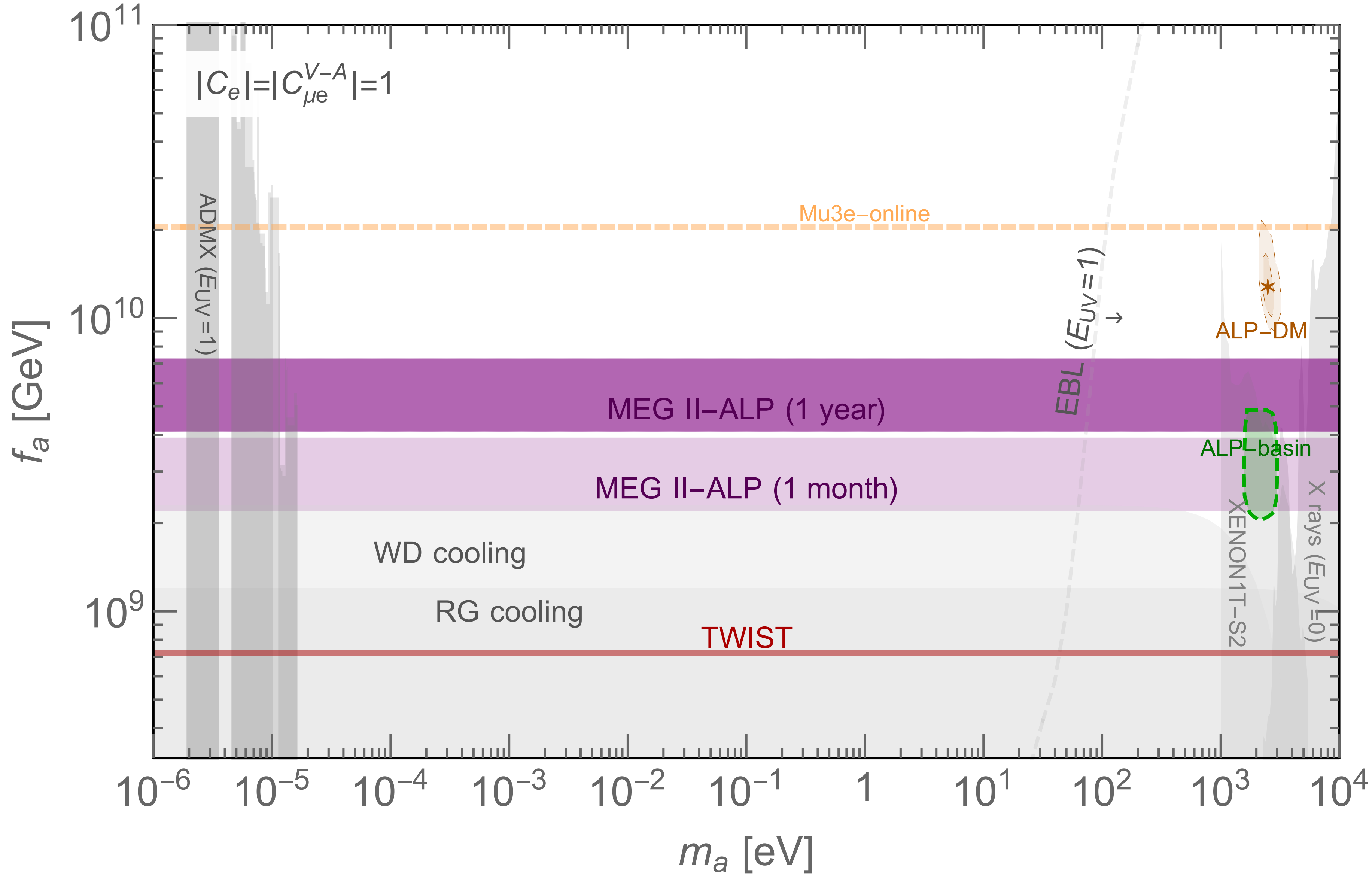
polarization

magnetic focusing

Jho Knapen Redigolo '22

- Prospect at MEG II for $\mu \to e\, a\, \gamma$

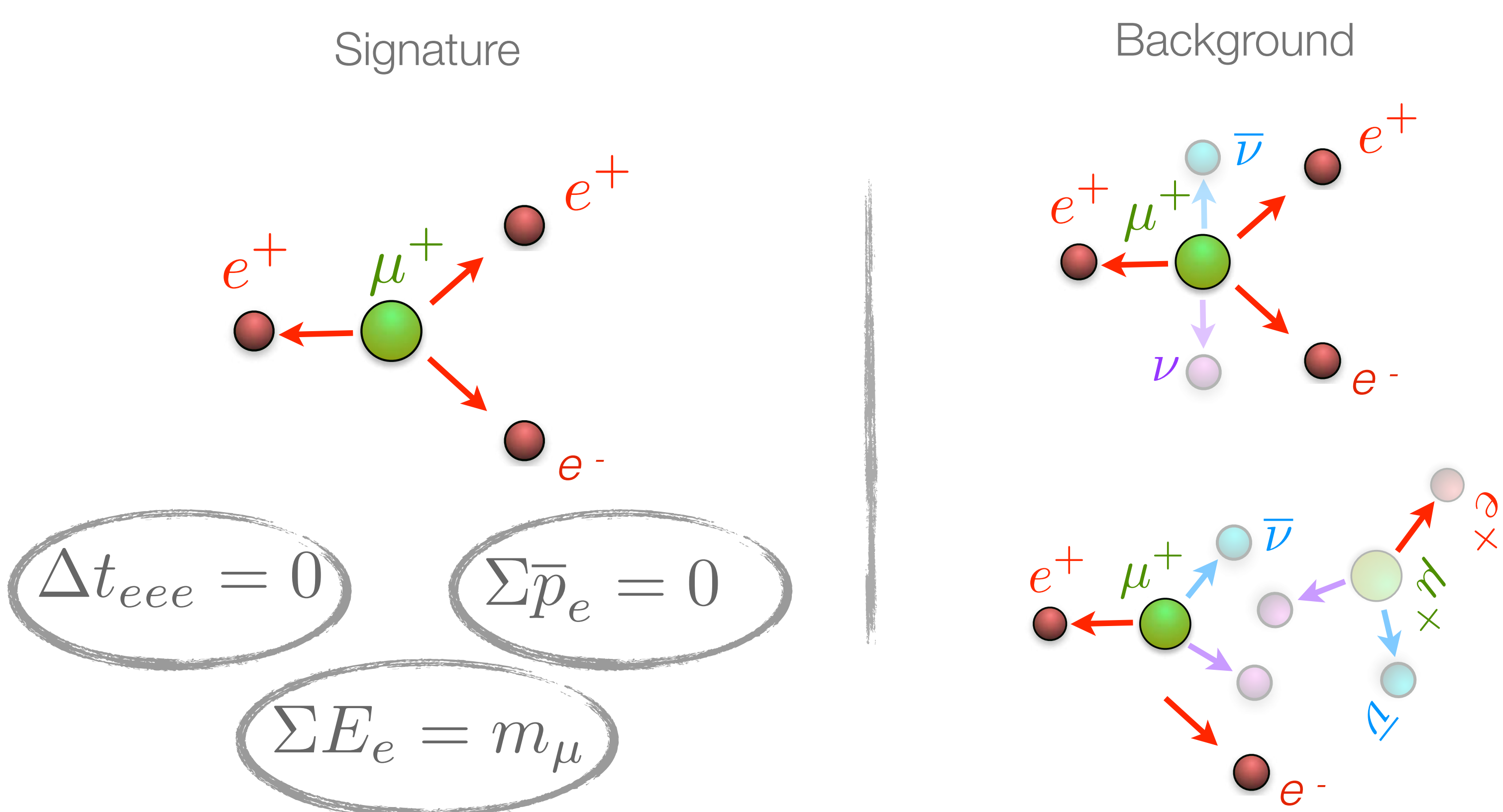
Search sensitive to V-A couplings too, requires a dedicated trigger:

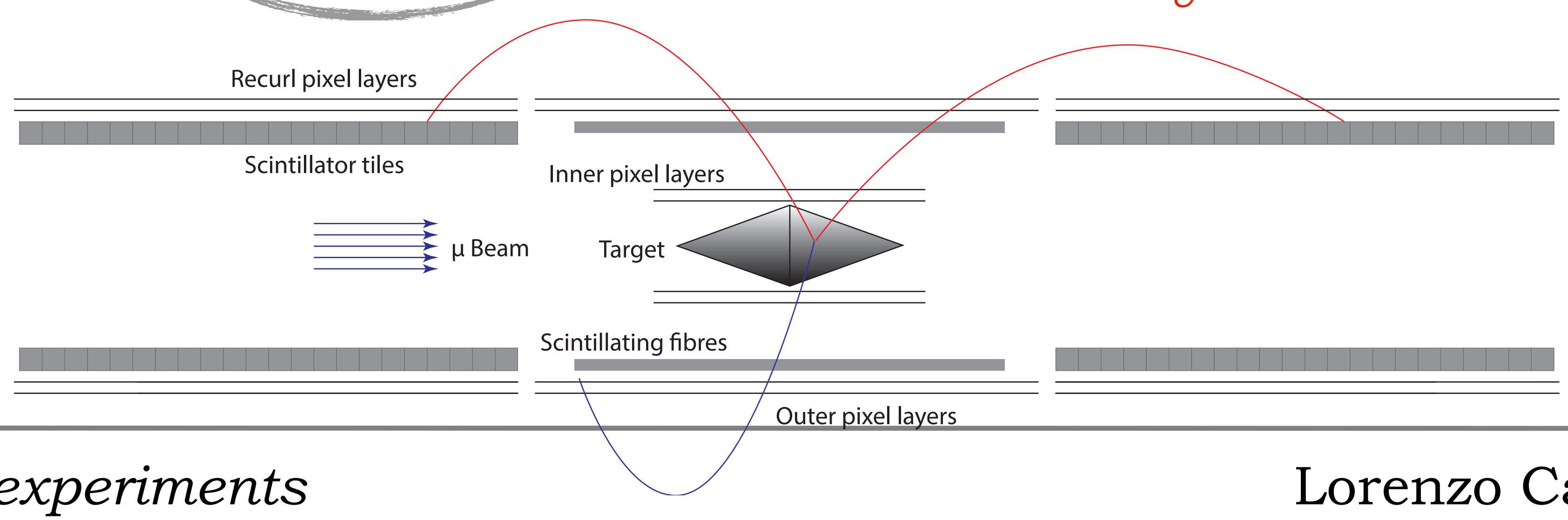


Mu3e: The $\mu^+ \to e^+e^+e^-$ search

slide borrowed from A. Papa

- The Mu3e experiment aims to search for $\mu^+ \to e^+ e^+ e^-$ with a sensitivity of ~10^{-15} (Phase I) up to down ~10^{-16} (Phase II). Previous upper limit $BR(\mu^+ \to e^+ e^+ e^-) \leq 1 \times 10^{-12}$ @90 C.L. by SINDRUM experiment)
- Observables (E_e, t_e, vertex) to characterize $\mu \to eee$ events



