Gravitational time dilation & free fall; Thomas precession

Outline:

- Ocean waves vs. de Broglie waves:
  - gravitational time dilation explains the free fall
  - motion along geodesics follows; not postulated

Gravitational time dilation, free fall, and matter waves

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Am. J. Phys. **89** (6), June 2021

- Thomas precession: the fate of the angular momentum

12th Bolyai–Gauss–Lobachevsky Conference, Budapest

Andrzej Czarnecki  University of Alberta

May 1, 2024

Part 1/2: Time dilation and free fall

Explanation why world-lines follow geodesics
(as mentioned in talks
by Francisco Lobo and Tiberiu Harko
this morning)

Gravitational time dilation

$$\Delta t = \frac{gh}{c^2} t$$

Example: a weekend trip from Seattle to Mount Rainier, t = 40 hours, elevation h = 1340 metres,

$$\Delta t = \frac{gh}{c^2} t = \frac{9.8 \cdot 1340}{9 \cdot 10^{16}} 40 \cdot 3600 \text{ s} = 21 \text{ ns}.$$

For h = 1 cm, $\Delta t/t = 10^{-18}$

This tiny difference of time flow makes things fall!

How does this tiny difference cause falling?

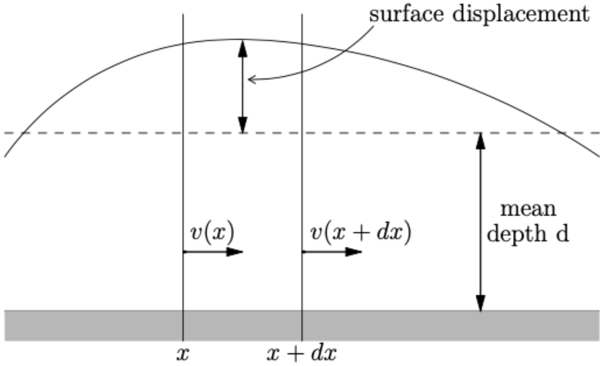
Matter waves evolve faster at a higher elevation.

Intuitive example: ocean waves; why do they always approach the beach?

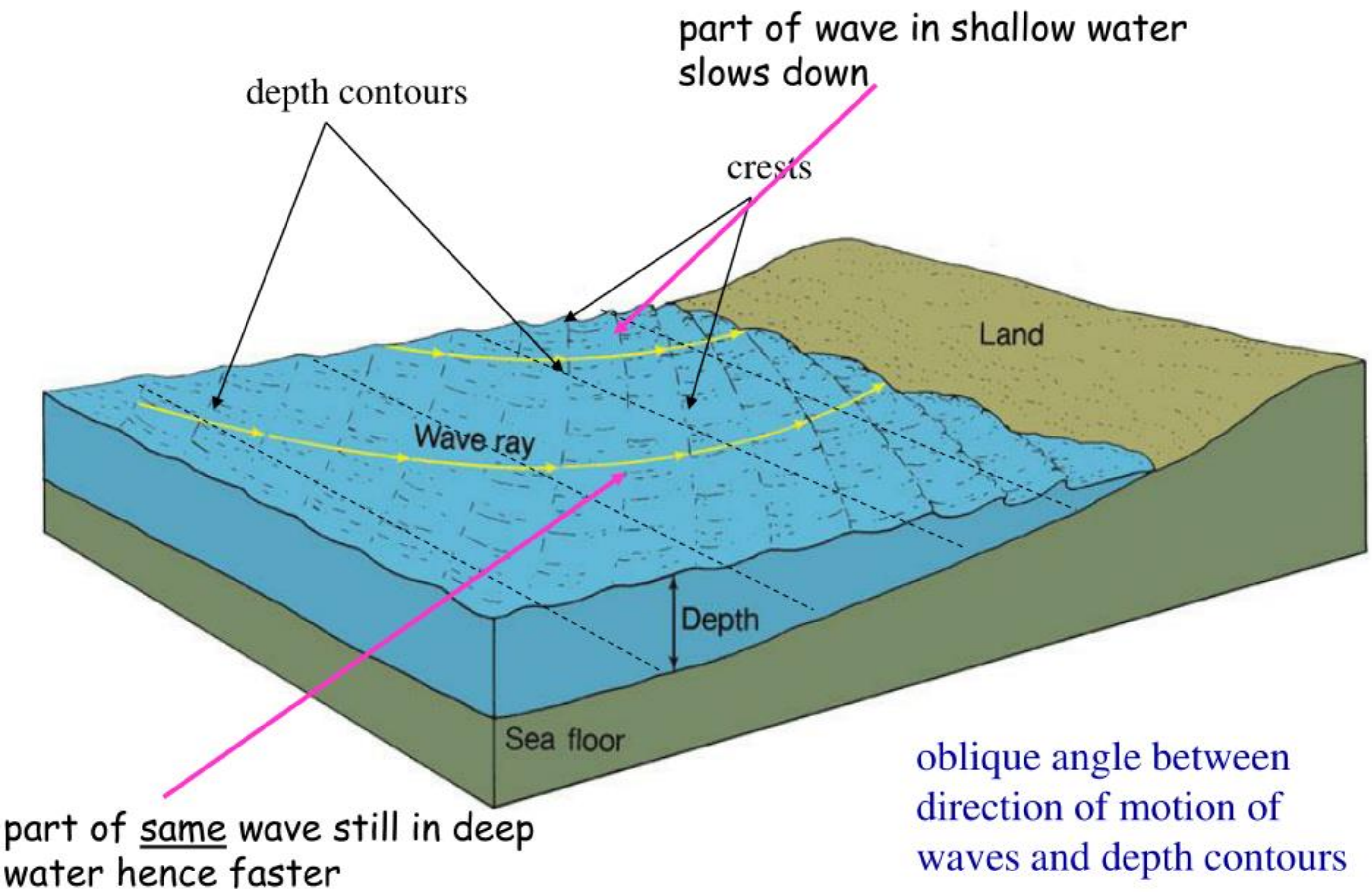
Reason: waves are slower in shallower water,

