Simulations of Electroweak Dumbbells and Symmetry Breaking

Teerthal Patel

8th May, 2024

Generation, evolution, and observations of cosmological magnetic fields

Bernoulli Center

Work with Tanmay Vachaspati, Paul Saffin & Zong-Gang Mou

ASU Department of Physics
ARIZONA STATE UNIVERSITY

Outline

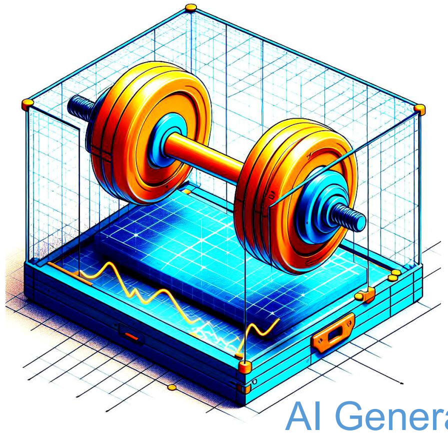
Part I

Relaxed configurations of electroweak dumbbells

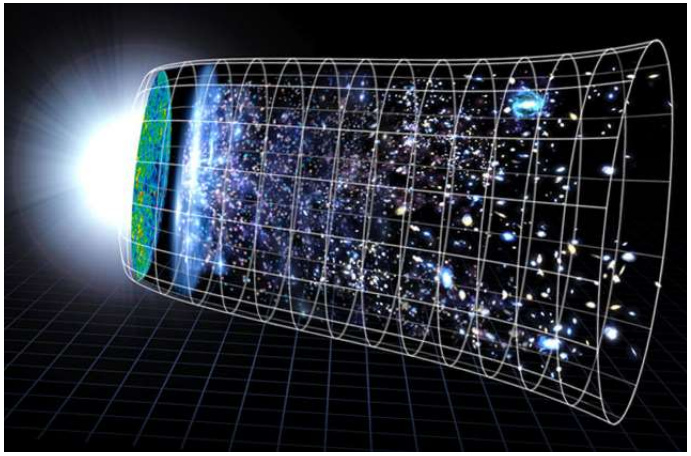
T.P & Vachaspati, T. (2023), *PRD*

Annihilation dynamics of electroweak dumbbells

T.P & Vachaspati, T. (2024), *JHEP*



AI Generated



Part II

Distribution of monopole-antimonopole pairs.

T.P & Vachaspati, T. (2022), *JCAP*

Cosmological magnetogenesis from EWSB

Ongoing with Tanmay, Paul and Mou

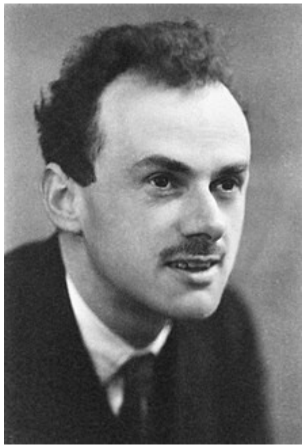
Image Credits: NASA/WMAP Science Team

Background

Why monopoles?

Elegant symmetrization of Maxwell's theory

Generic prediction in GUT theories



Originates all the way back to Dirac in 1931

The hunt continues

Background

Confined Monopoles / Dumbbells

G. ’t Hooft (1974), Nucl. Phys. A. M. Polyakov (1974), JETP Lett.	‘t Hooft-Polyakov Monopoles
H. B. Nielsen and P. Olesen (1973), Nucl. Phys. B	Infinite String
Y. Nambu (1977), Nucl. Phys. B	Dumbbells in the Electroweak theory

The Weinberg-Salam theory of electromagnetic and weak interactions admits classical configurations in which a pair of magnetic monopoles is bound by a flux string of the Z^0 field. They give rise to Regge trajectories of excitations with a mass scale in the TeV range.

Dumbbells undergoing relativistic rotation could be stable enough to be produced in accelerators

PART I : Dumbbells

In our simulations, we studied the monopole-antimonopole configurations in the electroweak theory (minus fermions)

Relaxed configurations of electroweak dumbbells

T.P & Vachaspati, T. (2023), *PRD*

Annihilation dynamics of electroweak dumbbells

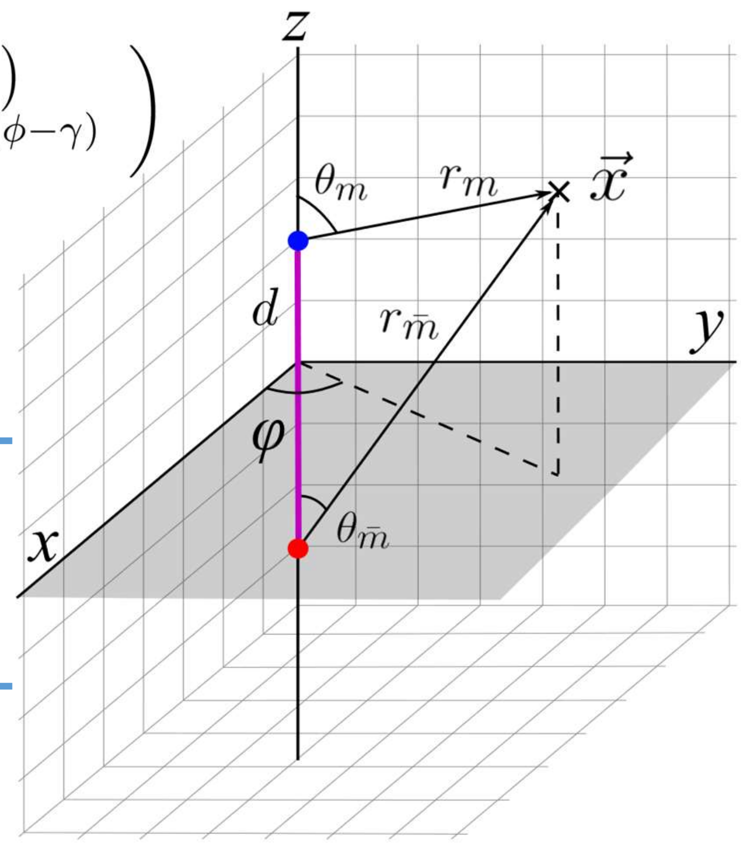
T.P & Vachaspati, T. (2024), *JHEP*

Dumbbell Configuration

$$\hat{\Phi}_{m\bar{m}}(\gamma) = \begin{pmatrix} \sin\left(\frac{\theta_m}{2}\right)\sin\left(\frac{\theta_{\bar{m}}}{2}\right)e^{i\gamma} + \cos\left(\frac{\theta_m}{2}\right)\cos\left(\frac{\theta_{\bar{m}}}{2}\right) \\ \sin\left(\frac{\theta_m}{2}\right)\cos\left(\frac{\theta_{\bar{m}}}{2}\right)e^{i\phi} - \cos\left(\frac{\theta_m}{2}\right)\sin\left(\frac{\theta_{\bar{m}}}{2}\right)e^{i(\phi-\gamma)} \end{pmatrix}$$

Vachaspati, T. and Field, G.B., 1994.
Electroweak string configurations with baryon number. Physical review letters, 73(3), p.373.

2 Parameters: Twist & Length



Goal: Spatial configuration of fields

$$\Phi = |\Phi|\hat{\Phi}_{m\bar{m}}$$

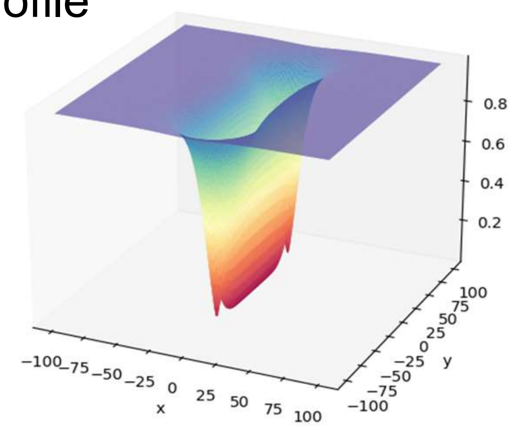
$|\Phi(\vec{x})|$ $\quad W_i^a(\vec{x})$ $\quad Y_i^a(\vec{x})$

Magnetic field definition (Vachaspati, T., 1991)

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\frac{2\sin\theta_w}{g\eta^2}(\partial_\mu\Phi^\dagger\partial_\nu\Phi - \partial_\nu\Phi^\dagger\partial_\mu\Phi)$$

Relaxation Outline

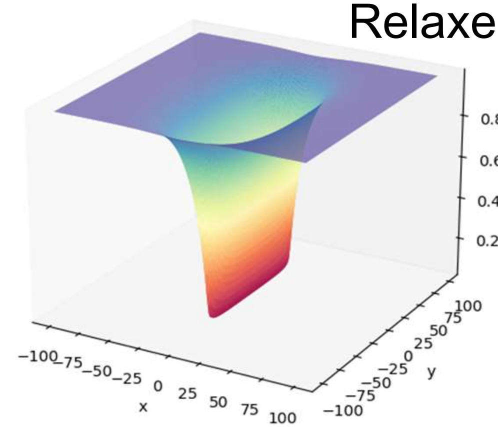
Guess Higgs profile



Numerical Relaxation

$$\Phi = |\Phi|\hat{\Phi}_{m\bar{m}}$$

Relaxed profile



13 Real variables $|\Phi| \quad W_i^a \quad Y_i$

Units

Time (t) & Space(x)	:	η^{-1}	3.8×10^{-27}s
Energy	:	η	174 GeV
Magnetic Field	:	η^2	1.5×10^{20} T