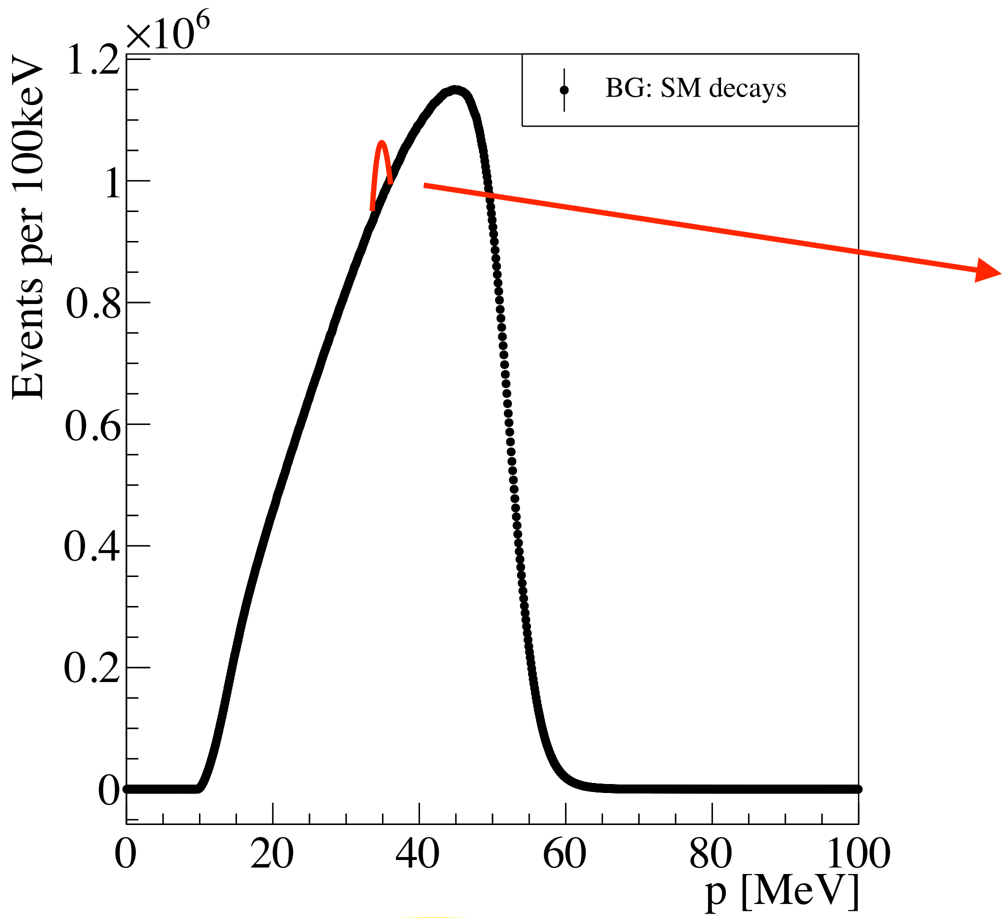


Future prospects: Mu3e

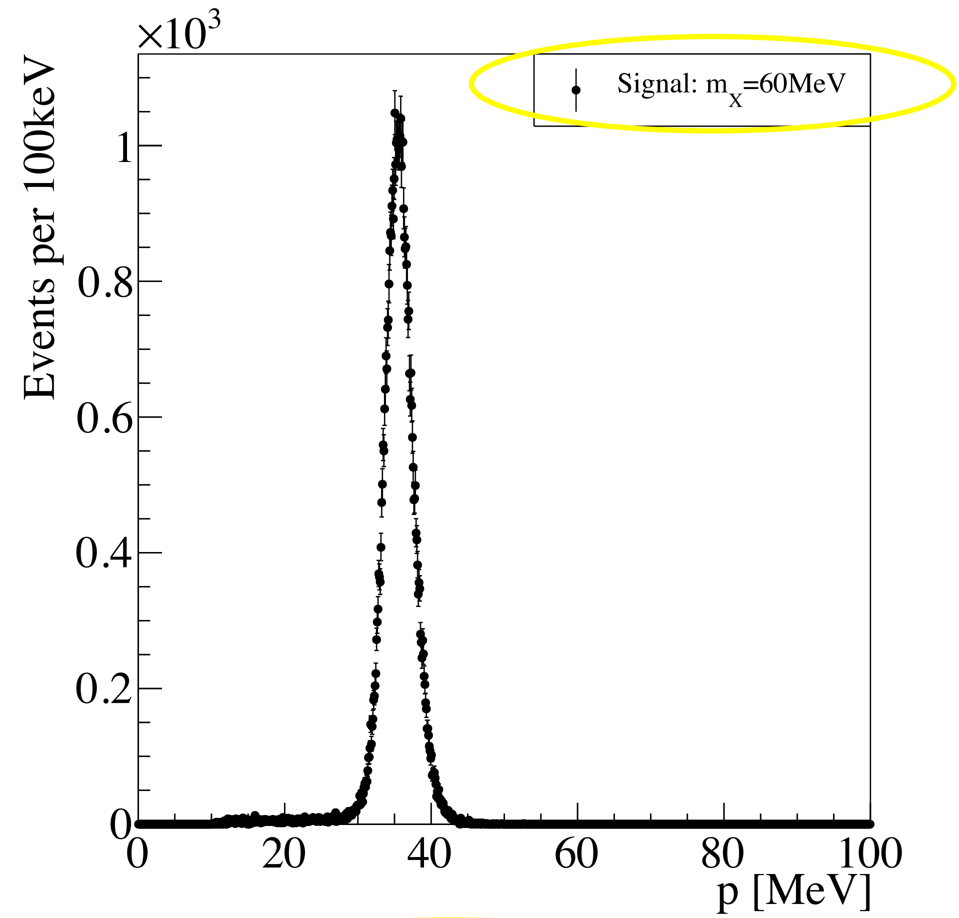
- Mu3e prospect for $\mu \to e\, a$

Perrevoort (Mu3e) '18

Potential search for performed on positron momentum histograms filled with *online* reconstructed short tracks



(a) Simulated background events.



(b) Simulated $\mu \to eX$ signal events.

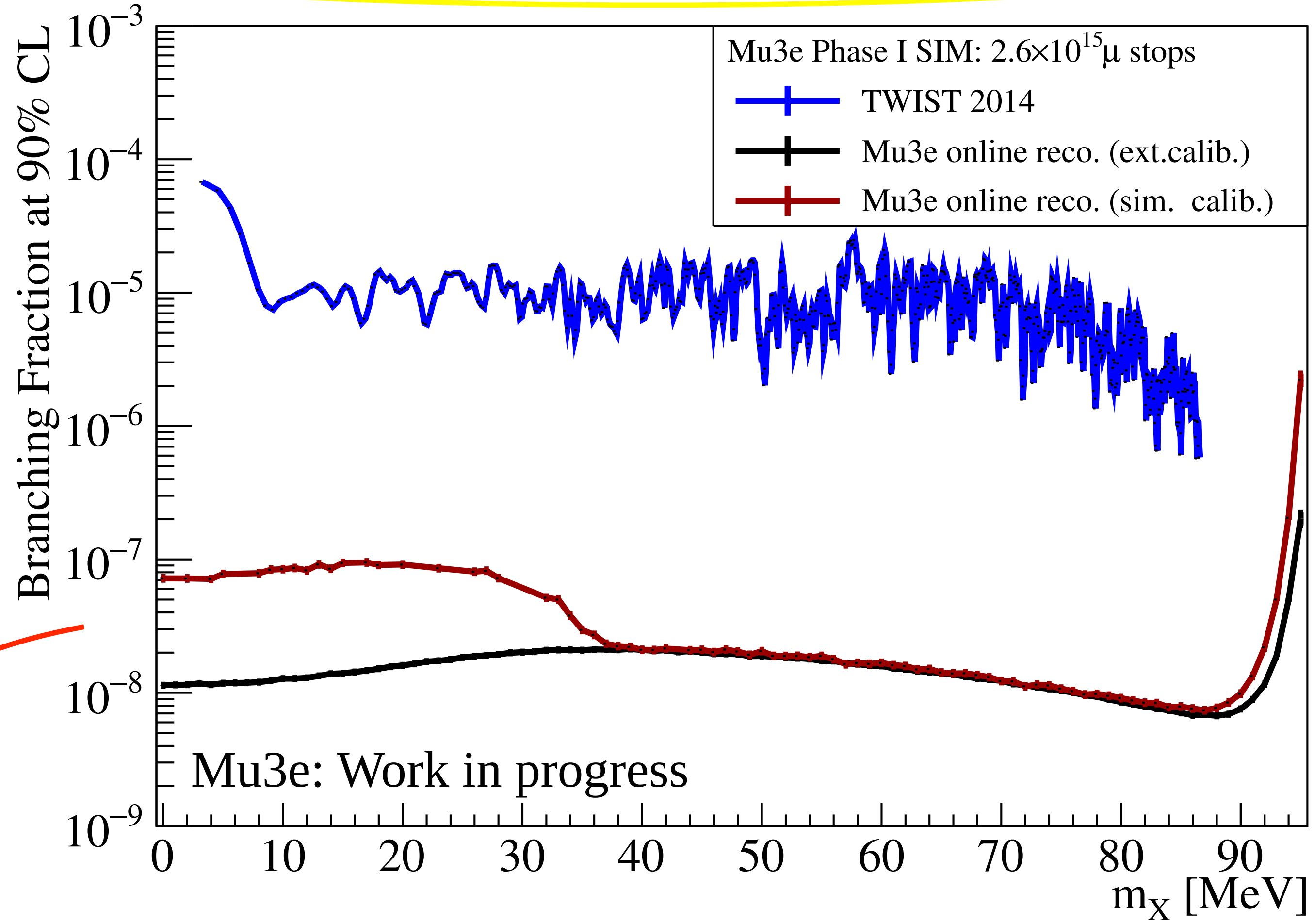
Future prospects: Mu3e

- Mu3e prospect for $\mu \to e\, a$

Perrevoort (Mu3e) '18

Potential search for performed on positron momentum histograms filled with *online* reconstructed short tracks

Expected limit for phase I ($2.6\times10^{15}\ \mu^+$):

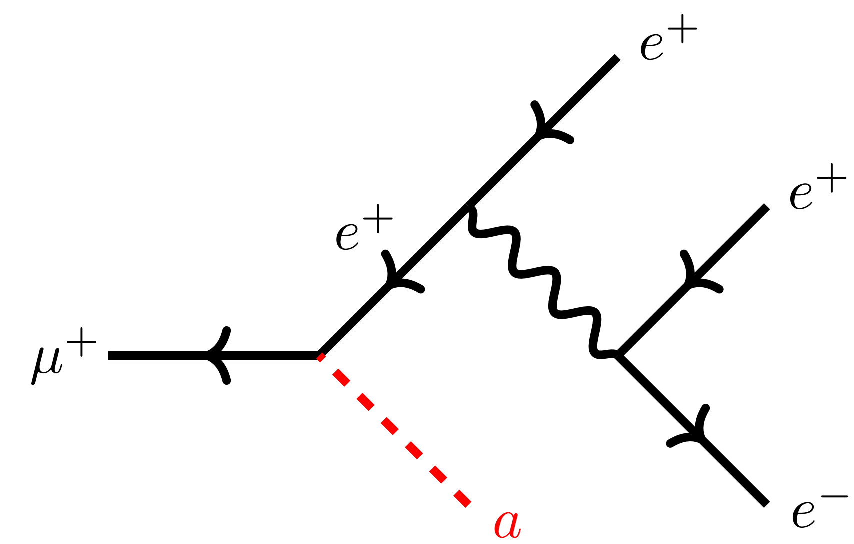


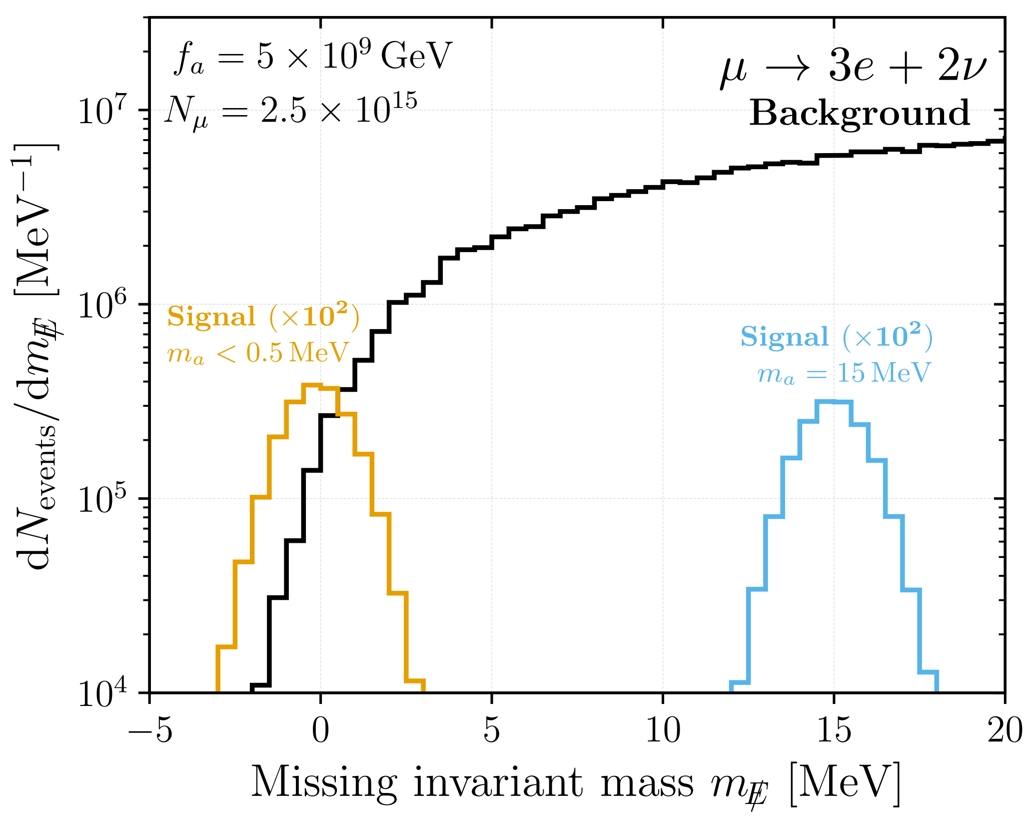
$$m_a \approx 0 : \quad \text{BR}(\mu \to ea) < 7 \times 10^{-8} \quad \Rightarrow \quad F_{\mu e} \gtrsim 3 \times 10^{10}\ \text{GeV}$$