$$gd\frac{\partial^2 \zeta}{\partial x^2} = \frac{\partial^2 \zeta}{\partial t^2}$$

$$u = \sqrt{gd}$$



Wave Refraction – slowing and bending of waves as they approach shore at an angle



What does this mean for matter waves?

Consider a free particle traveling vertically: a plane wave

$$\psi = \exp \frac{ipz - i\sqrt{m^2c^4 + p^2c^2}\,t}{\hbar}$$

$$\simeq \exp \frac{1}{\hbar}\left(ipz - imc^2t - i\frac{p^2}{2m}t\right)$$

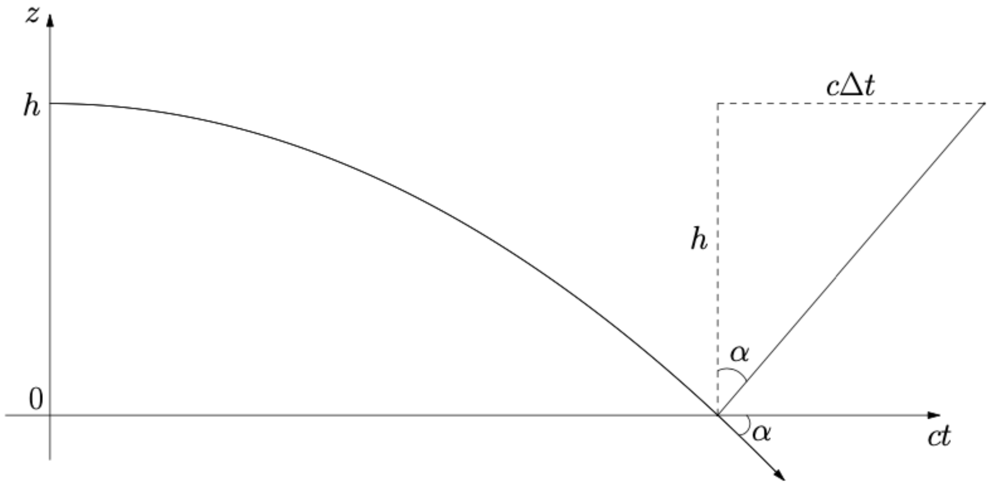
we usually neglect this term: an overall phase,
independent of particle's motion;
but near the earth, time *t* has a slight z-dependence

$$t \to t\left(1 + \frac{gz}{c^2}\right)$$

$$\psi \simeq \exp \frac{1}{\hbar}\left(i(p - mgt)z - imc^2t - i\frac{p^2}{2m}t\right)$$

this is just the free-fall evolution of momentum

Phase of matter waves in space-time



Note: this plot shows a one-dimensional motion (only vertical), not a two-dimensional projectile motion.