EOMs from

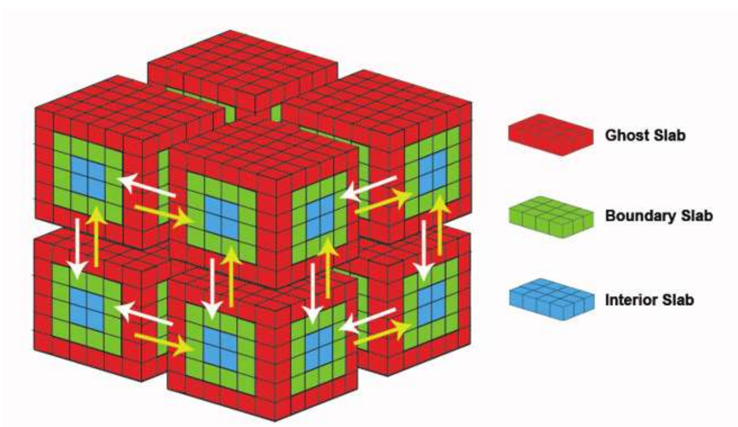
$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W^{a\,\mu\nu} - \frac{1}{4}Y_{\mu\nu}Y^{\mu\nu} + |D_\mu\Phi|^2 - \lambda(|\Phi|^2 - \eta^2)^2$$

SM $\quad g = 0.65,\ \sin^2\theta_w = 0.22\,,\ g' = g\tan\theta_w\,,\ \lambda = 0.129$

Relaxation Algorithm

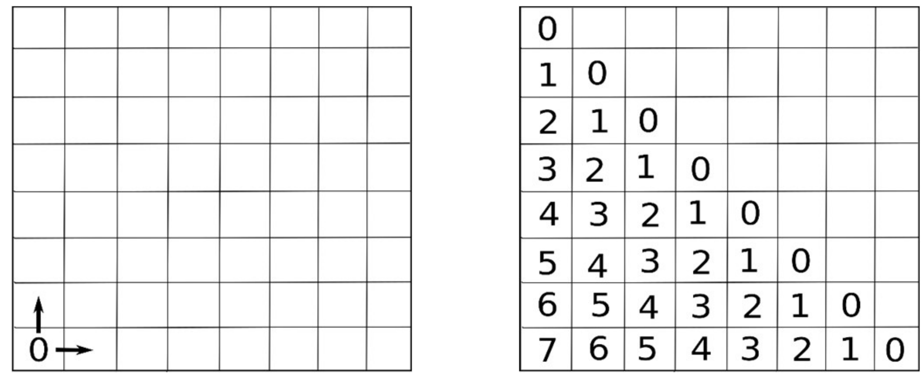
Parallelization

- Divide lattice in sub-domains
- Assign a subdomain to a unique CPU processor
- Compute update and exchange boundary data



Domain decomposition

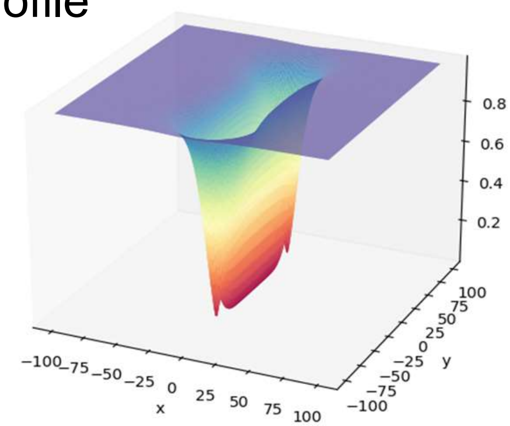
Asynchronous Parallelization



- Gauss-Seidel relaxation requires updated field values at neighboring lattice points
- Asynchronous parallelization scheme developed by Ayush Saurabh

Relaxation Outline

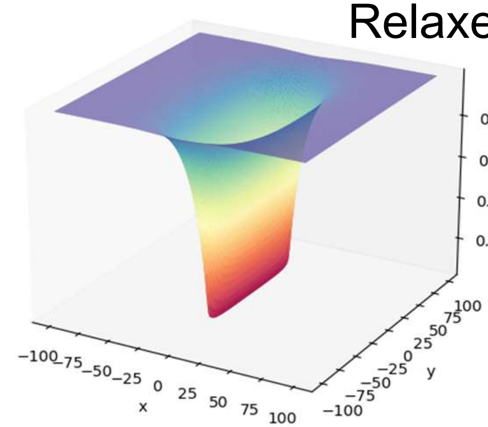
Guess Higgs profile



Numerical Relaxation

$$\Phi = |\Phi|\hat{\Phi}_{m\bar{m}}$$

Relaxed profile



2 Parameters: Twist & Length

13 Real variables $|\Phi| \quad W_i^a \quad Y_i$

Units

Time (t) & Space(x) : η^{-1} 3.8×10^{-27}s

Energy : η 174 GeV

Magnetic Field : η^2 1.5×10^{20} T

EOMs from

$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W^{a\,\mu\nu} - \frac{1}{4}Y_{\mu\nu}Y^{\mu\nu} + |D_\mu\Phi|^2 - \lambda(|\Phi|^2 - \eta^2)^2$$