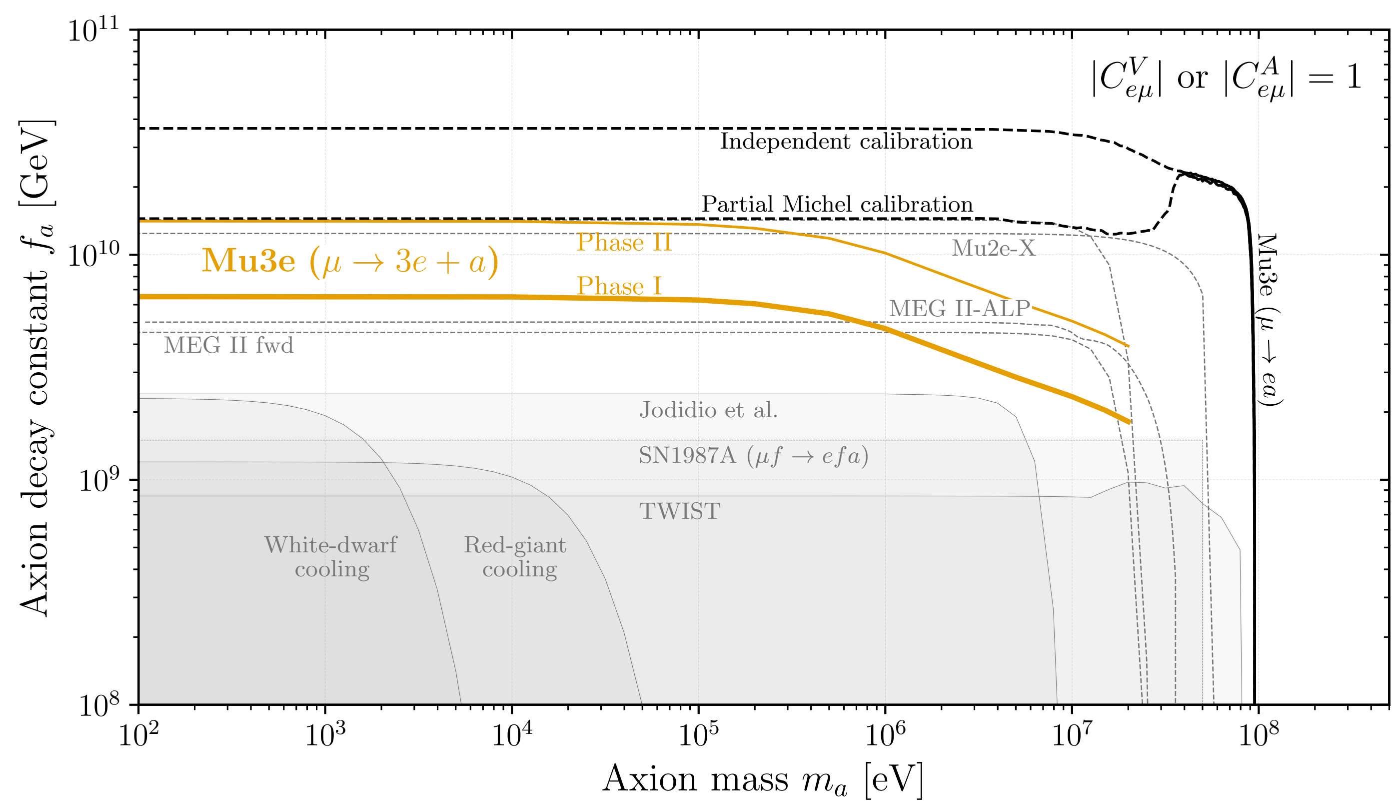
- Mu3e prospect for $\mu \to 3e\, a$

Knapen Langhoff Opferkuch Redigolo '23

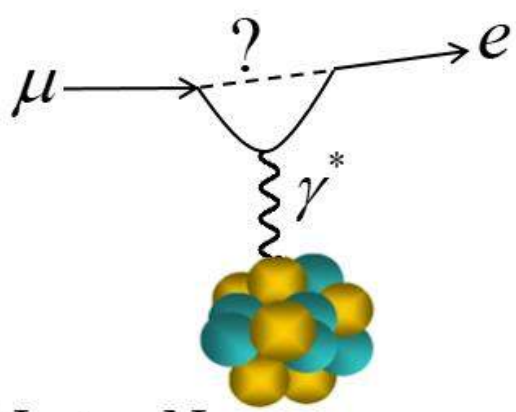
Search for $3e$ + invisible from the internal conversion of virtual photon into a e^+e^- pair:



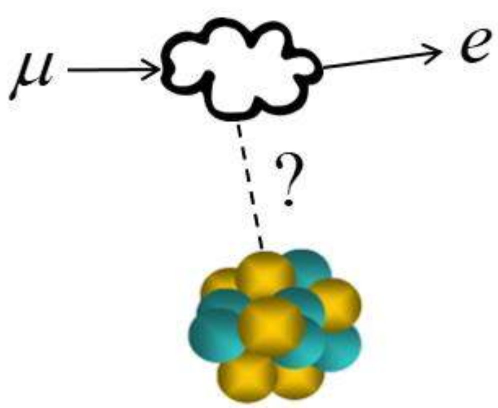


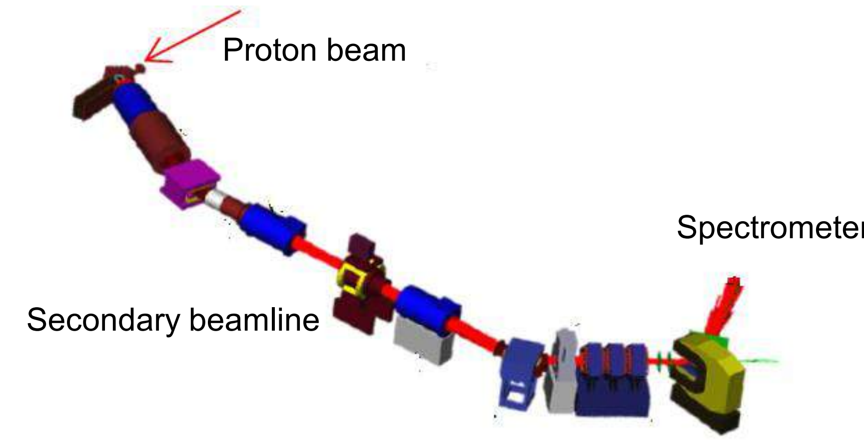


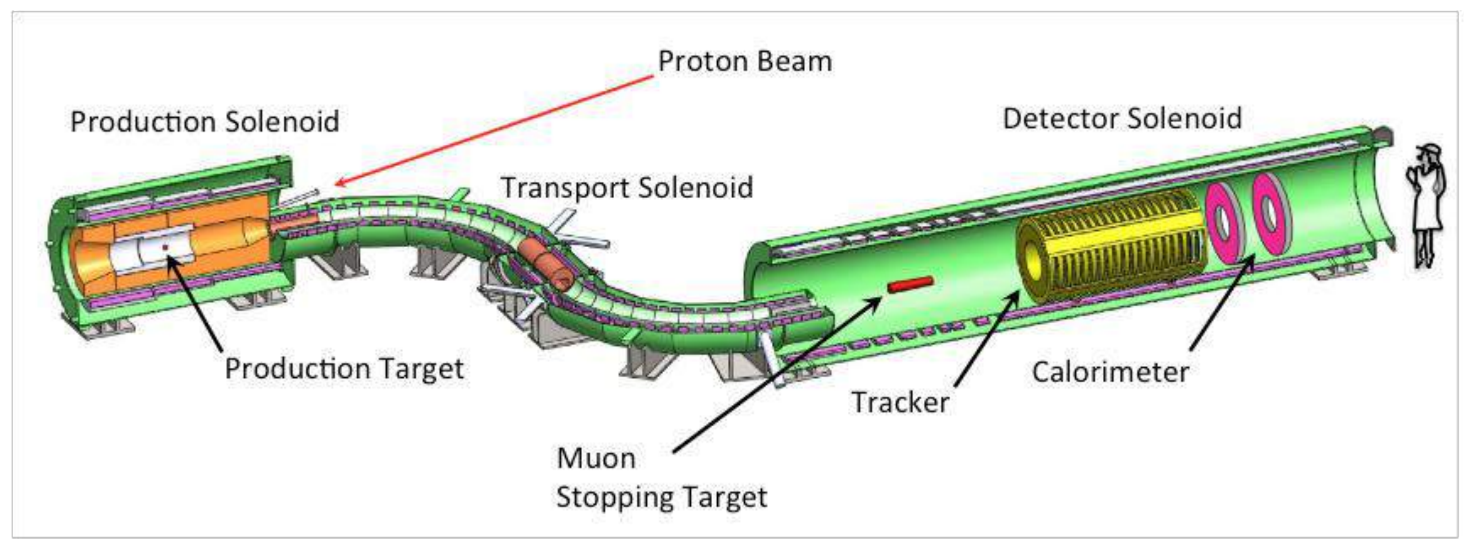
$\mu \to e$ conversion in nuclei experiments

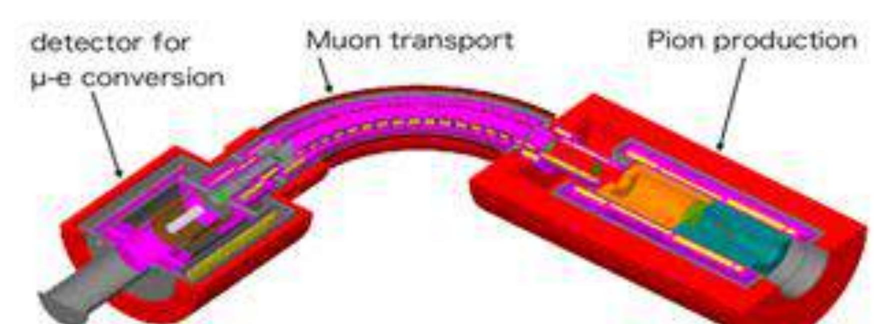


$\mu N \to e N$







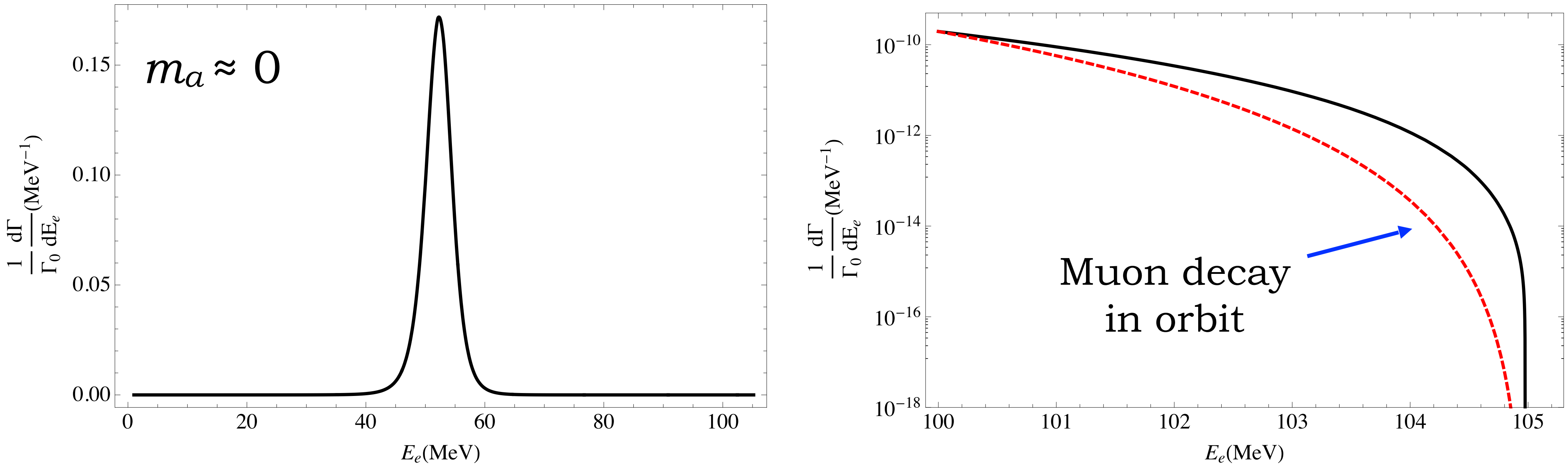


David Hitlin
Beijing CLFV School
June 3-7, 2019
Lecture 1

- Prospect at $\mu \to e$ conversion experiments

Garcia i Tomo et al. '11

Spectrum of $\mu \to ea$ emission in orbit (for Al):