Velocity from the slope: $$\frac{v}{c} = -\tan\alpha$$

$$\frac{v}{c} = -\frac{c\Delta t}{h}$$ h cancels with $\Delta t = \frac{gh}{c^2}t$

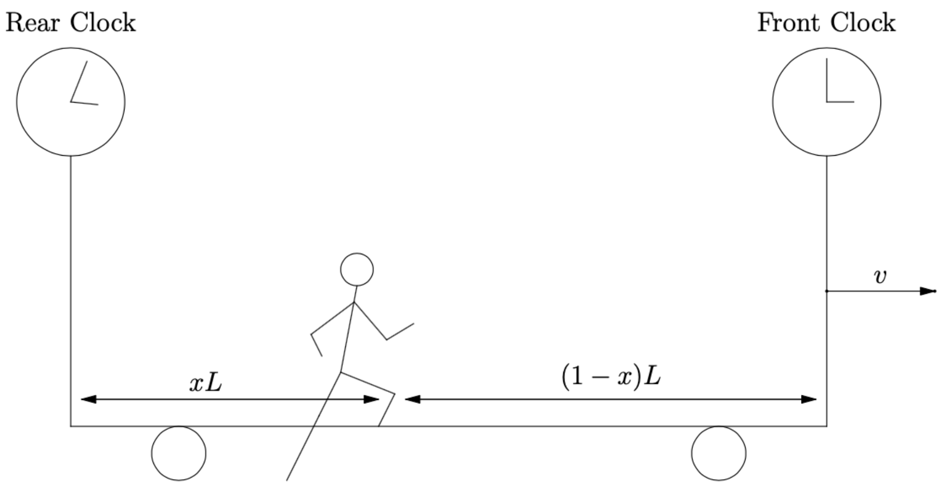
We reproduce the familiar result, $$\boxed{v = -gt}$$

First described by William Wootters (2003)
https://doi.org/10.1023/A:1026052631476

Application: twin paradox (local version)

Rear clock is ahead by

$$\Delta t = \frac{Lv}{c^2}$$



Clocks are synchronized in the train's frame.

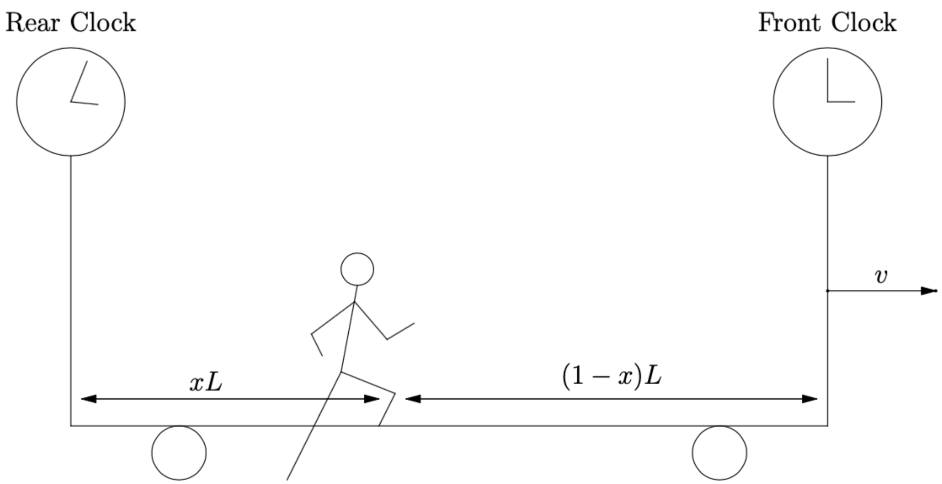
Observer on the ground sees them out of synch.

What happens if the observer decides to get on the train?

Application: twin paradox (local version)

Rear clock is ahead by

$$\Delta t = \frac{Lv}{c^2}$$



duration of the spurt is $t = v/g$ (here "g" is the average acceleration)

$$\Delta t_R = g\,xL\,t/c^2 \qquad \Delta t_F = g(1-x)Lt/c^2$$

$$\Delta t = \Delta t_F + \Delta t_R = \frac{gLt}{c^2} = \frac{Lv}{c^2}$$

After settling on the train,
the observer sees both clocks
show the same time.