SM $g = 0.65,\ \sin^2\theta_w = 0.22\,,\ g' = g\tan\theta_w\,,\ \lambda = 0.129$

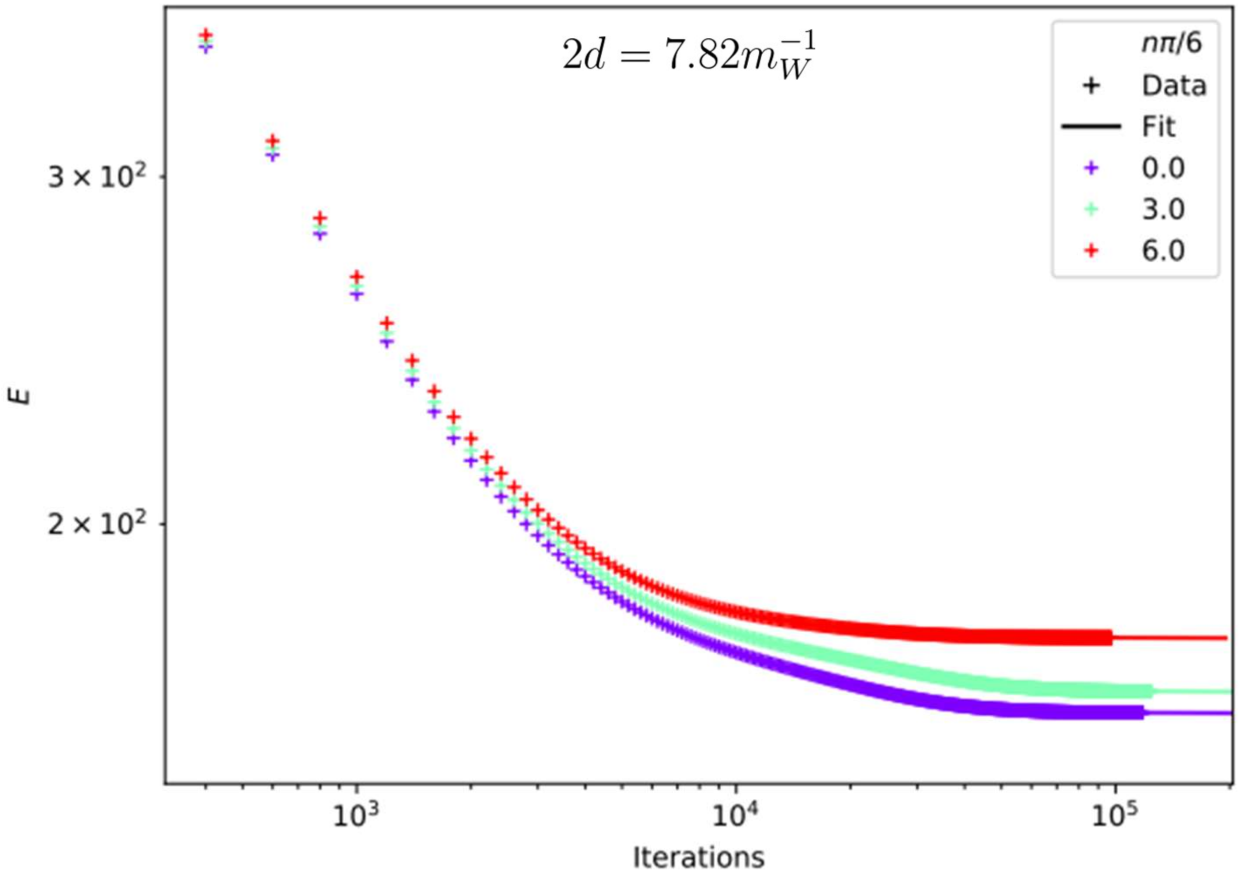
Numerical Relaxation

Relaxed Until

$$\frac{\Delta E}{E} = 10^{-6}$$

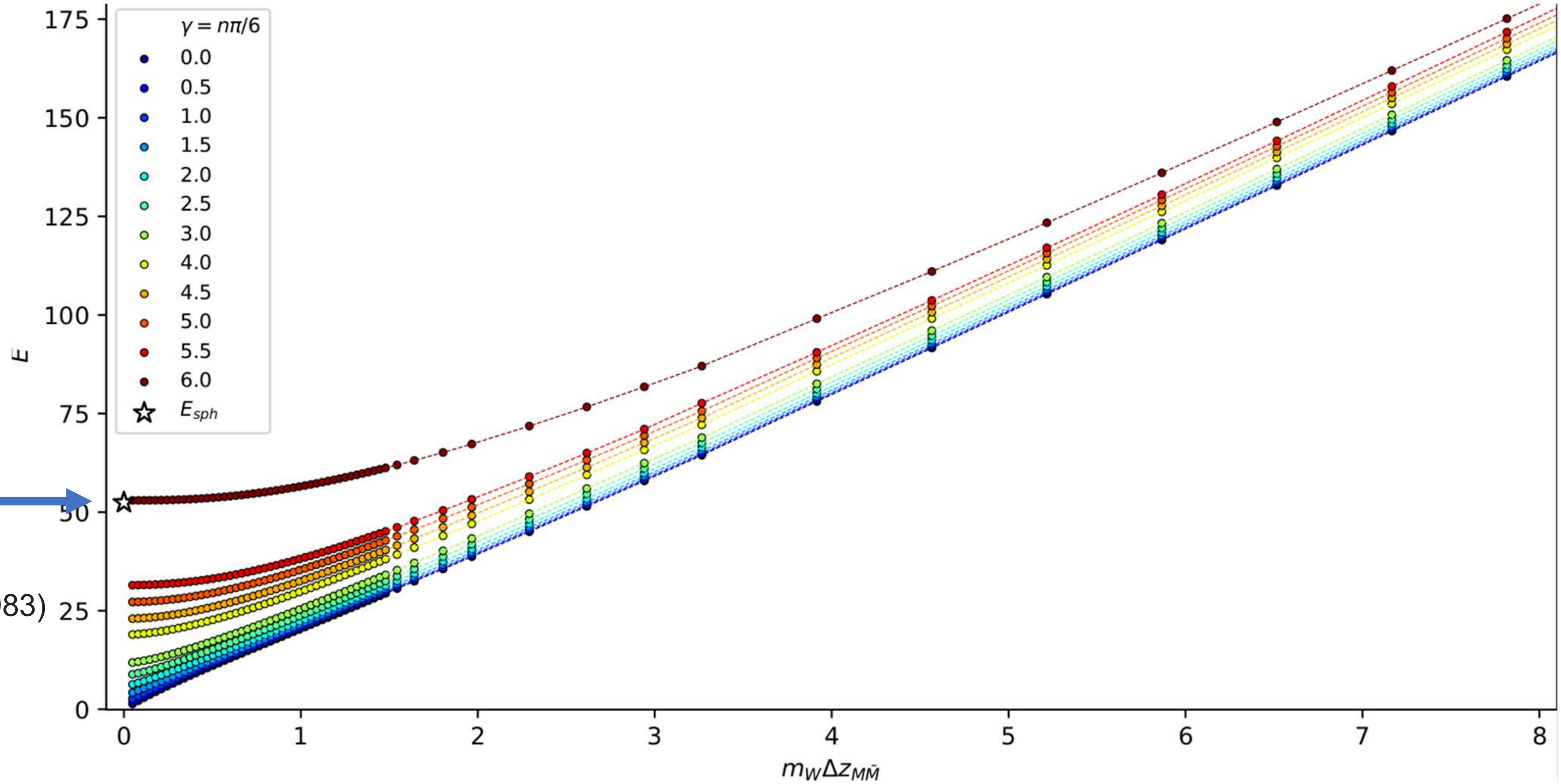
$$N^3 = (740)^3$$

$$m_W^{-1} = \sqrt{2}\,\eta^{-1}/g \approx 44\,\delta$$



$(m_W)^{-1} \sim$ Monopole width

Results: Energy v separation

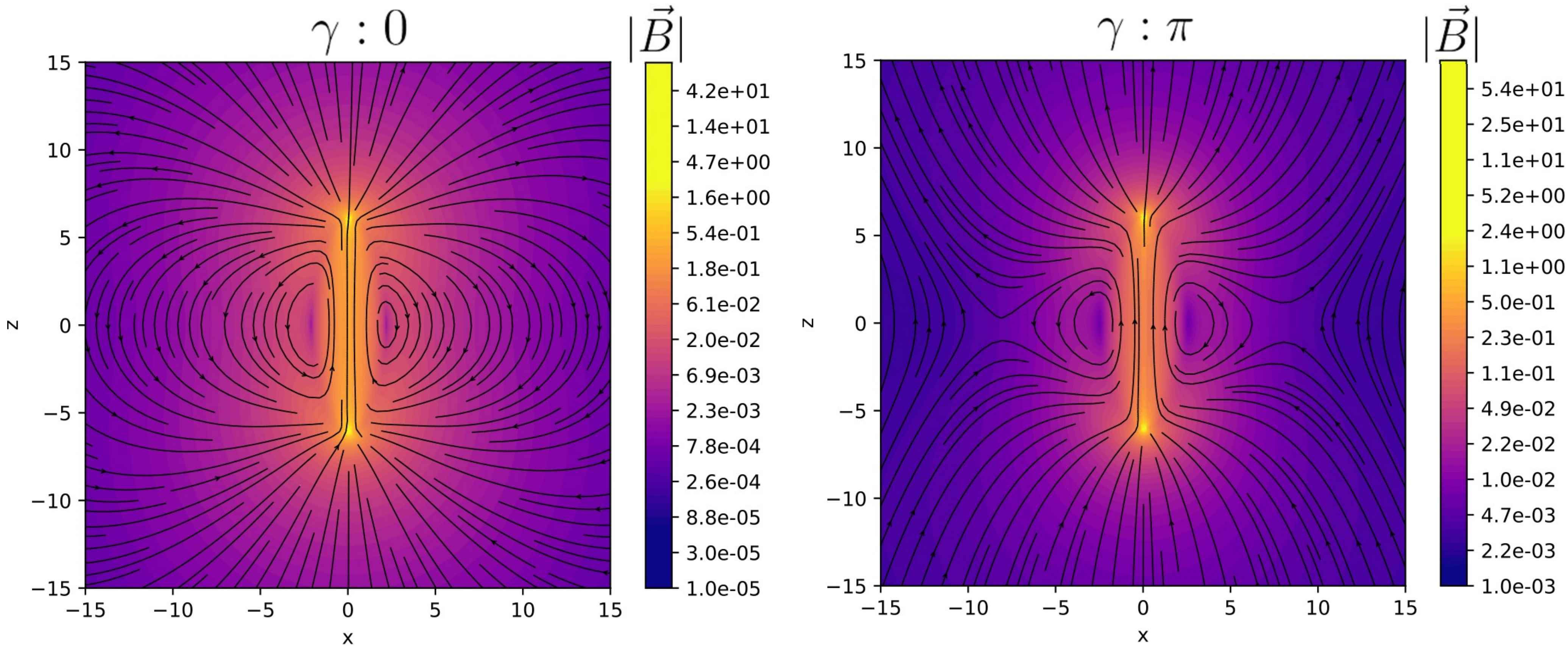


EW Sphaleron

Manton, N. S. (1983)
PRD

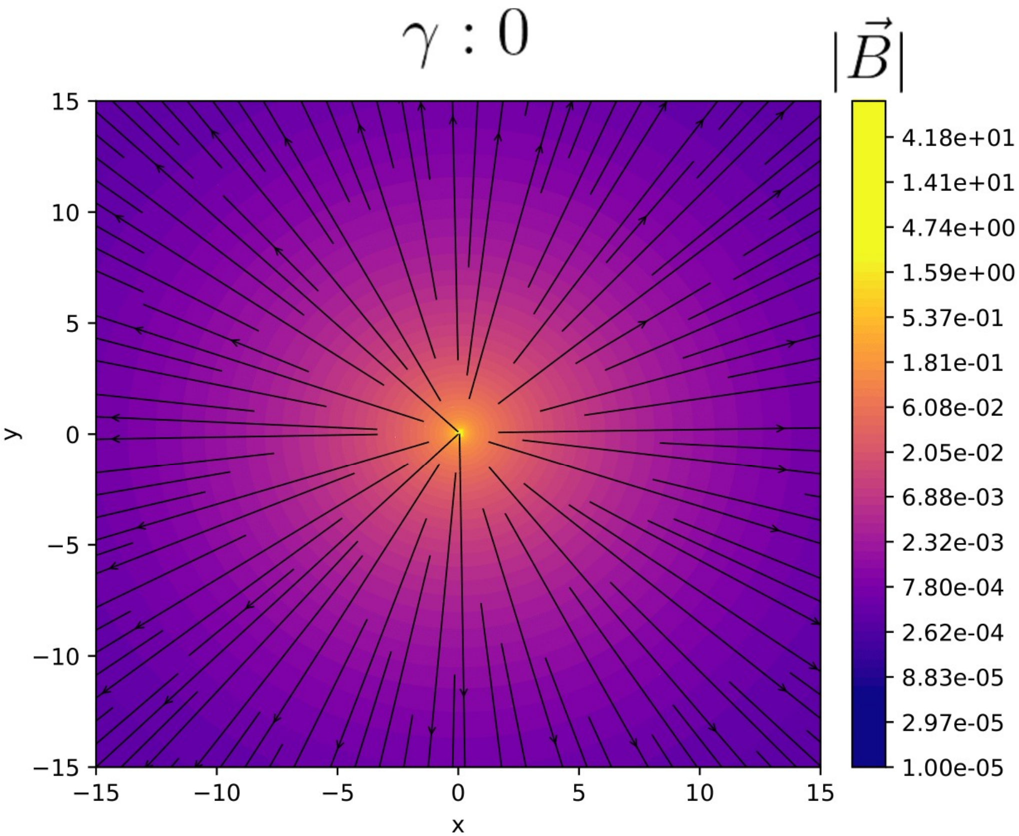
$(m_W)^{-1} \sim \text{Monopole width}$

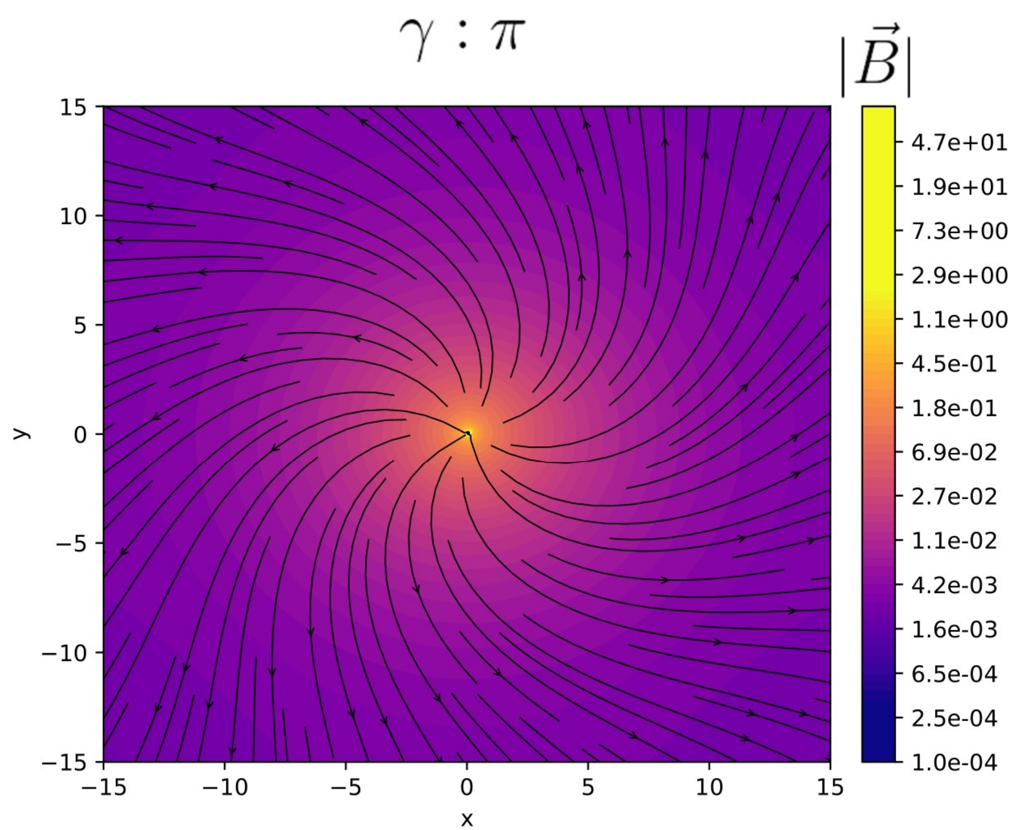
Results: Magnetic fields



$$B_r\Big|_{r \gg z_m} = \kappa(1-\cos\gamma)\frac{\cos\theta}{r^2}$$

Results: Magnetic fields





$$B_\phi|_{r \gg z_m} = -\kappa z_m \sin\gamma \frac{\sin\theta\cos\theta}{r^3}$$

PART I: Dynamics

With the numerically relaxed field configuration, we can now simulate the dynamics

Relaxed configurations of electroweak dumbbells

T.P & Vachaspati, T. (2023), *PRD*