Sensitivity in terms of the $\mu \to e$ conv. limit:

$$B(\mu \to eJ) \sim \frac{N_R R_{\mu e}}{f_J}\frac{\Gamma_{\text{capture}}}{\Gamma(\mu \to e\nu_\mu\bar{\nu}_e)} \sim \frac{N_R R_{\mu e}}{f_J} 1.5 \Rightarrow 2\times10^{-6}$$

- N_R: Phase-space correction factor
- $\Gamma_{\text{capture}}/\Gamma(\mu \to e\nu_\mu\bar{\nu}_e)$: 27 (in Al)
- $R_{\mu e}$: 10^{-17}
- f_J: Fraction of $\mu \to ea$ events in the signal region: 2×10^{-10} (E_e > 100 MeV)

Possible bound at the level of Jodidio et al.
Limited by the $\mu \to e$ conv. signal region (only the tail included): dedicated search?

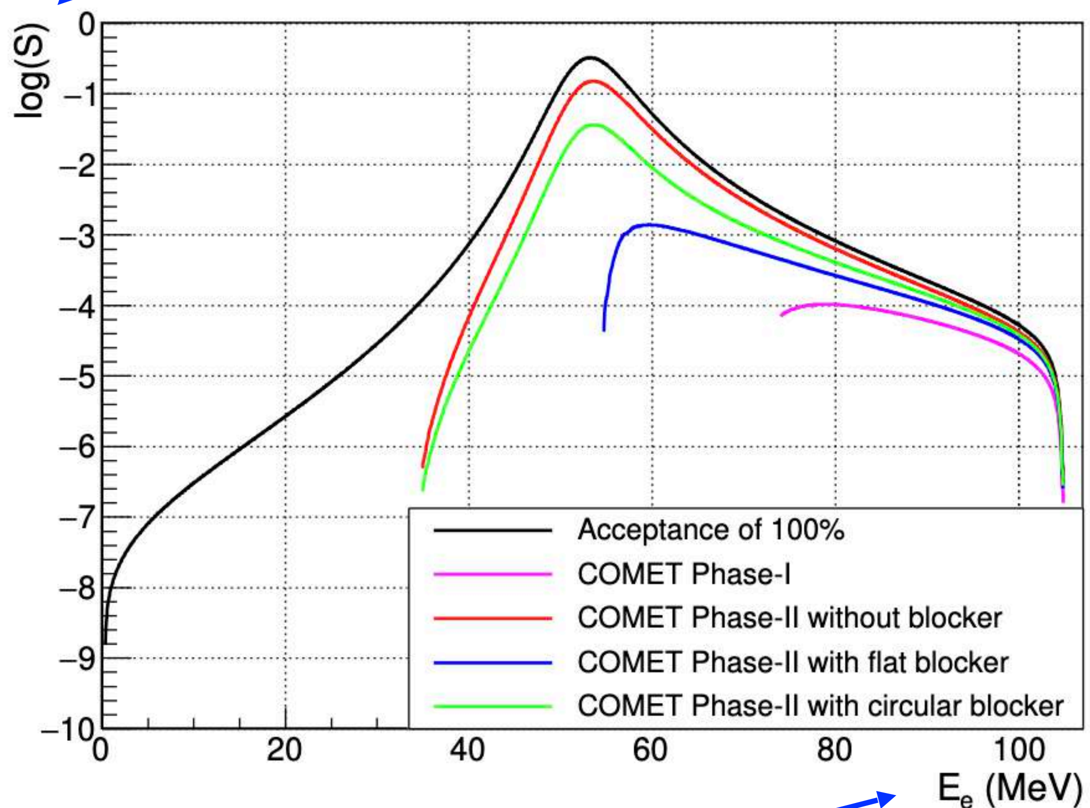
Future prospects: a COMET study

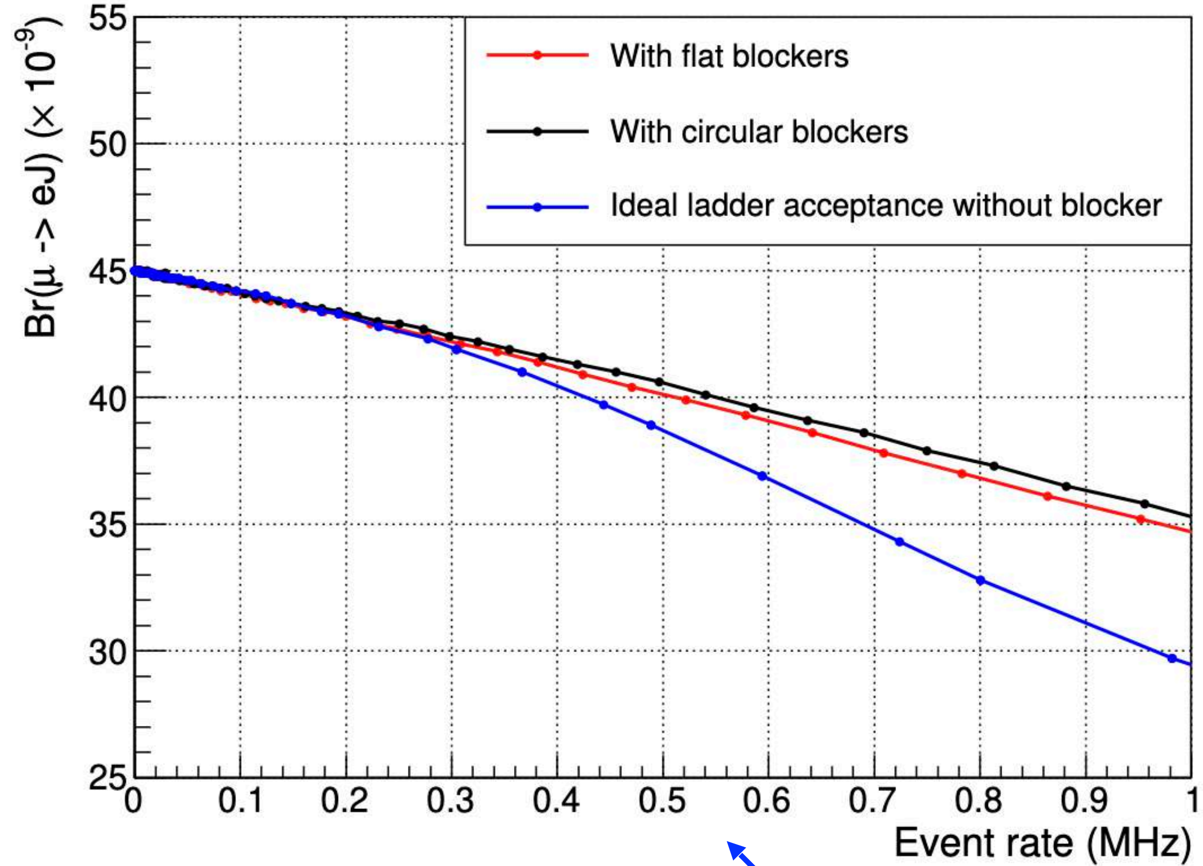
- Prospect at $\mu \to e$ conversion experiments

Xing et al. '22

One can try to lower the lower limit of the signal window
(very large background from muons decaying in orbit)

Significance factor





lower limit of
signal window

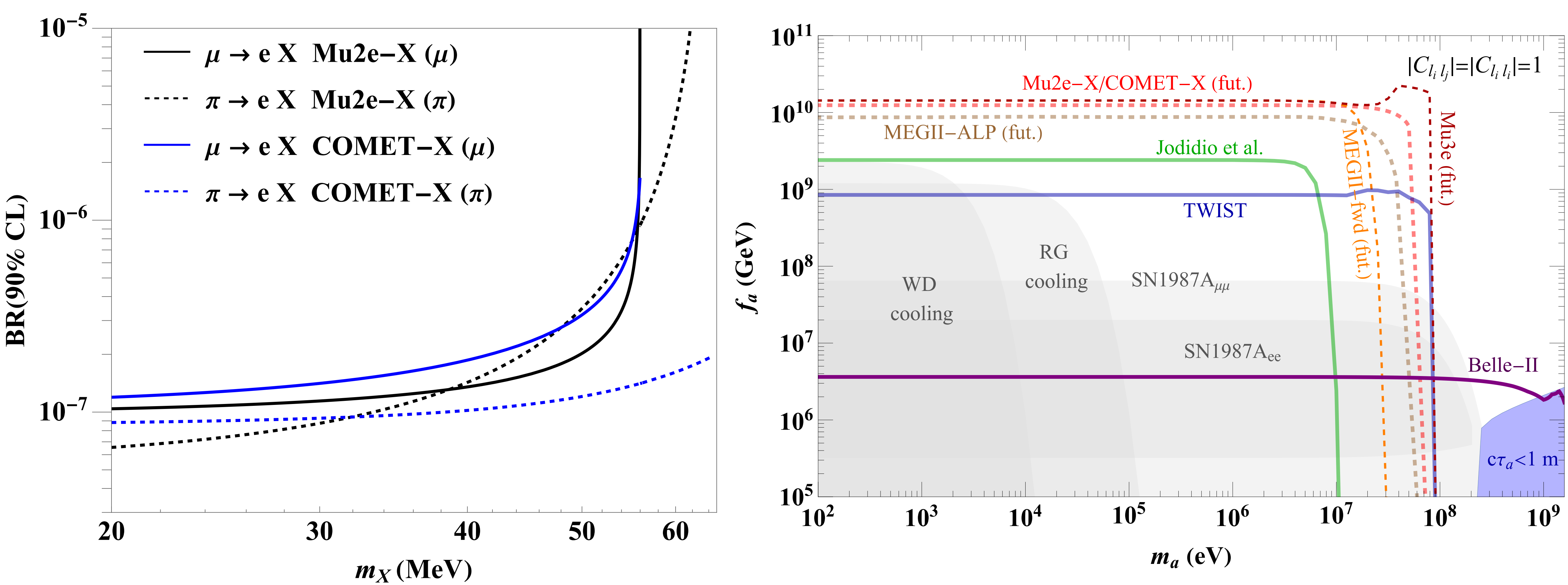
⇨ COMET Phase II can reach ~10^{-8} coping with a bg rate ~O(100)kHz

- Prospect at $\mu \to e$ conversion experiments

Hill Plestid Zupan '23

Conversion experiments employ μ^- beams to form muonic atoms: in the signal window for $\mu \to e$ conversion, the SM bg is suppressed, but $\mu \to ea$ would be too