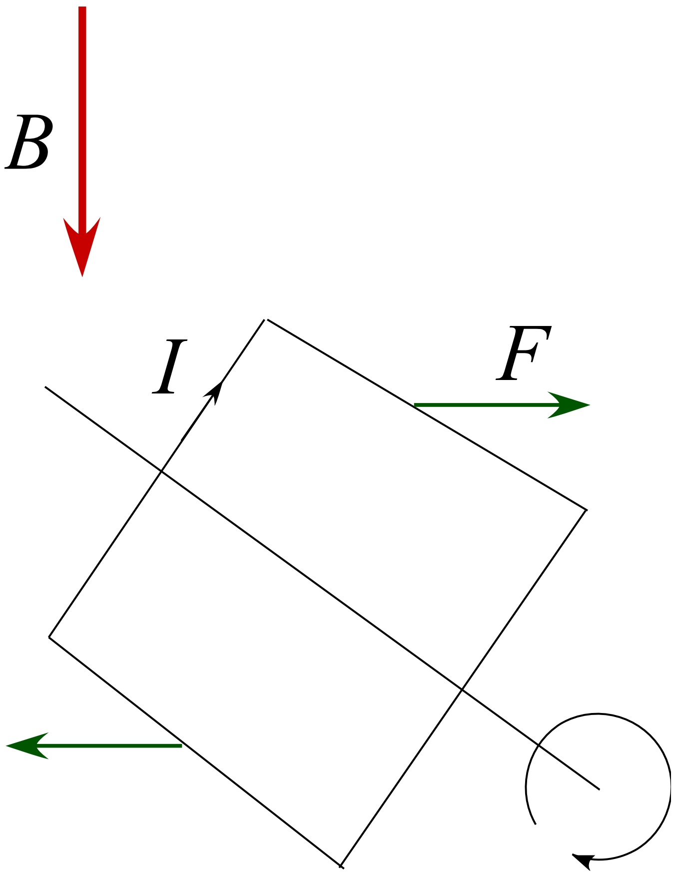
Part 2/2:

Thomas precession

Magnetic moment

A loop with a current
in a magnetic field:

torque acts along
the axis

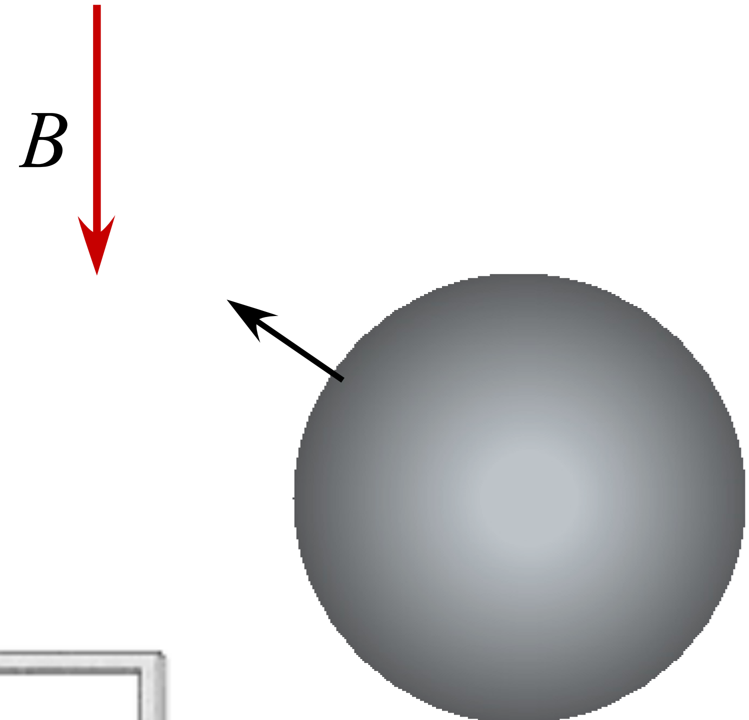


What happens when a charged particle rotates?