Annihilation dynamics of electroweak dumbbells

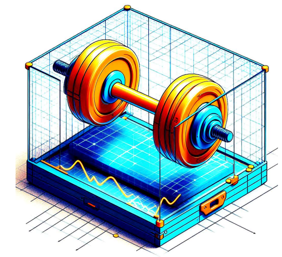
T.P & Vachaspati, T. (2024), *JHEP*

Dumbbell Dynamics

$$\partial_0^2\Phi = D_iD_i\Phi - 2\lambda(|\Phi^2| - \eta^2)\Phi$$
$$\partial_0^2 Y_i = -\partial_j Y_{ij} + g'\,\mathrm{Im}[\Phi^\dagger(D_i\Phi)]$$
$$\partial_0^2 W_i^a = -\partial_j W_{ij}^a - g\epsilon^{abc}W_j^b W_{ij}^c + g\mathrm{Im}[\Phi^\dagger\sigma^a(D_i\Phi)]$$

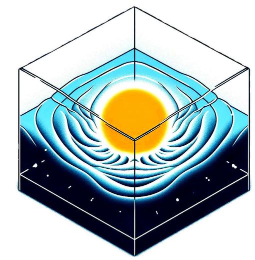
} EOMs

Initial: Relaxed configuration



↓ Dirichlet Boundaries

Simulate evolution



Numerical Relativity method
(PDE approach from Paul's talk)

$$\Xi = \partial_i Y_i \qquad \Gamma^a = \partial_i W_i^a$$

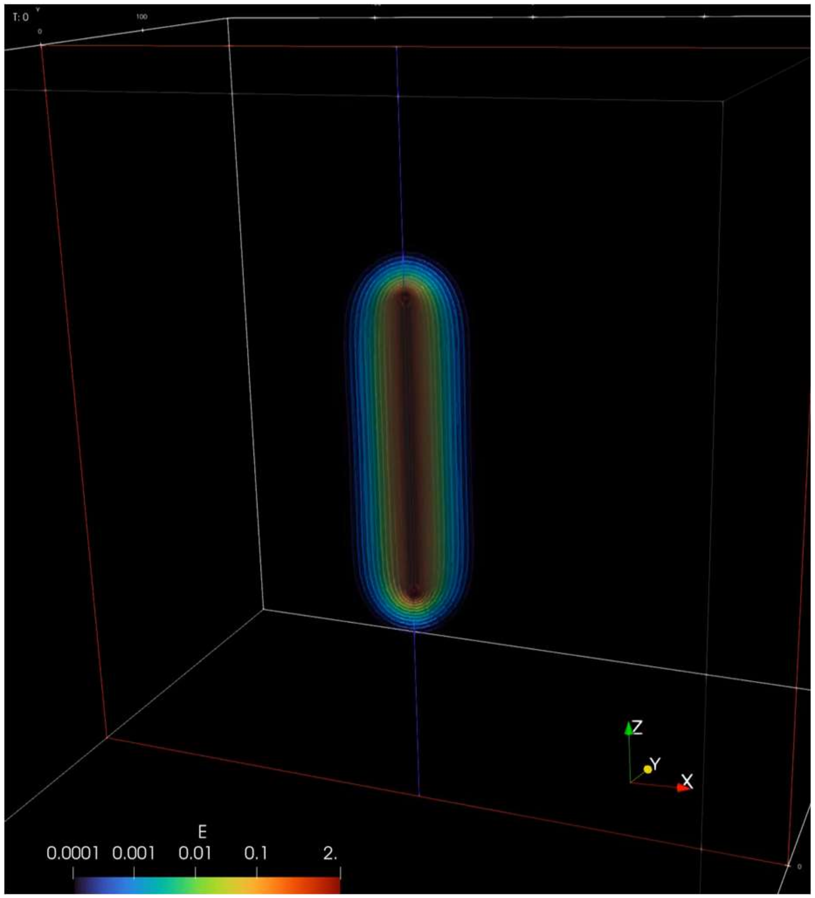
$$\partial_0\Xi = \partial_i Y_{0i} - g_p^2\{\partial_i Y_{0i} - g'\,\mathrm{Im}[\Phi^\dagger(\partial_0\Phi)]\}$$
$$\partial_0\Gamma^a = \partial_i W_{0i}^a - g_p^2\{\partial_i W_{0i}^a + g\epsilon^{abc}W_i^b\partial_0 W_i^c - g\mathrm{Im}[\Phi^\dagger\sigma^a(\partial_0\Phi)]\}$$