Idea: search for a mono-energetic positron excess over the Michel spectrum, using data from calibration runs employing *positive* muon beams (10^{13} - $10^{14} \mu^+$)



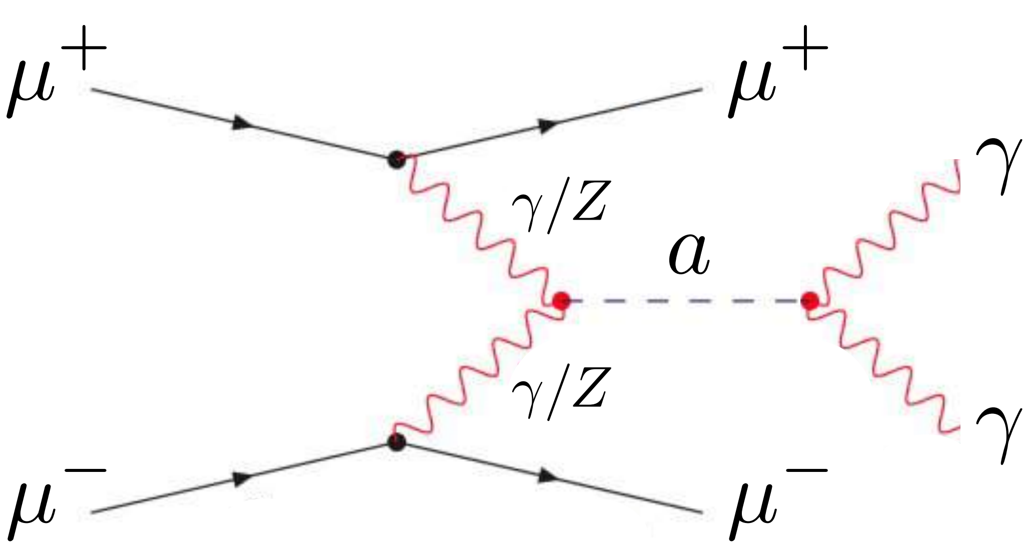
What about high-energy leptonic colliders?

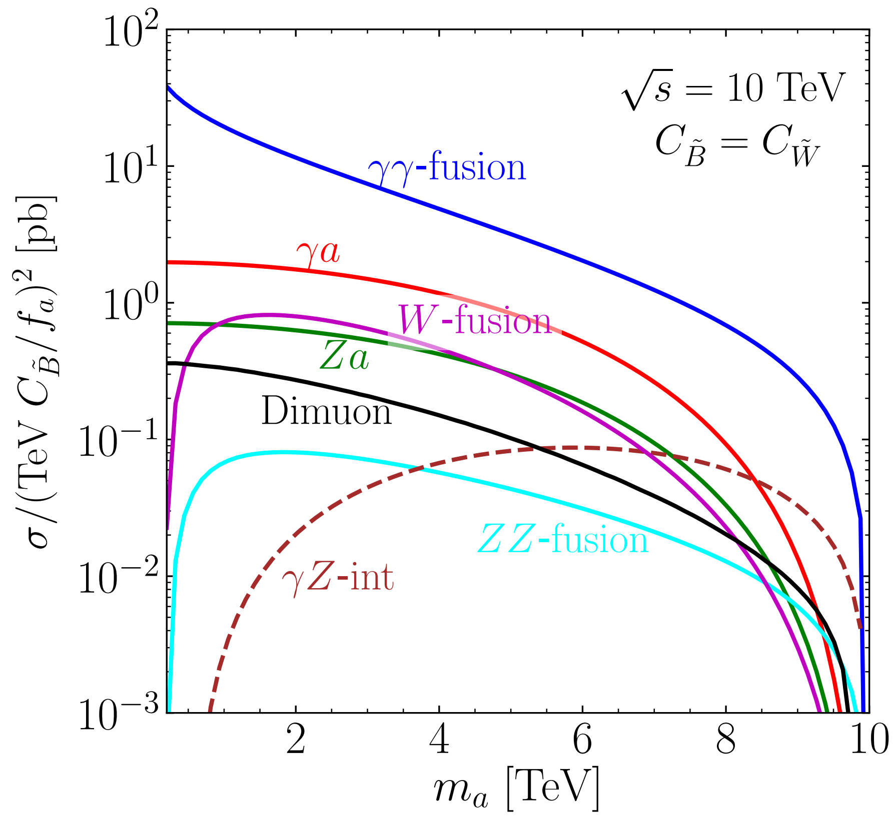
At a high-energy muon collider, one could e.g. test the ALP couplings to EW gauge bosons:

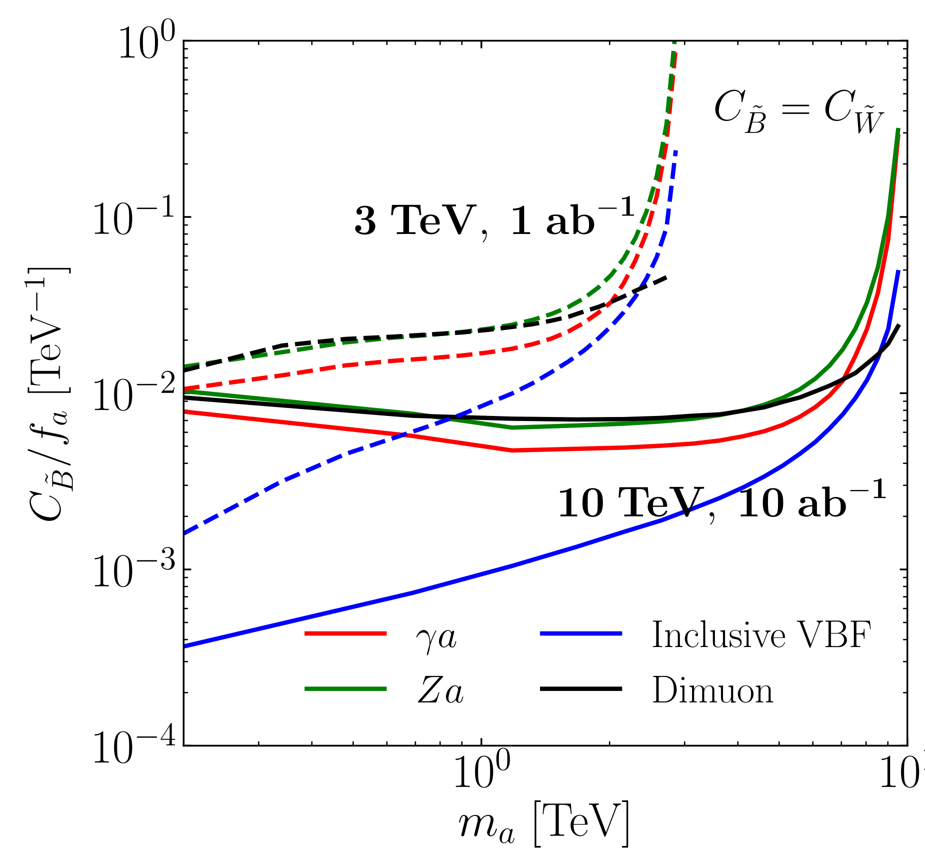
$$\mathcal{L}_{eff} \supset -\frac{g_{agg}}{4} a G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a - \frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{g_{a\gamma Z}}{4} a F_{\mu\nu}\tilde{Z}^{\mu\nu} - \frac{g_{aZZ}}{4} a Z_{\mu\nu}\tilde{Z}^{\mu\nu} - \frac{g_{aWW}}{4} a W_{\mu\nu}\tilde{W}^{\mu\nu} \,,$$

$$g_{agg} = \frac{4}{f_a} C_{\tilde{G}} \,, \quad g_{a\gamma\gamma} = \frac{4}{f_a}(s^2_\theta C_{\tilde{W}} + c^2_\theta C_{\tilde{B}}) \,, \quad g_{aZZ} = \frac{4}{f_a}(c^2_\theta C_{\tilde{W}} + s^2_\theta C_{\tilde{B}})$$

$$g_{a\gamma Z} = \frac{8}{f_a} s_\theta c_\theta (C_{\tilde{W}} - C_{\tilde{B}}) \,, \quad g_{aWW} = \frac{4}{f_a} C_{\tilde{W}} \,,$$



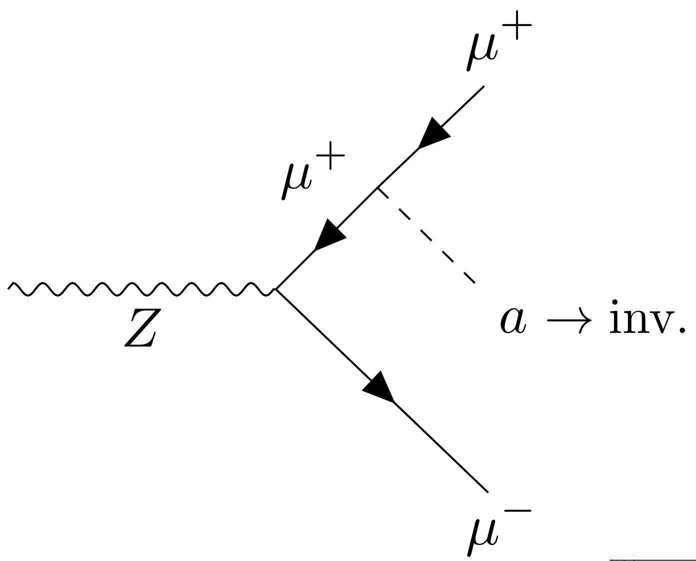




Han Li Wang '22

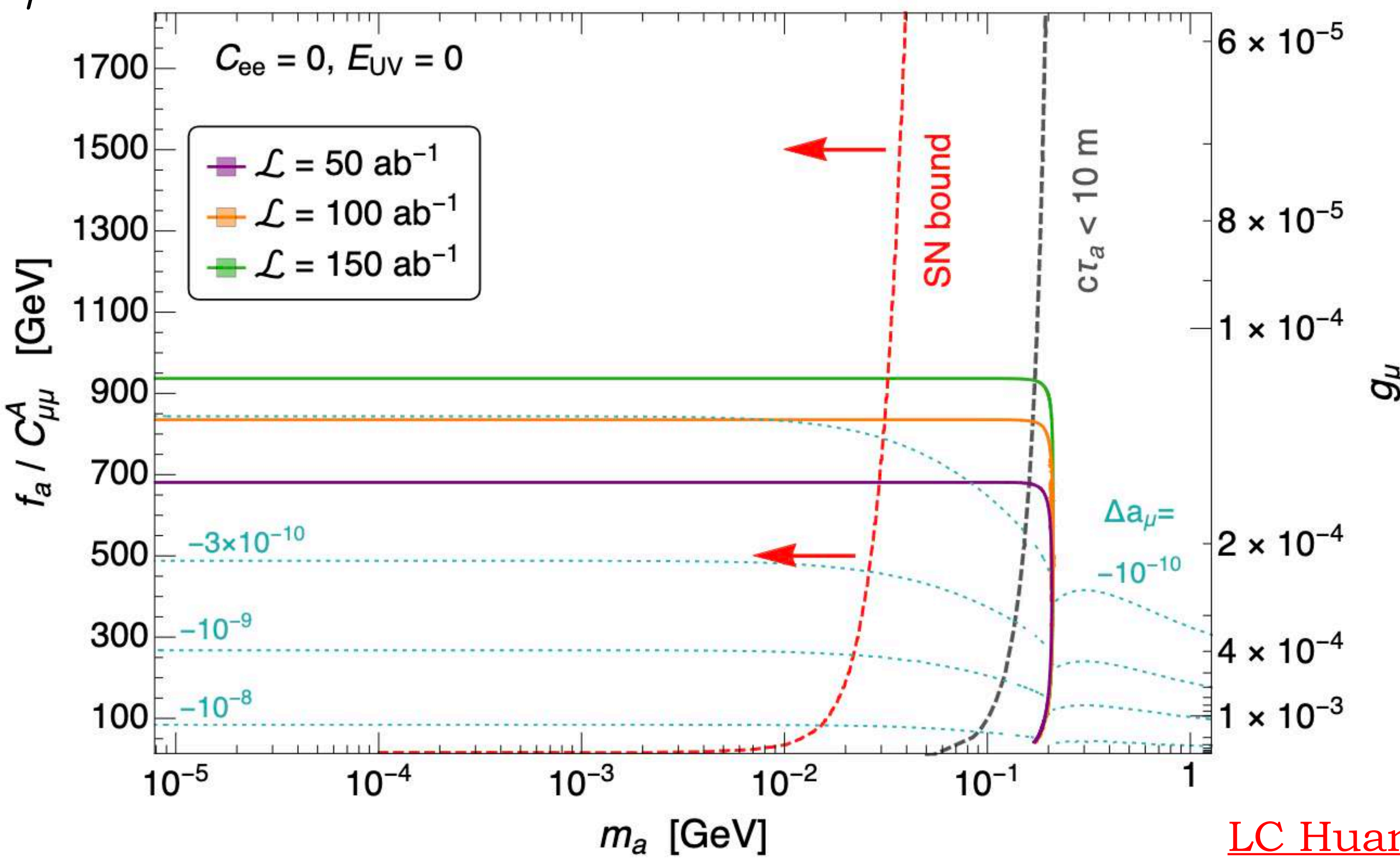
What about high-energy leptonic colliders?