Rotation of charges equivalent to currents;

Magnetic field exerts torque

Precession results



Magnetic moments

Spinning charged particle: magnetic dipole

$$\vec{\mu} = g\frac{q}{2m}\vec{s}$$

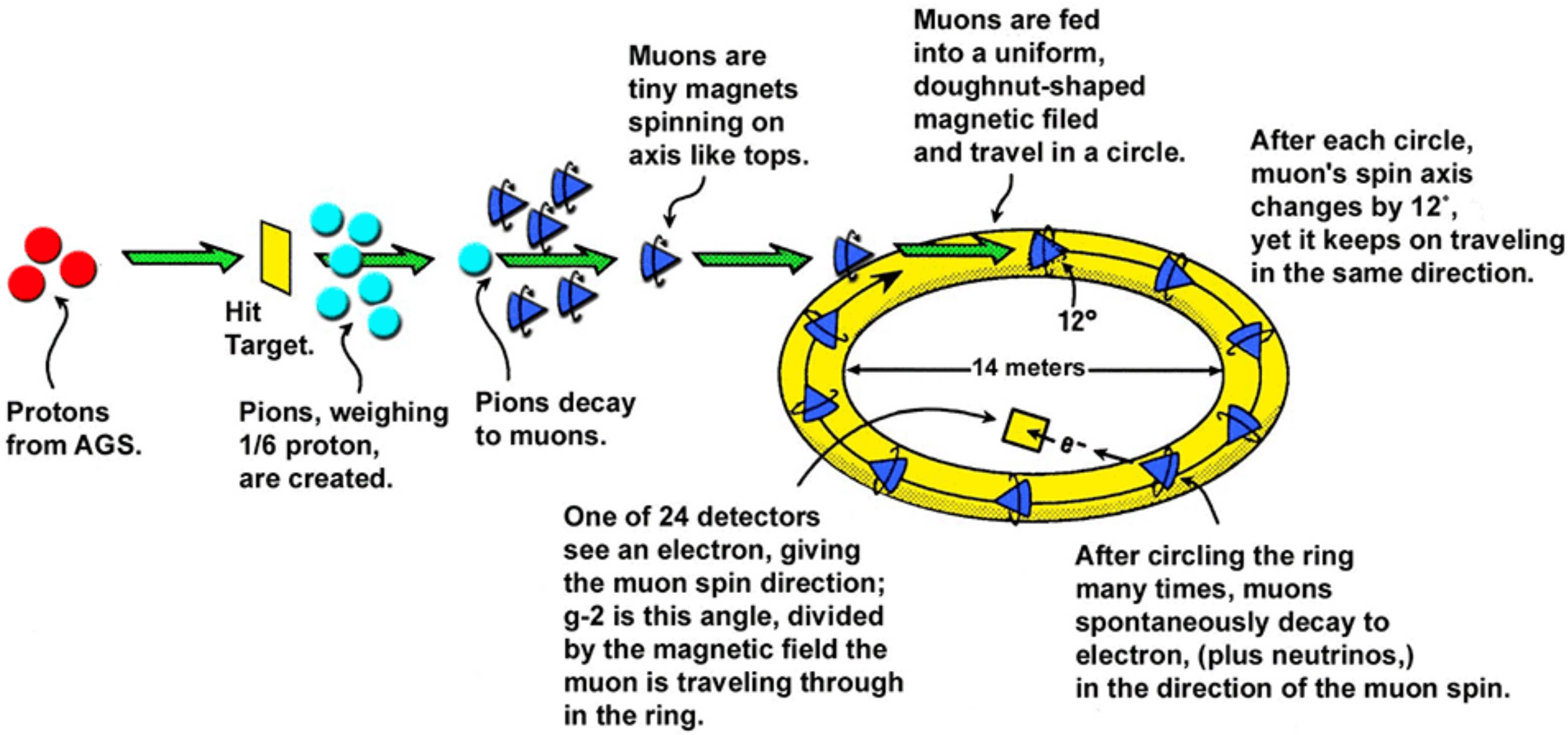
For elementary fermions (electron, muon),
Dirac equation predicts $g = 2$

This is a special value;
if g were exactly 2,
spin and velocity would stay parallel.

If no spin, only orbital angular momentum: g = 1.

Thomas precession for muon g-2: g/2 or g-1?

Muon g-2 experiments: CERN / Brookhaven / Fermilab:



Thomas precession: 2-1, not 2/2

Precession of velocity and of spin

$$\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{qB}{\gamma m}$$

$$\frac{1}{s}\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{qB}{m}\left(\frac{g}{2} - \frac{\gamma - 1}{\gamma}\right)$$

How can there be a g-independent term??

$$\frac{1}{s}\frac{\mathrm{d}s}{\mathrm{d}t} - \frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{qB}{m}\left(\frac{g}{2} - 1\right)$$

$a = \frac{g}{2} - 1 = \frac{g-2}{2}$ anomalous magnetic moment

Thomas precession: 2-1, not 2/2

Precession of velocity and of spin

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$$\frac{1}{s}\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{qB}{m}\left(\frac{g}{2} - \frac{\gamma - 1}{\gamma}\right)$$

How can there be a g-independent term??