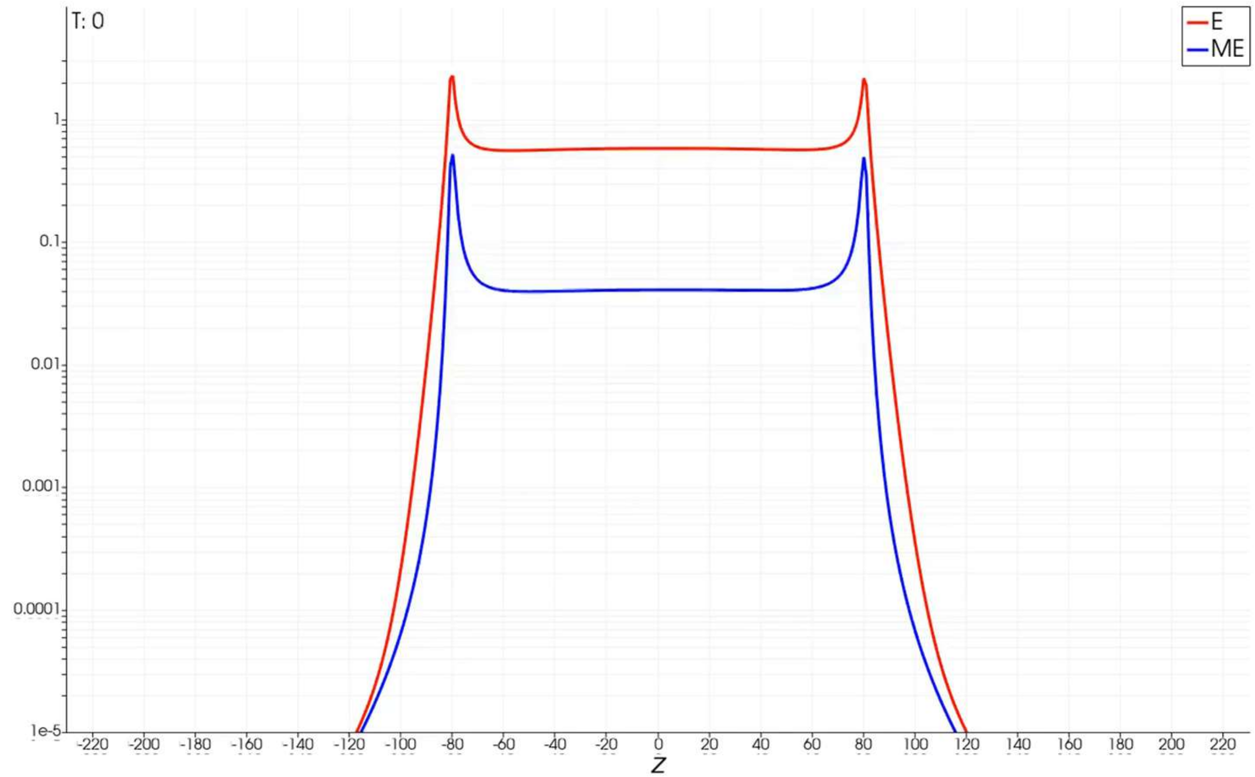
← Gauss constraints

g_p :Numerical stability parameter

Vachaspati '15
Baumgarte & Shapiro

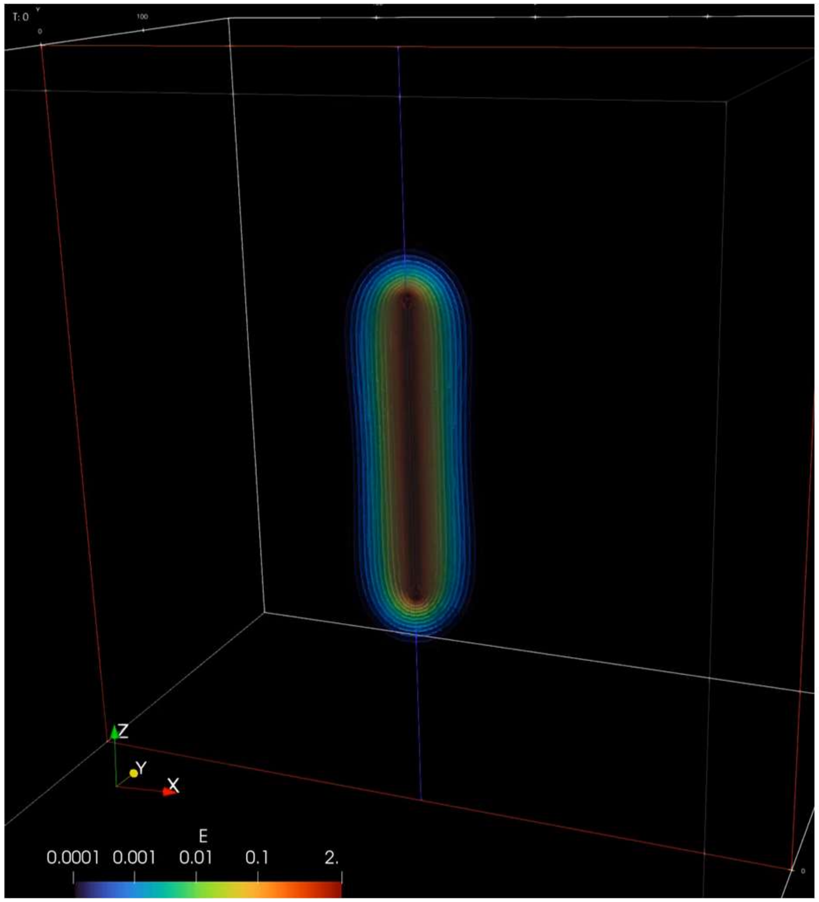
Dumbbell Annihilation

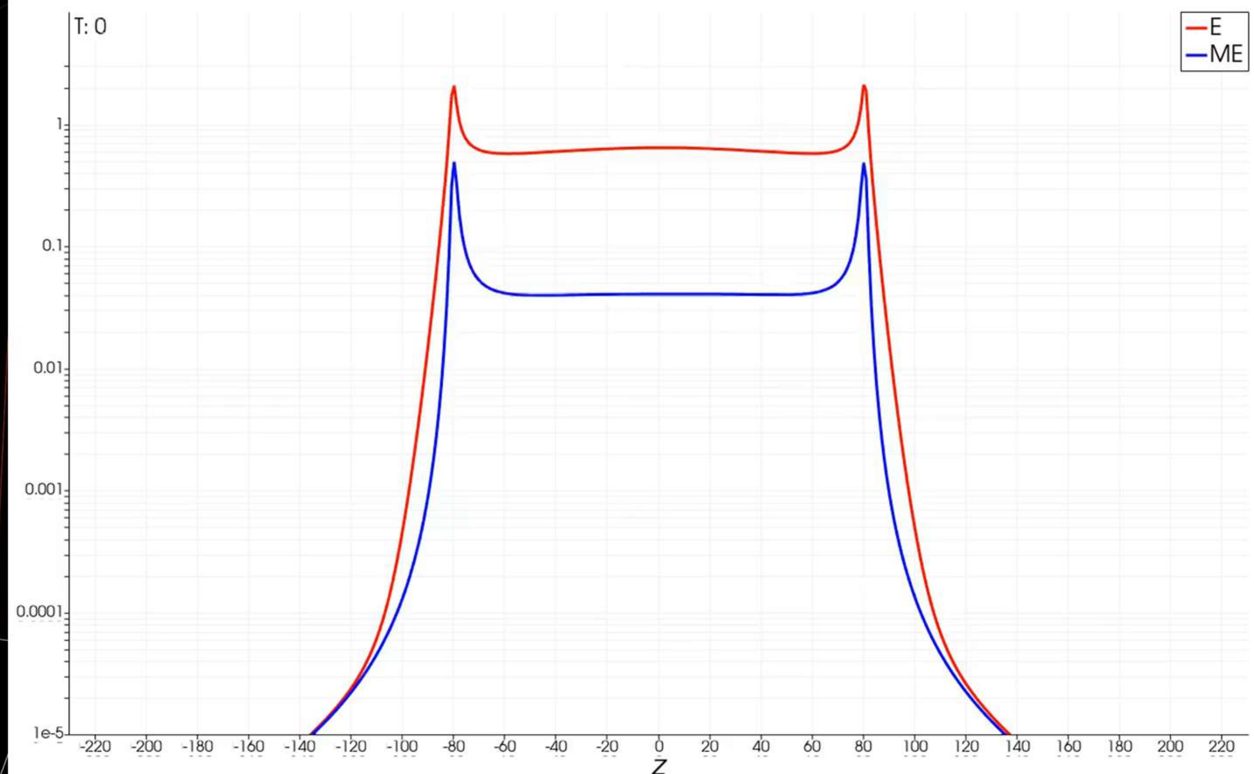




Energy density isosurfaces $\gamma = 0$

Dumbbell Annihilation

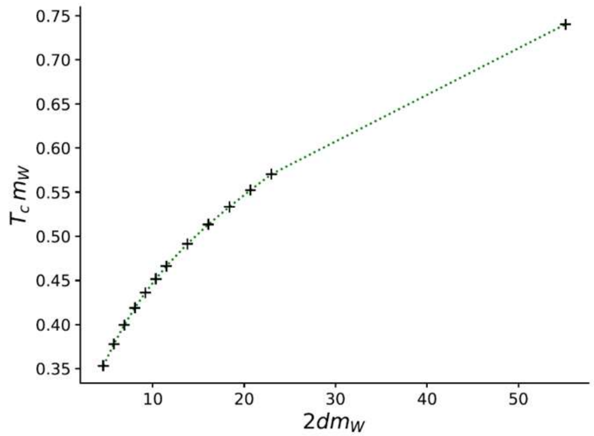


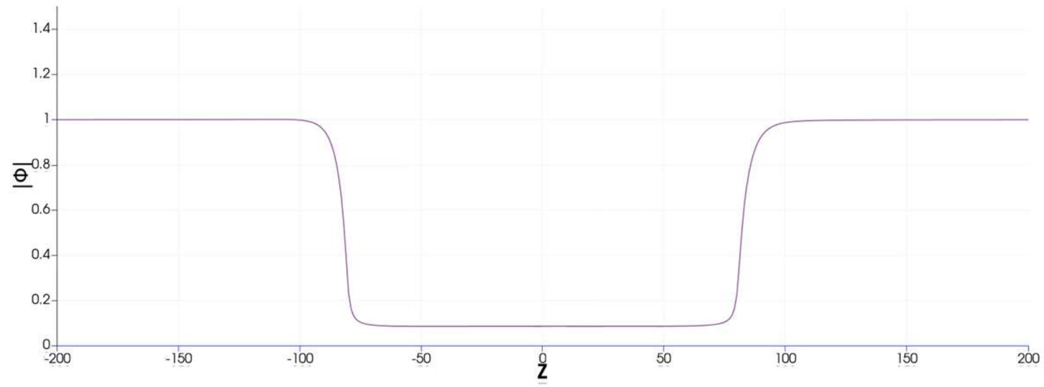


Energy density isosurfaces $\gamma = \pi$