Flavour-conserving couplings to muons can be also tested at colliders, e.g. at the Tera-Z factory runs of future e⁺e⁻ colliders such as the CEPC:



	$\mathcal{L} = 50\,\text{ab}^{-1}$	$\mathcal{L} = 100\,\text{ab}^{-1}$	$\mathcal{L} = 150\,\text{ab}^{-1}$
σ_{95} ($\sigma_{5\sigma}$) [$\times 10^{-6}$ pb]	1.7 (5.6)	1.1 (3.4)	0.87 (2.6)
f_{95} ($f_{5\sigma}$) [GeV]	680 (370)	834 (473)	936 (541)
BR_{95} ($\text{BR}_{5\sigma}$) [$\times 10^{-11}$]	5.4 (18)	3.6 (11)	2.8 (8.5)

Table 3: Projected 95% CL limit (5σ-discovery sensitivity) on the signal cross section, the ALP-muon coupling $f_a/C^A_{\mu\mu}$, and $\text{BR}(Z \to \mu^+\mu^- a)$. See the text for details.



LC Huang Qin Yang Yin '22

Summary

A wide class of new physics models entails axions/ALPs with flavour-violating couplings to SM leptons

Old searches for muon decays into invisible ALPs already tested new physics scales up to 10^9 GeV

These searches are complementary to tau decays, astrophysical/cosmological bounds and are sensitive to regions of the parameter space where ALP can be dark matter

Plenty of new ideas to improve the limit on BR($\mu \to e\, a$) by 2-3 orders of magnitude (testing scales above 10^{10} GeV). Future high-energy colliders can play an important role too

We have huge room for improvement over old limits: next generation experiments may discover axions with muons!

Thanks!

谢谢!

Additional slides

Summary of the model-independent bounds

Comparison in the case $m_a \approx 0$

$$\mathcal{L}_{a\ell\ell} = \frac{\partial^\mu a}{2f_a}\left(C^V_{ij}\ \bar{\ell}_i\gamma_\mu\ell_j + C^A_{ij}\ \bar{\ell}_i\gamma_\mu\gamma_5\ell_j\right) \qquad F^{V,A}_{ij} \equiv \frac{2f_a}{C^{V,A}_{ij}} \qquad F_{ij} \equiv \frac{2f_a}{\sqrt{|C^V_{ij}|^2 + |C^A_{ij}|^2}}$$

Present best limits				
Process	BR Limit	Decay constant	Bound (GeV)	Experiment
Star cooling	–	F^A_{ee}	4.6×10^9	WDs [44]
	–	$F^A_{\mu\mu}$	1.6×10^6	SN1987A$_{\mu\mu}$ [45]
	4×10^{-3}	$F_{\mu e}$	1.4×10^8	SN1987A$_{\mu e}$ (Sec. 6.1)
$\mu \to e\,a$	$2.6 \times 10^{-6*}$	$F_{\mu e}$ (V or A)	4.8×10^9	Jodidio at al. [9]
$\mu \to e\,a$	$2.5 \times 10^{-6*}$	$F_{\mu e}$ ($V + A$)	4.9×10^9	Jodidio et al. [9]
$\mu \to e\,a$	$5.8 \times 10^{-5*}$	$F_{\mu e}$ ($V - A$)	1.0×10^9	TWIST [10]
$\mu \to e\,a\,\gamma$	$1.1 \times 10^{-9*}$	$F_{\mu e}$	$5.1 \times 10^{8\#}$	Crystal Box [46]
$\tau \to e\,a$	$2.7 \times 10^{-3**}$	$F_{\tau e}$	4.3×10^6	ARGUS [43]
$\tau \to \mu\,a$	$4.5 \times 10^{-3**}$	$F_{\tau\mu}$	3.3×10^6	ARGUS [43]

Past searches: $\mu \to e\,\gamma\,a$

- Crystal Box 1988

PHYSICAL REVIEW D VOLUME 38, NUMBER 7 1 OCTOBER 1988

Search for rare muon decays with the Crystal Box detector

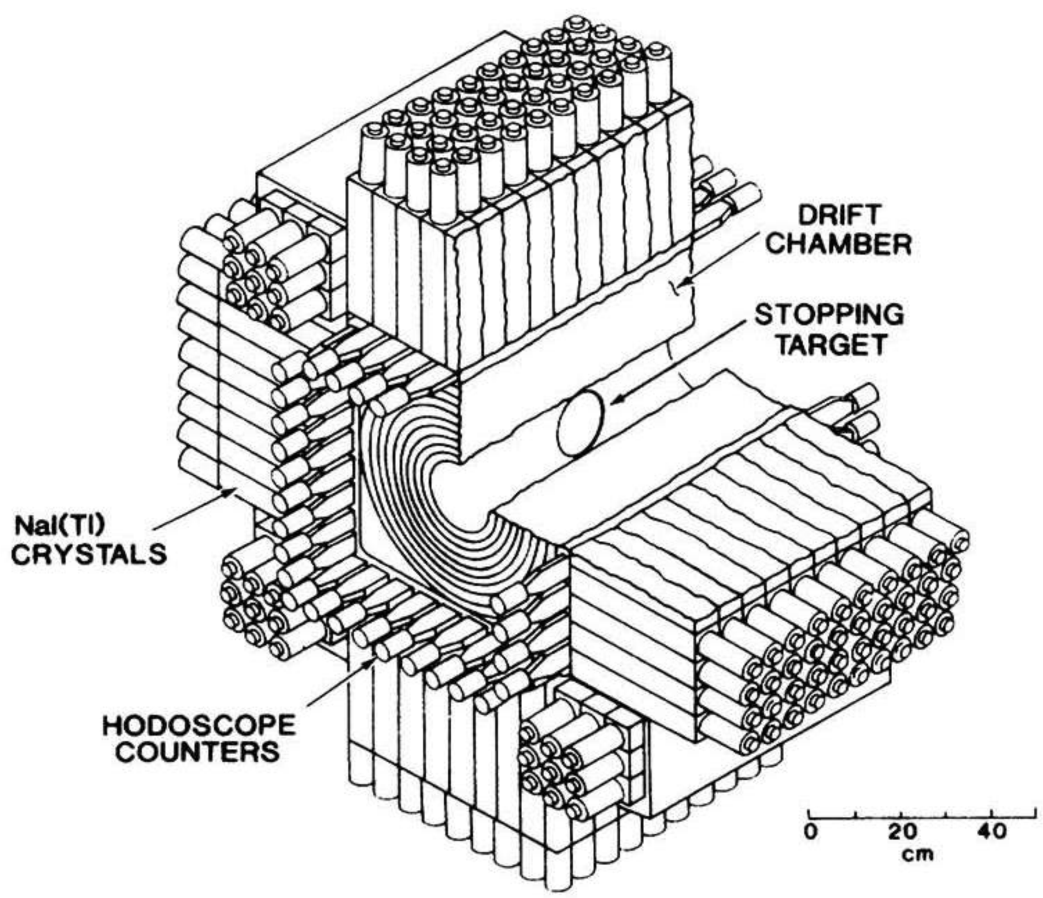


TABLE I. Types of events generated with the Monte Carlo program.

Process	Trigger
$\mu^+ \to e^+\gamma$	e-γ
$\mu^+ \to e^+\gamma\nu\bar{\nu}$	e-γ, 1-γ
$\mu^+ \to e^+\gamma\gamma$	e-γ-γ, e-γ
$\mu^+ \to e^+e^+e^-$	e-e-e
$\mu^+ \to e^+e^+e^-\nu\bar{\nu}$	e-e-e
$\mu^+ \to e^+\nu\bar{\nu}$	1-e
$\mu^+ \to e^+\gamma f$ (f = familon)	e-γ
$\pi^0 \to \gamma\gamma$	γ-γ, 1-γ
$\pi^- p \to n\gamma$	1-γ

- Crystal Box 1988

Analysis for massless familon $m_a \approx 0$
(with 1.4×10^{12} stopped μ^+) yields:

$$\mathrm{BR}(\mu \to e\,a\,\gamma) < 1.1 \times 10^{-9} \quad (90\%\ \mathrm{CL})$$

$$\mathrm{BR}(\mu \to e\,a\,\gamma) \approx \frac{\alpha_{\rm em}}{2\pi}\mathcal{I}(x_{\rm min}, y_{\rm min})\mathrm{BR}(\mu \to e\,a)$$

Hirsch et al. '09

$$\mathcal{I}(x_{\rm min}, y_{\rm min}) = \int_{x_{\rm min}, y_{\rm min}}^{1} \mathrm{d}x\mathrm{d}y \frac{(x-1)(2-xy-y)}{y^2(1-x-y)}$$

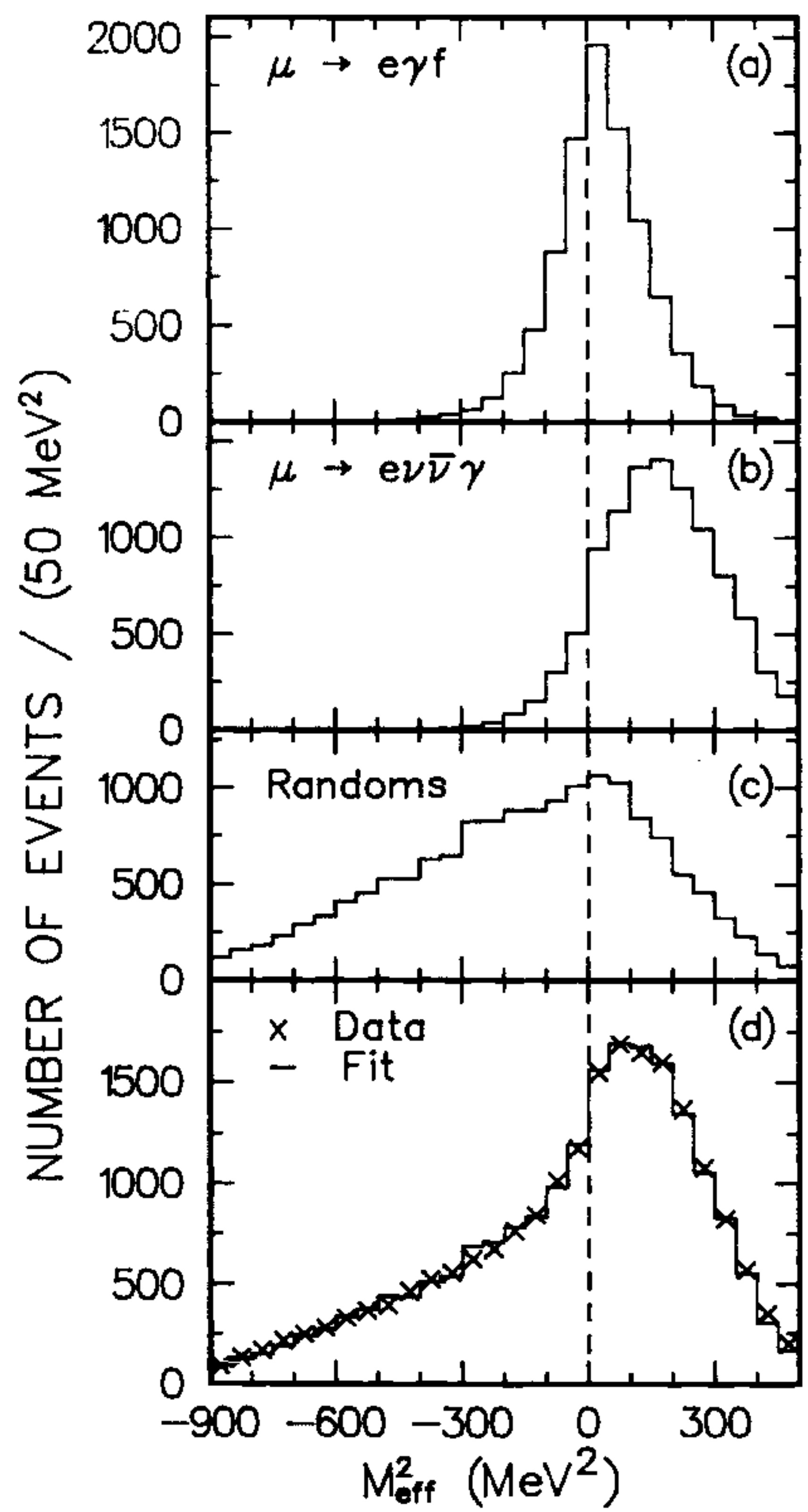
$$x = 2E_e/m_\mu \qquad y = 2E_\gamma/m_\mu$$