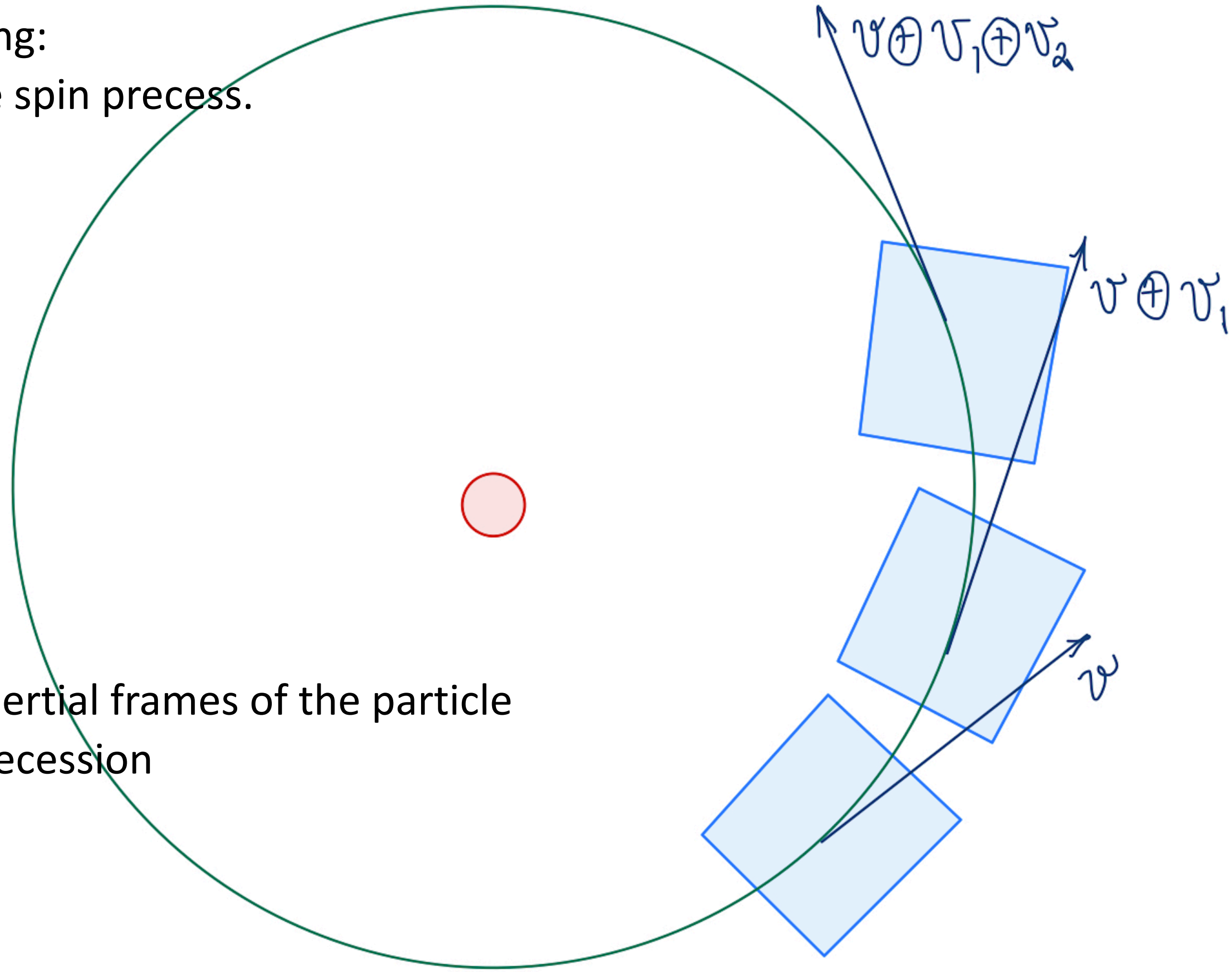
This is the Thomas precession.

$$\frac{1}{s}\frac{\mathrm{d}s}{\mathrm{d}t} - \frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{qB}{m}\left(\frac{g}{2} - 1\right)$$

$a = \frac{g}{2} - 1 = \frac{g-2}{2}$ anomalous magnetic moment

What is Thomas precession?