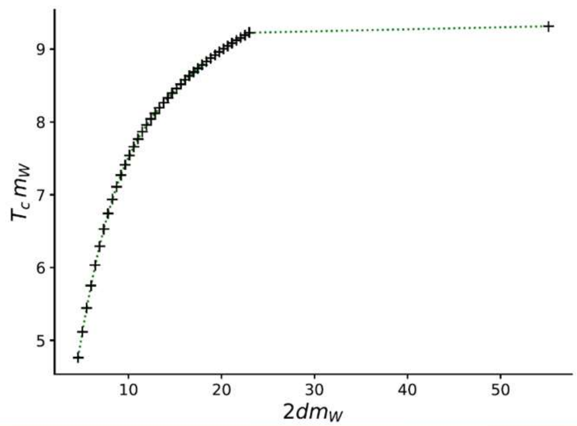
Separation-Lifetime Relation

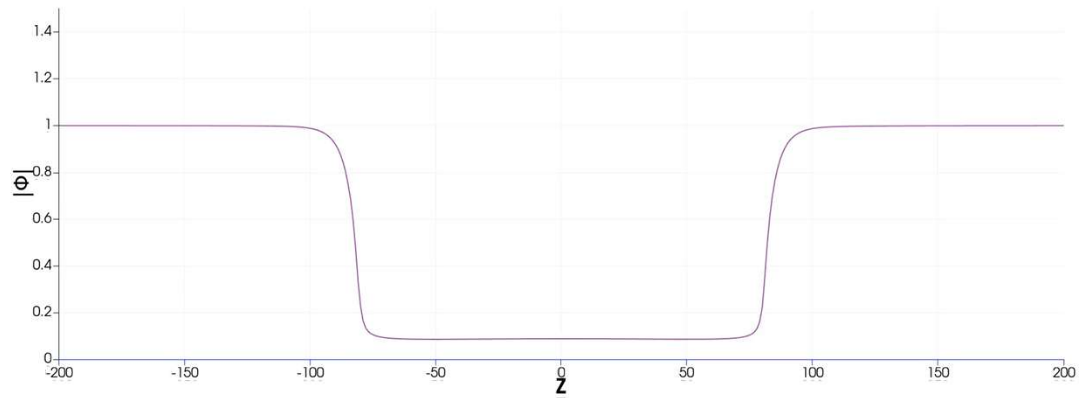
$\gamma = 0$



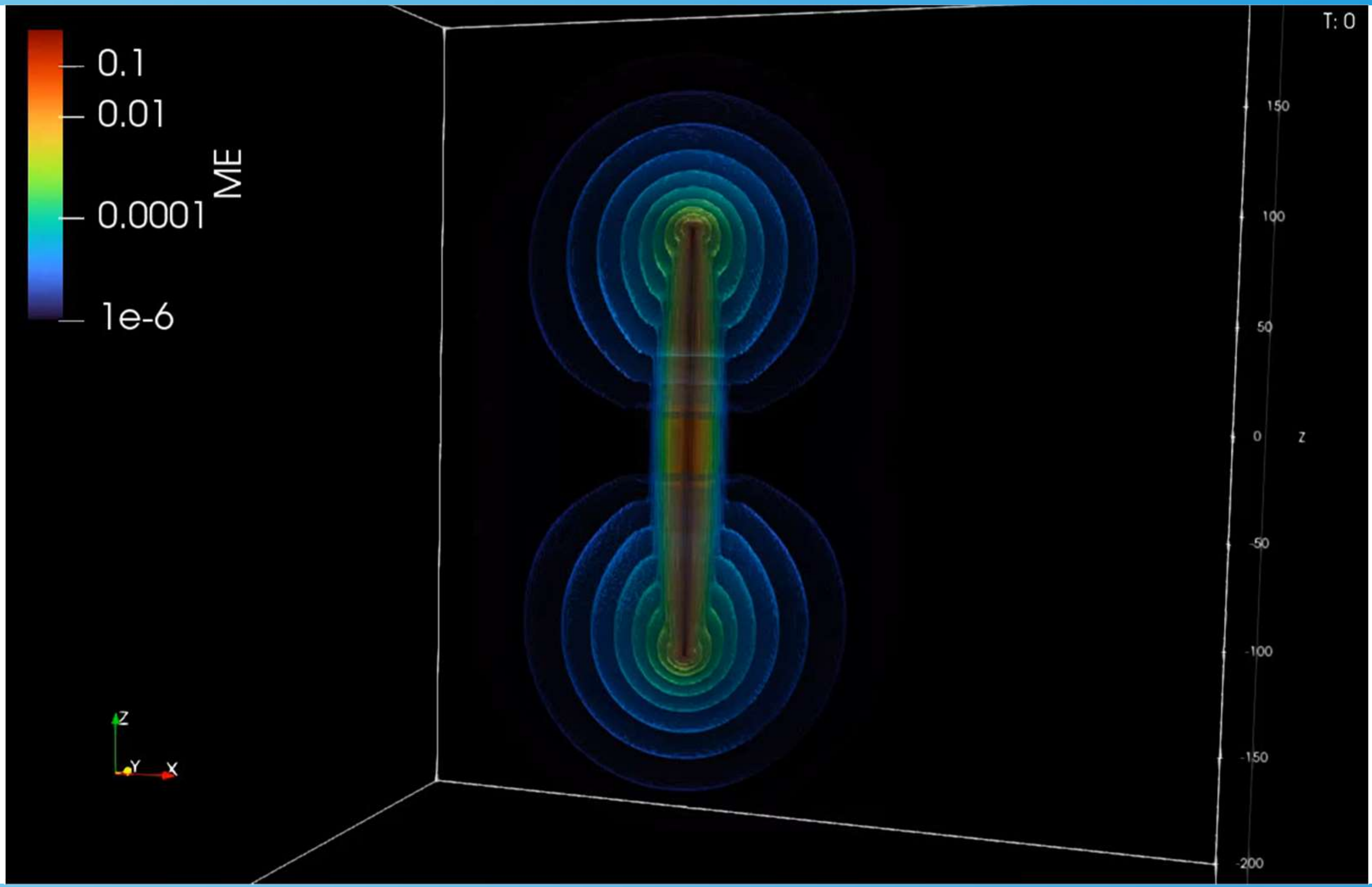


$\gamma = \pi$



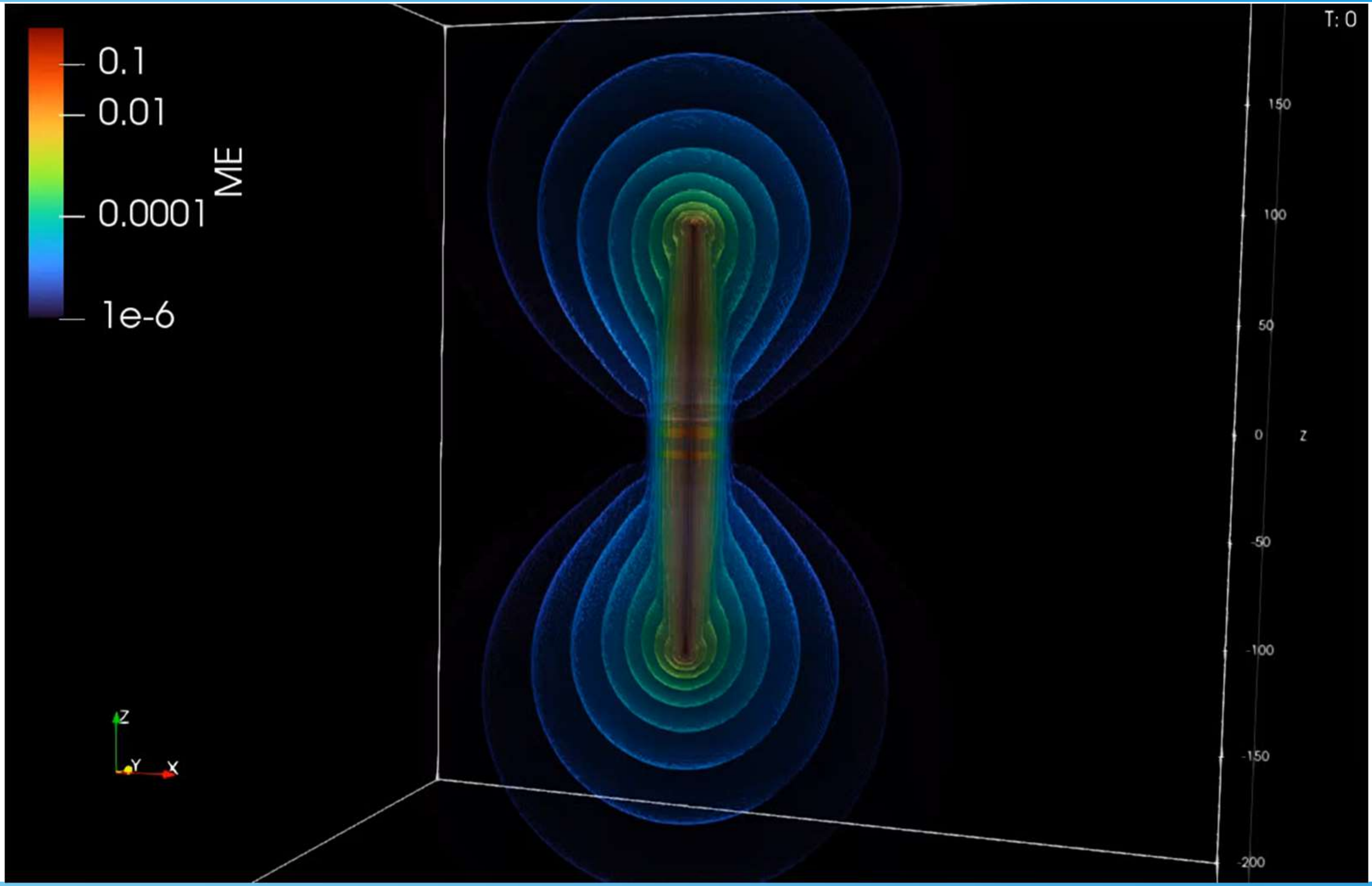


Magnetic relics



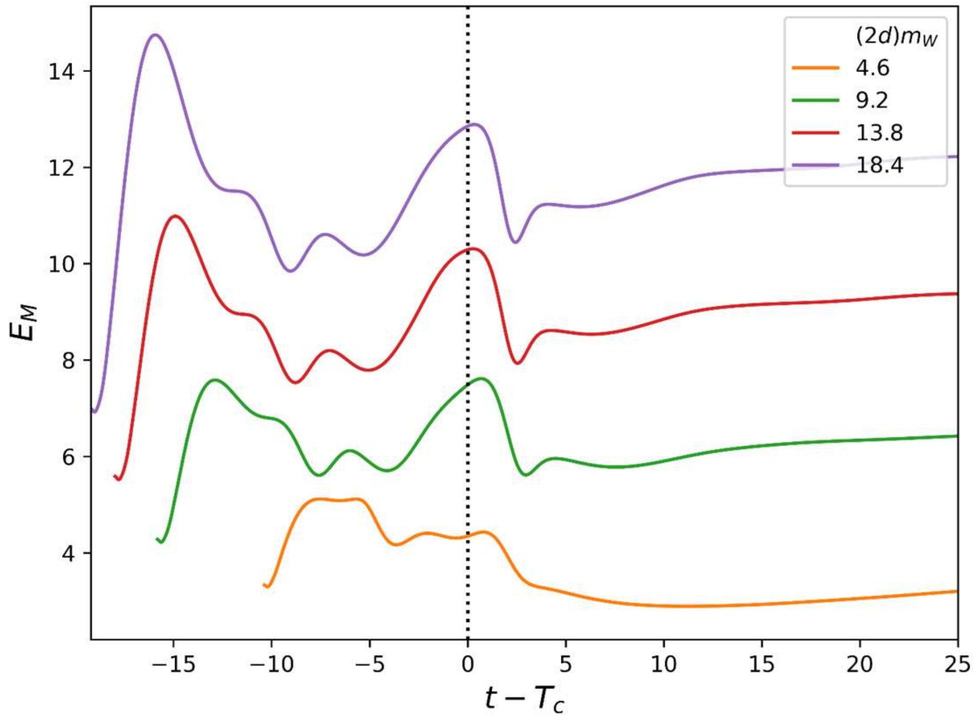
$\gamma = 0$

Magnetic relics

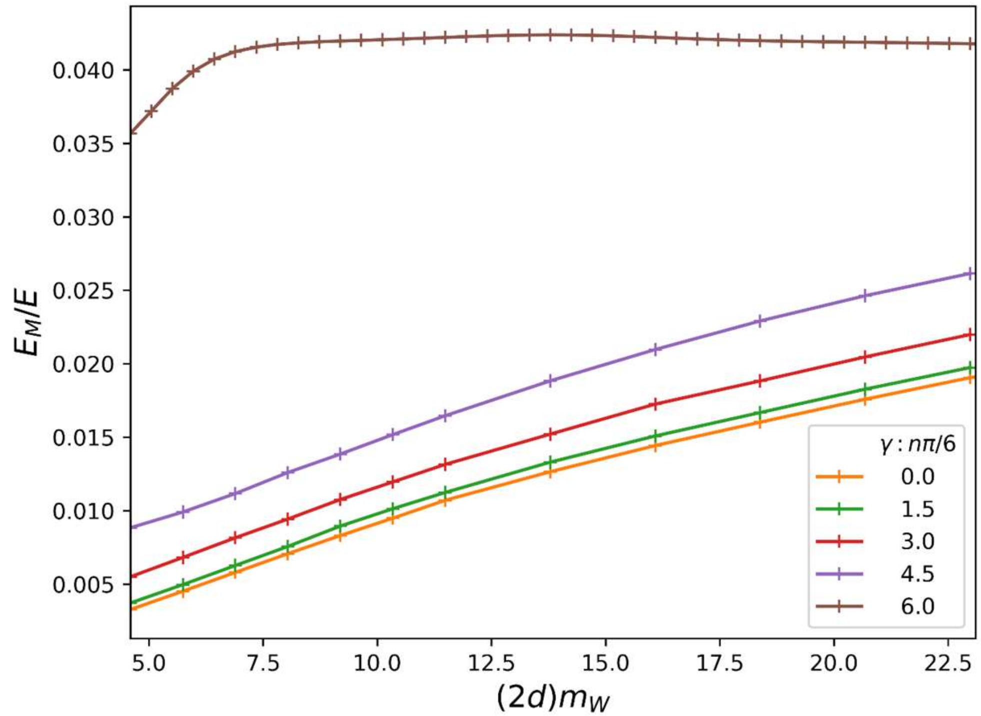


Magnetic relics

Magnetic energy over time