Crystal Box energy thresholds:

$$E_e > 38-43\ \mathrm{MeV}\,, \quad E_\gamma > 38\ \mathrm{MeV} \quad \Rightarrow \quad x_{\rm min} = 0.72-0.81\,,\ y_{\rm min} = 0.72$$

$$\Rightarrow \quad F_{e\mu} > (5.1-8.3)\times 10^8\ \mathrm{GeV}$$

weaker but independent
of V/A nature of the couplings



Models for LFV ALPs

- How generic is a PNGB with flavour-violating couplings to leptons?
- Can we test ALPs with LFV *beyond stars*?
- That is, how are FC and FV couplings related (F_{ee}, $F_{\mu e}$, etc.) ?

To answer these questions, we need to consider specific models

- LFV QCD axion:

QCD axion (DSFZ type) with leptons carrying non-universal PQ

- LFV axiflavon:

QCD axion obtained by identifying PQ = Froggatt-Nielsen U(1)
(FV axion-quark couplings suppressed by an additional flavour SU(2))

- Leptonic familon

PNGB from spontaneously broken Froggatt-Nielsen U(1) (acting on leptons only)

- Majoron

spontaneously broken lepton number (in the context of low-energy seesaw)

LFV QCD axion

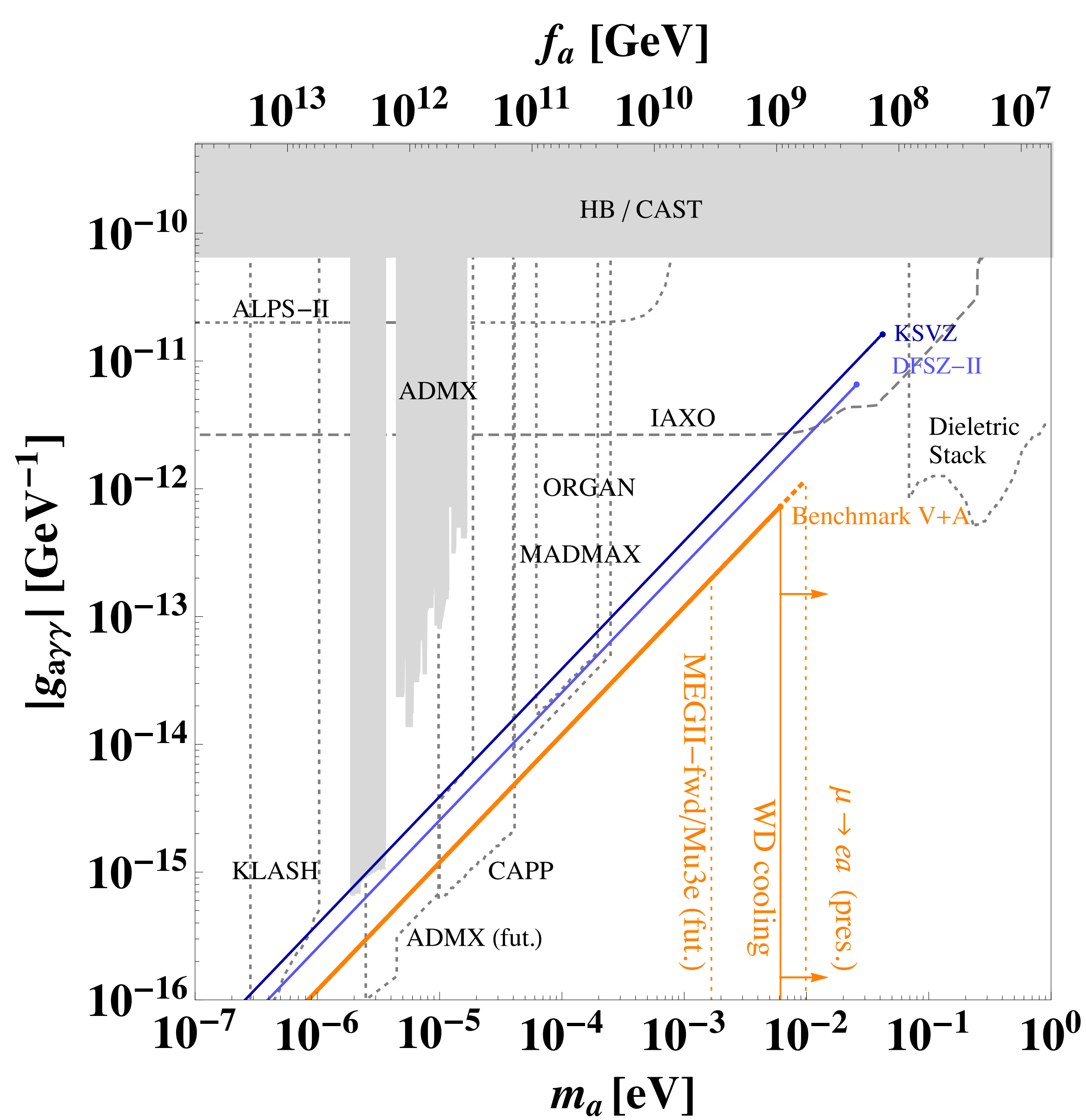
flavor non-universal charges
→ flavor-violating couplings

$$C^{V,A}_{f_i f_j} = \frac{1}{2N}\left(V_R^{f\,\dagger} X_{f_R} V_R^f \pm V_L^{f\,\dagger} X_{f_L} V_L^f\right)_{ij}$$

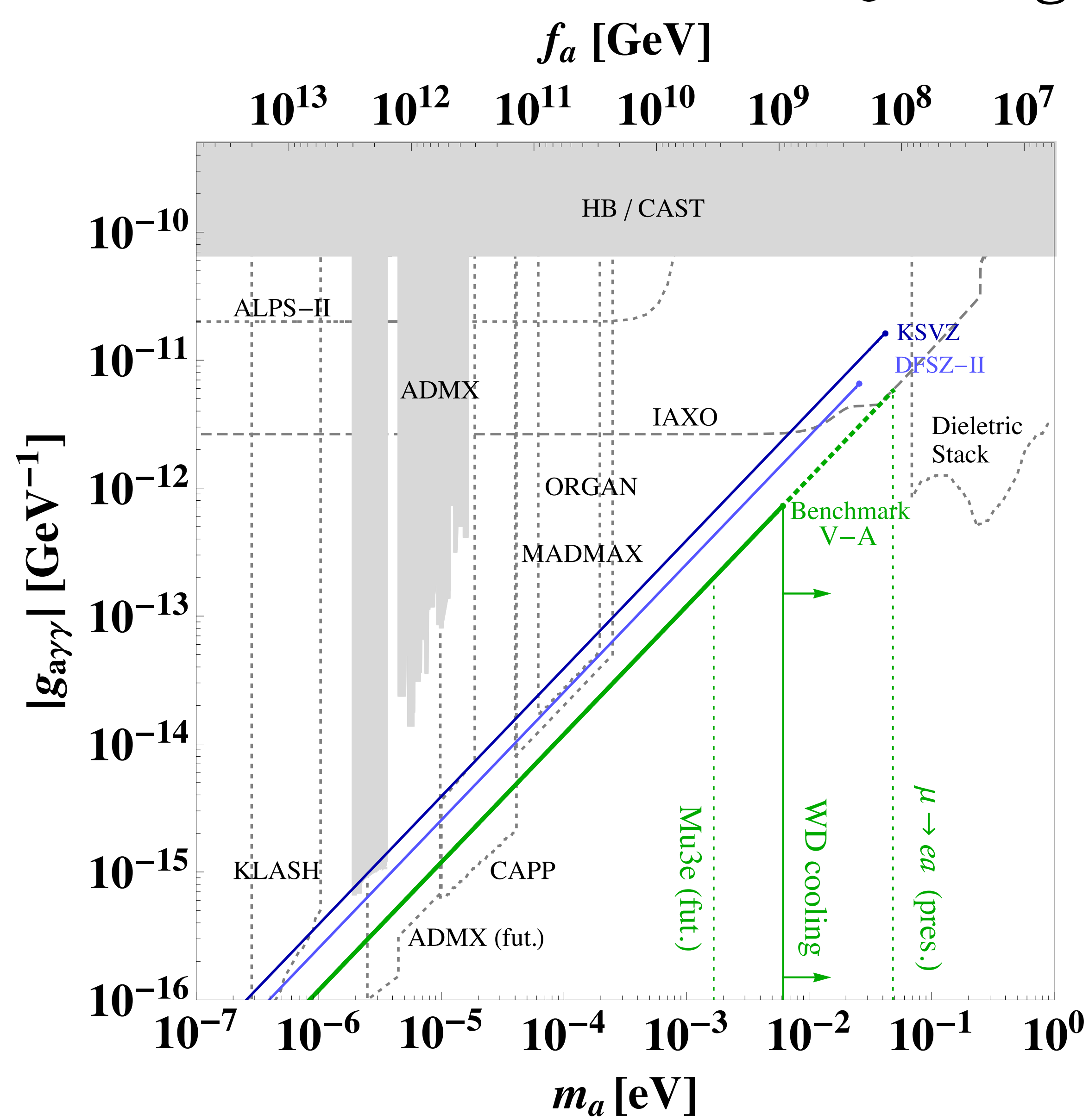
$$V_L^\dagger Y^e V_R = Y^e_{diag}$$

L and R unitary rotations to the lepton mass basis

matrices of PQ charges



V+A axion (large R rotations)



V-A axion (large L rotations)