Spinning electron orbiting a nucleus,
or a muon in a storage ring:
both the velocity and the spin precess.



Subsequent co-moving inertial frames of the particle
undergo a retro-grade precession

Thomas precession: discovery

Ludwik Silberstein (1872-1948) with Niels Bohr

THE THEORY OF RELATIVITY

L. SILBERSTEIN, PH.D.

LECTURER IN NATURAL PHILOSOPHY AT THE UNIVERSITY OF ROME

1914

* In four-dimensional language the case under consideration may be expressed as follows. Call **t** the time-axis in Minkowski's world. Then $L(\mathbf{v}_1)$ will be a rotation in the plane **t**, $\mathbf{v}_1$; similarly, $L(\mathbf{v}_2)$ will be a rotation in the plane **t**, $\mathbf{v}_2$. Now, if $\mathbf{v}_2 \parallel \mathbf{v}_1$, the resultant transformation $L(\mathbf{v}_2)L(\mathbf{v}_1)$ will again be a rotation in the plane **t**, $\mathbf{v}_1$. But if $\mathbf{v}_1$ and $\mathbf{v}_2$ are not parallel, the resultant four-dimensional rotation will also have a component 'round **t**,' *i.e.* $L(\mathbf{v}_2)L(\mathbf{v}_1)$ will involve also a pure (three-dimensional) space-rotation.

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Around the same time:
Borel; Foppl & Daniell; Fermi; Walker; de Sitter
see Krivoruchenko, Успехи физических наук, 2009

Thomas precession: last discovery

Thomas and **Wallace J. Eckart**,
Director of Watson Lab, at work

Llewellyn Hilleth Thomas 1903-1992

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

[SEVENTH SERIES.]

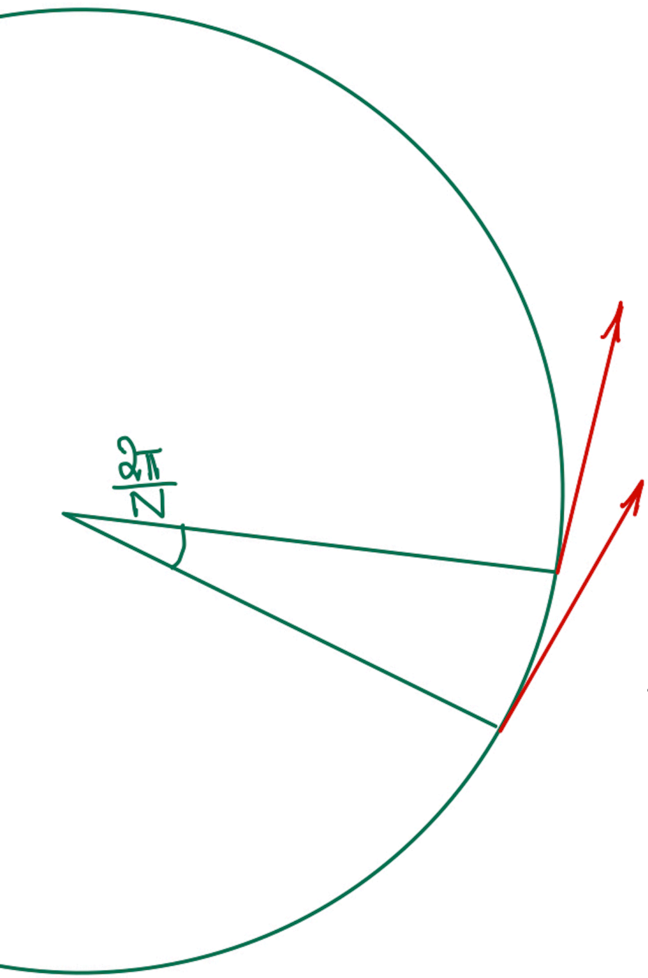
JANUARY 1927.

I. *The Kinematics of an Electron with an Axis.*
By L. H. THOMAS, *B.A.*, *Trinity College, Cambridge* *.

Thomas precession: elementary derivation

Thomas precession: Where is the torque?