Relic ME after annihilation



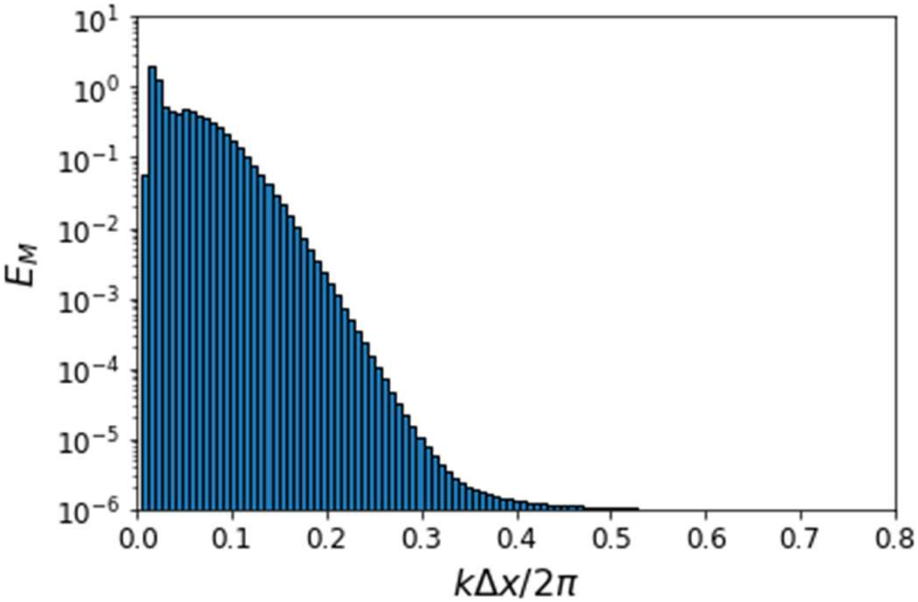
$\gamma = \pi$

Part I: Summary

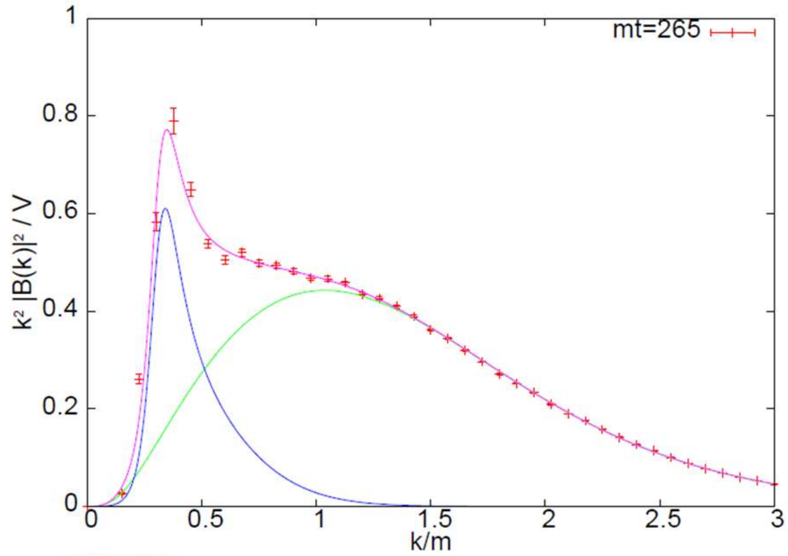
- Resolved the static dumbbell configurations
- Dumbbell annihilation : Lifetimes and magnetic relics
- Maximally twisted dumbbells form sphaleron-like configurations before decay
- Chains of twisted dumbbells could have interesting cosmological consequences (Talk by Tanmay)