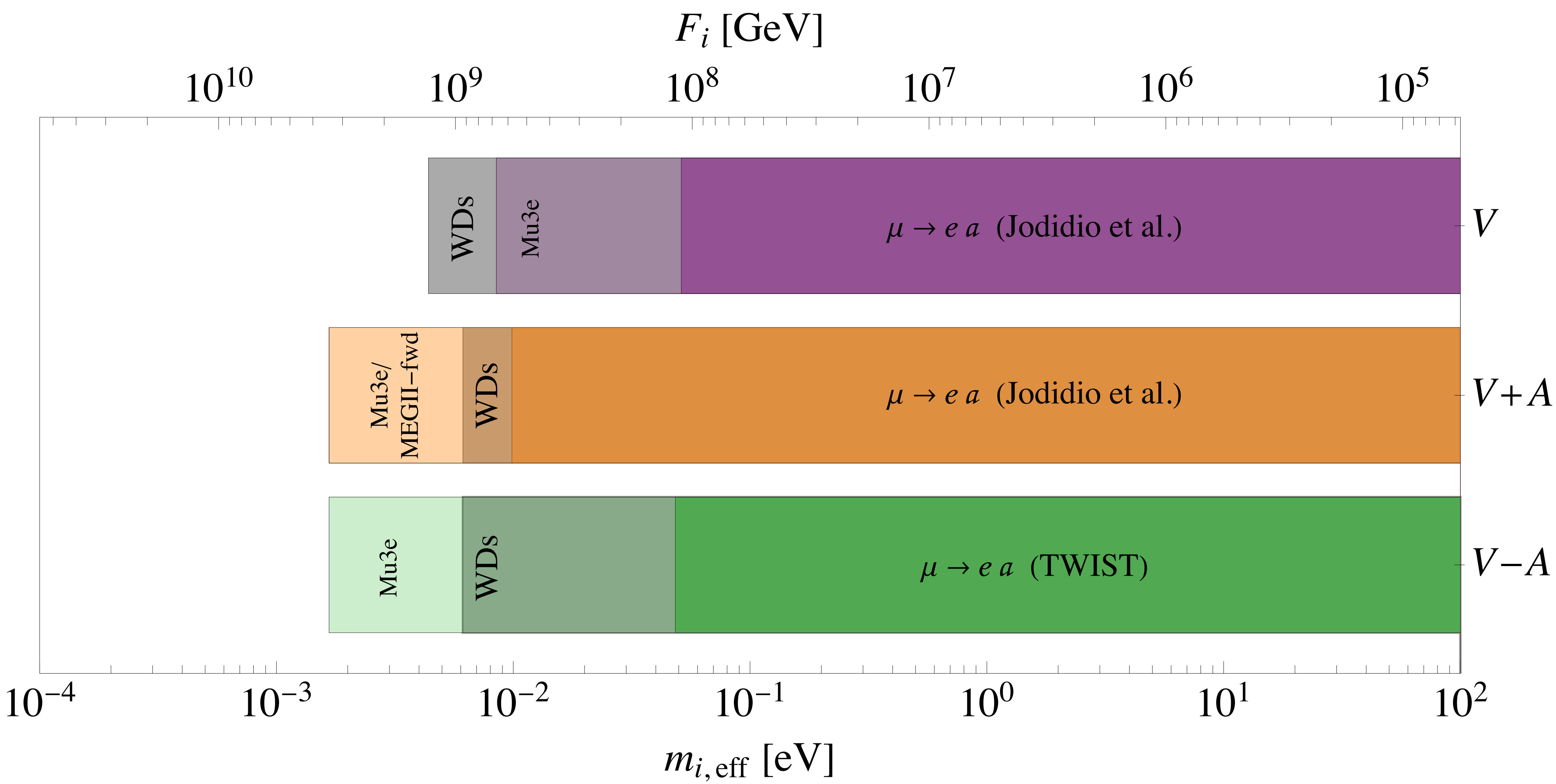
LFV QCD axion

flavor non-universal charges
→ flavor-violating couplings

$$C^{V,A}_{f_i f_j} = \frac{1}{2N}\left(V_R^{f\,\dagger} X_{f_R} V_R^f \pm V_L^{f\,\dagger} X_{f_L} V_L^f\right)_{ij}$$

$V_L^\dagger Y^e V_R = Y^e_{diag}$ L and R unitary rotations to the lepton mass basis

matrices of PQ charges



Type I seesaw: $\mathcal{L} = \mathcal{L}_{\rm SM} + i\overline{N}\partial\!\!\!/ N - \left(Y_N \overline{N}\widetilde{\Phi}^\dagger L + \frac{1}{2}M_N \overline{N} N^c + \text{h.c.}\right)$

L-breaking term

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & Y_N^T v/\sqrt{2} \\ Y_N v/\sqrt{2} & M_N \end{pmatrix} \overset{M_N \gg Y_N v}{\Longrightarrow} \quad m_\nu = -\frac{v^2}{2} Y_N^T M_N^{-1} Y_N$$

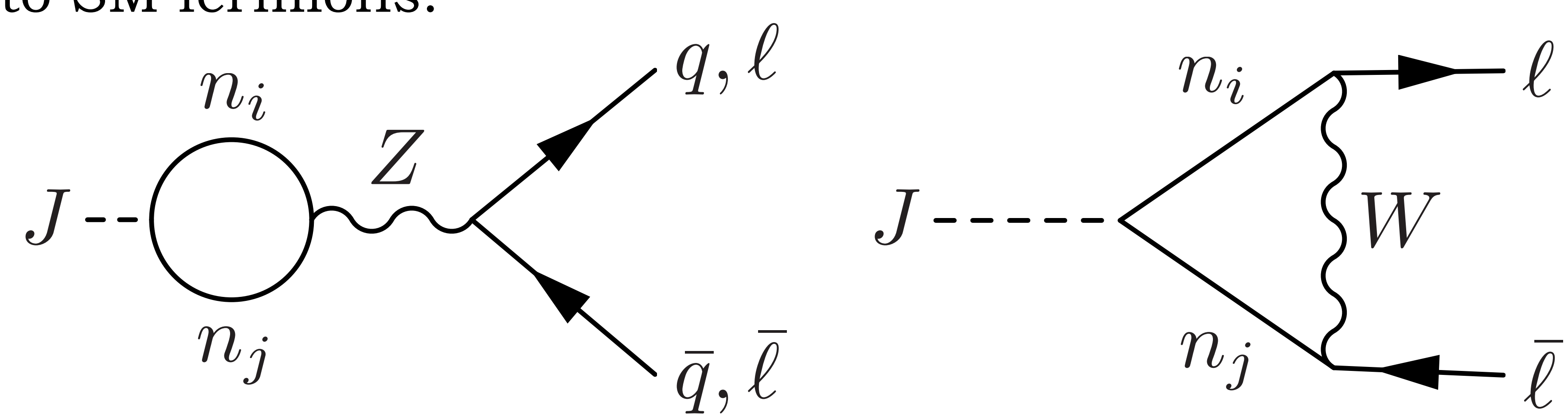
Spontaneous breaking of the lepton number:

$$\frac{1}{2}\lambda_N\, \sigma \bar{N}^c N, \quad \sigma = \frac{f_N + \hat{\sigma}}{\sqrt{2}} e^{iJ/f_N} \Longrightarrow \quad M_N = \lambda_N f_N/\sqrt{2}$$

PNGB: Majoron!

Chikashige Mohapatra Peccei '80

Couplings to SM fermions:



Majoron

Type I seesaw: $\mathcal{L} = \mathcal{L}_{\rm SM} + i\overline{N}\partial\!\!\!/ N - \left(Y_N \overline{N}\widetilde{\Phi}^\dagger L + \frac{1}{2} M_N \overline{N} N^c + \text{h.c.}\right)$

L-breaking term

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & Y_N^T v/\sqrt{2} \\ Y_N v/\sqrt{2} & M_N \end{pmatrix} \xRightarrow{M_N \gg Y_N v} m_\nu = -\frac{v^2}{2} Y_N^T M_N^{-1} Y_N$$

Spontaneous breaking of the lepton number:

$$\frac{1}{2}\lambda_N\, \sigma \bar{N}^c N, \quad \sigma = \frac{f_N + \hat{\sigma}}{\sqrt{2}} e^{iJ/f_N} \Longrightarrow M_N = \lambda_N f_N/\sqrt{2}$$

PNGB: Majoron!

Chikashige Mohapatra Peccei '80

Couplings to SM fermions:

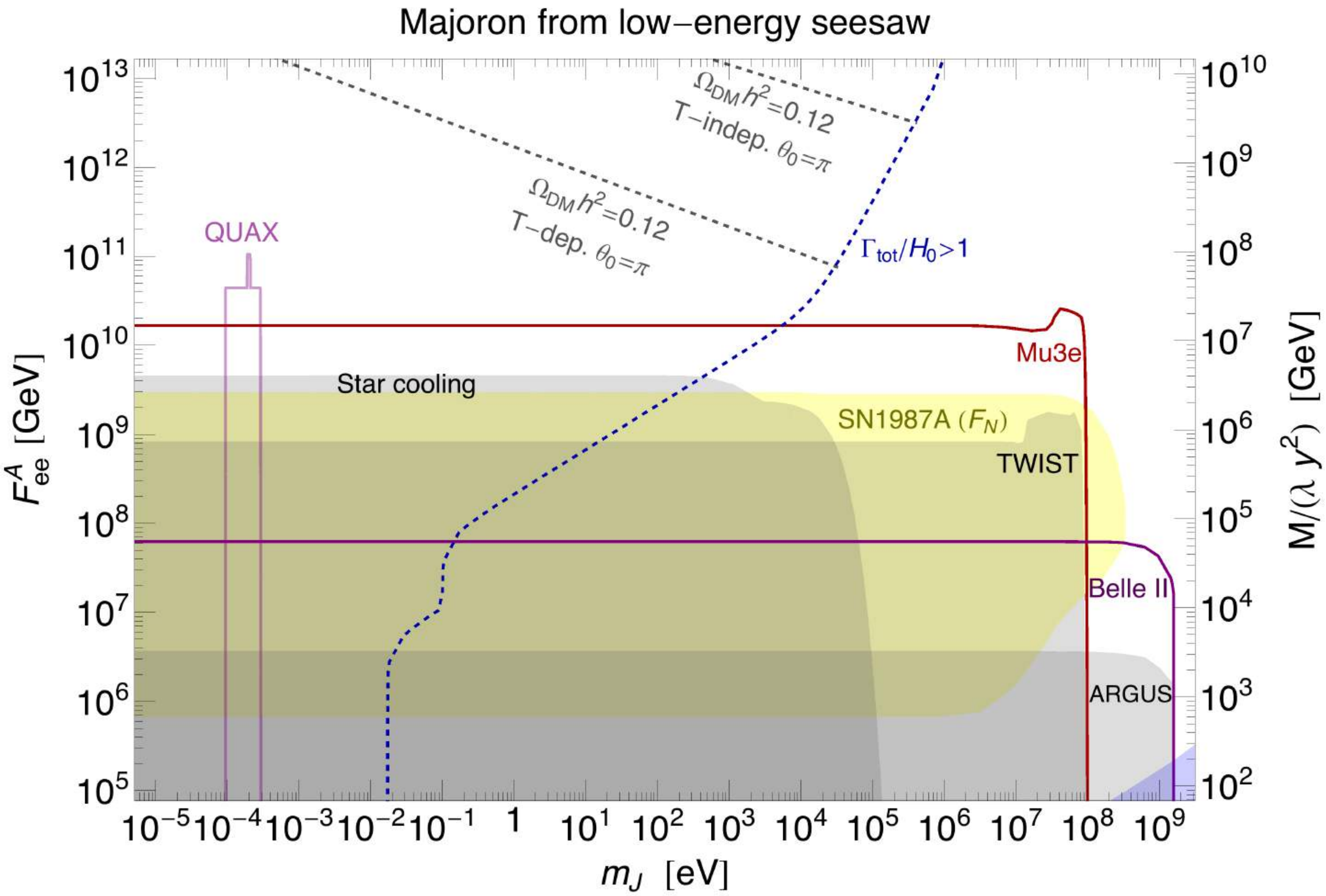
$$C^V_{q_i q_j} = 0\,, \qquad C^A_{q_i q_j} = -\frac{T_3^q}{16\pi^2}\delta_{ij}\,\text{Tr}\left(Y_N Y_N^\dagger\right)\,,$$

$$C^V_{\ell_i \ell_j} = \frac{1}{16\pi^2}\left(Y_N Y_N^\dagger\right)_{ij}\,, \qquad C^A_{\ell_i \ell_j} = \frac{1}{16\pi^2}\left[\frac{\delta_{ij}}{2}\,\text{Tr}\left(Y_N Y_N^\dagger\right) - (Y_N Y_N^\dagger)_{ij}\right]$$

Generically flavour-violating, (V-A)

Pilaftsis '94
Garcia-Cely Heeck '17

Majoron



Lepton number anomaly free: suppressed coupling to photons ($E_{\rm UV}$=0)

$$\Gamma(a\to\gamma\gamma)=\frac{\alpha_{\rm em}^2E_{\rm eff}^2}{64\pi^3}\frac{m_a^3}{f_a^2},\qquad m_a\ll m_{\ell_i}:\ E_{\rm eff}\simeq E_{\rm UV}\qquad \mathcal{L}_{\rm eff}=E_{\rm UV}\frac{\alpha_{\rm em}}{4\pi}\frac{a}{f_a}F\tilde{F}$$