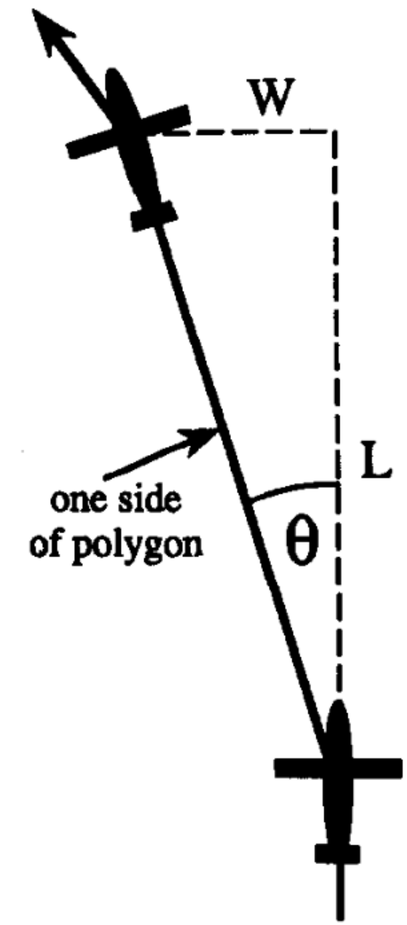
Richard A. Muller



In the comoving frame of the airplane, longitudinal distances are contracted. In its rest frame, airplane rotates by a larger angle,

$$\theta' = \gamma\theta$$

$$\gamma = \frac{1}{\sqrt{1 - v^2}}$$



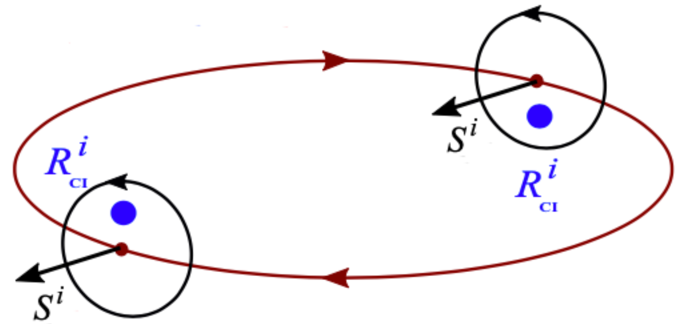
For the precession to have the same frequency as the velocity, the rest frame must be counter-rotating with

$$\omega_{\text{Thomas}} = (\gamma - 1)\omega$$

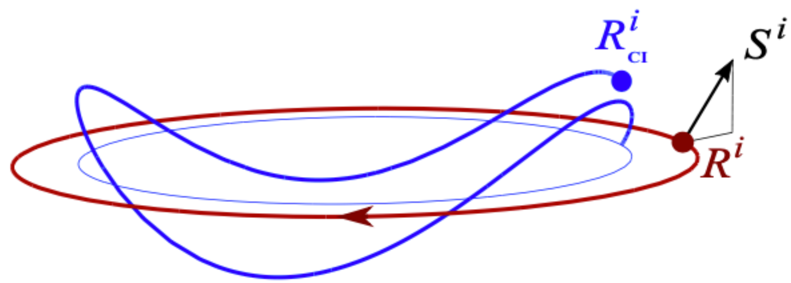
Thomas precession: what happens with angular momentum?

work in progress with Andrei Zelnikov

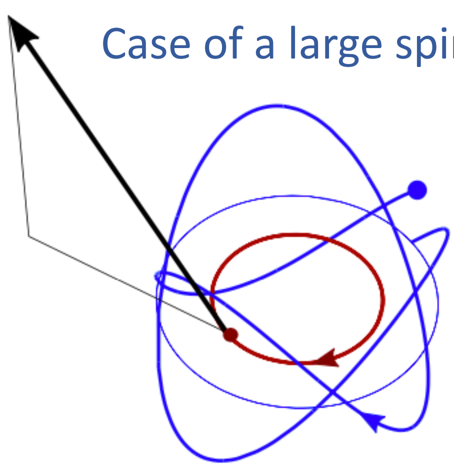
Displacement of the center of mass with respect to the center of inertia



Typical trajectory of the center of inertia



Case of a large spin



Summary

— Free fall explained by the evolution rate of matter waves at various altitudes.

— Analogy: ocean waves near beaches.

— Take-home message: non-relativistic matter follows geodesics because of the phase evolution of its wave function.

— The role of the mc^2t term in the phase: discussed previously, Sagnac effect:

Am. J. Phys. 58 (7), July 1990

Relativistic aspects of nonrelativistic quantum mechanics

Dennis Dieks[a] and Gerard Nienhuis[b]

— Thomas precession: not only does the spin precess but also the orbit is distorted;

— Hypothesis: chaotic effects may result for large spin