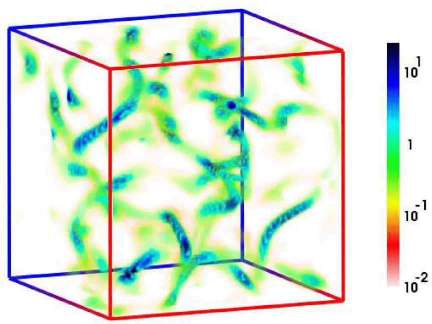
Dumbbell is unstable and we can’t simulate rotation before it decays!

EWSB Magnetogenesis Simulations



Zhang, Y., Vachaspati, T., & Ferrer, F. (2019). *PRD*





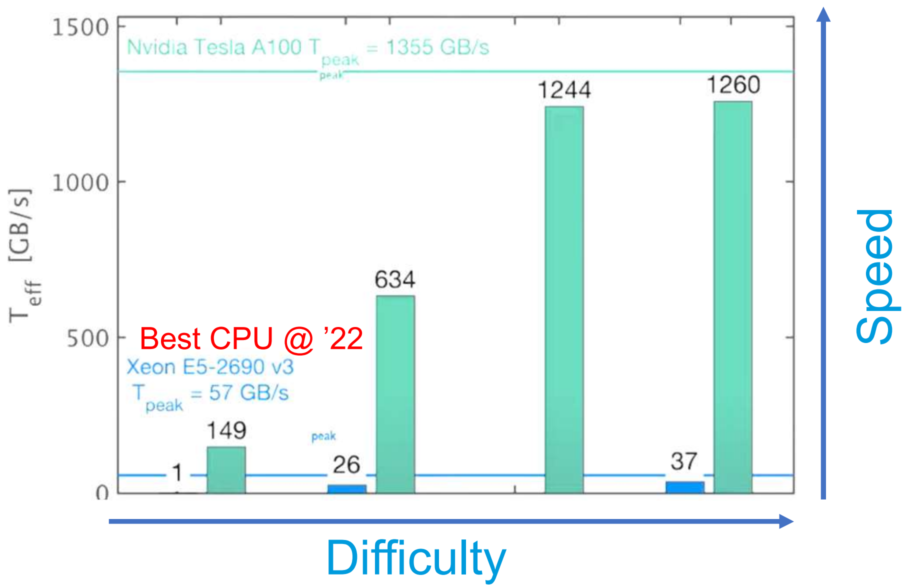
Diaz-Gil, A., Garcia-Bellido, J., Perez, M. G., & Gonzalez-Arroyo, A. (2008). *JHEP*

Our EWSB simulations

w TV, Paul Saffin & Zong-Gang Mou

- Developed a GPU code for studying EWSB (Coming to Github Soon)
  - PDE Approach
  - Peak scale in large lattices
  - CP violation: Helical magnetic fields

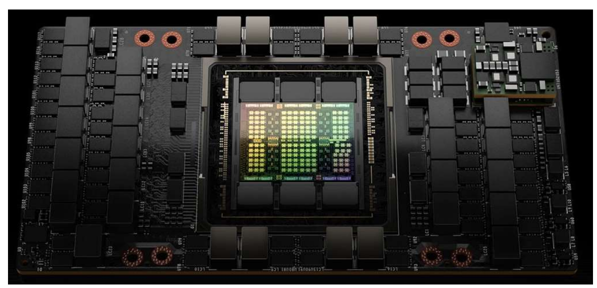
Side Spiel : Parallelization/GPU



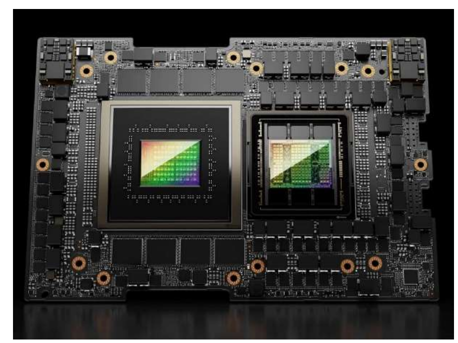
- *CPU* – **C**entral **P**rocessing **U**nit
- Low compute density
- Complex control logic

- *GPU* – **G**raphics **P**rocessing **U**nit
- High compute density
- Simple control logic

- Explosive growth due to AI development
- Inherently parallelized
- Need to write GPU kernels

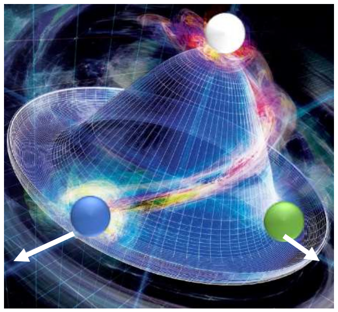


(2022) H100 : x3

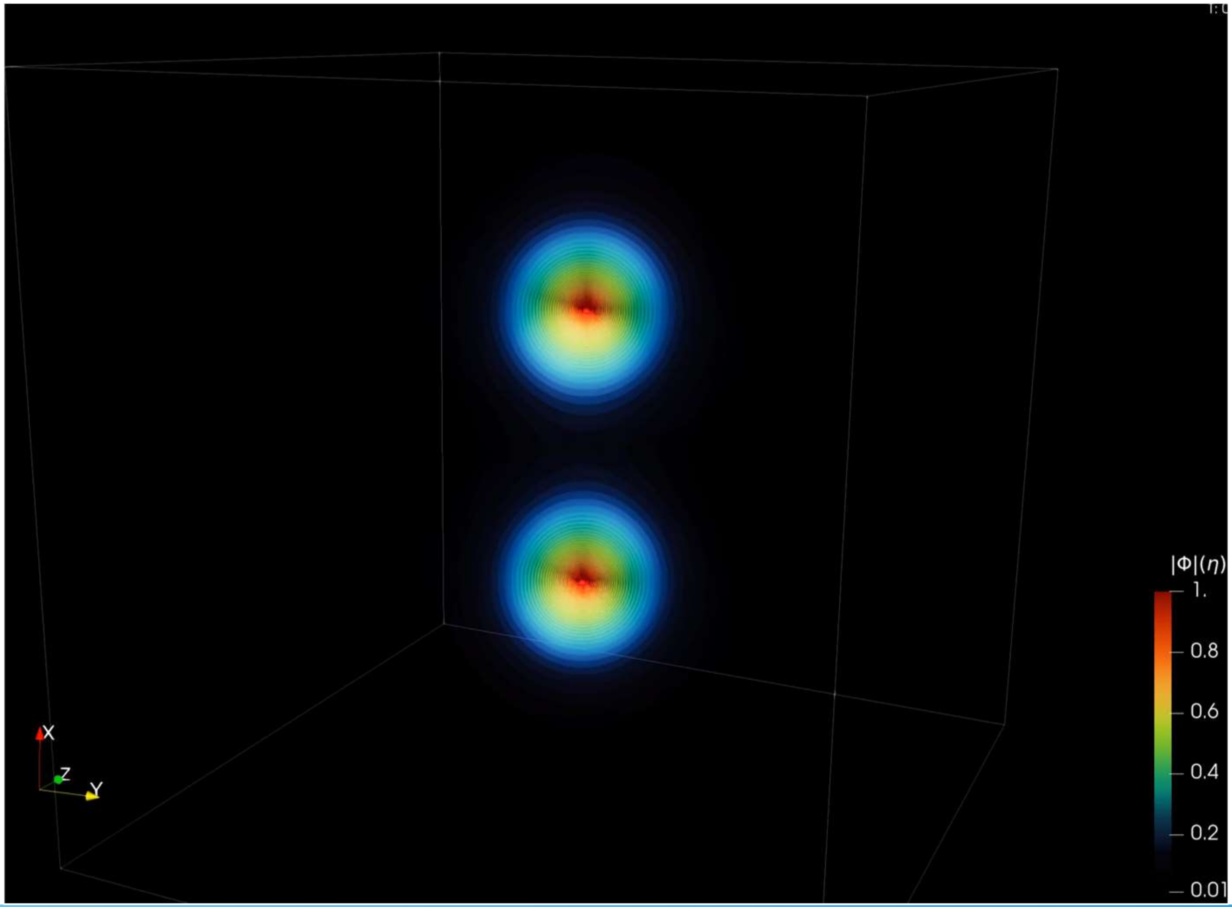


(2024) H200 : 4.8Tb/s

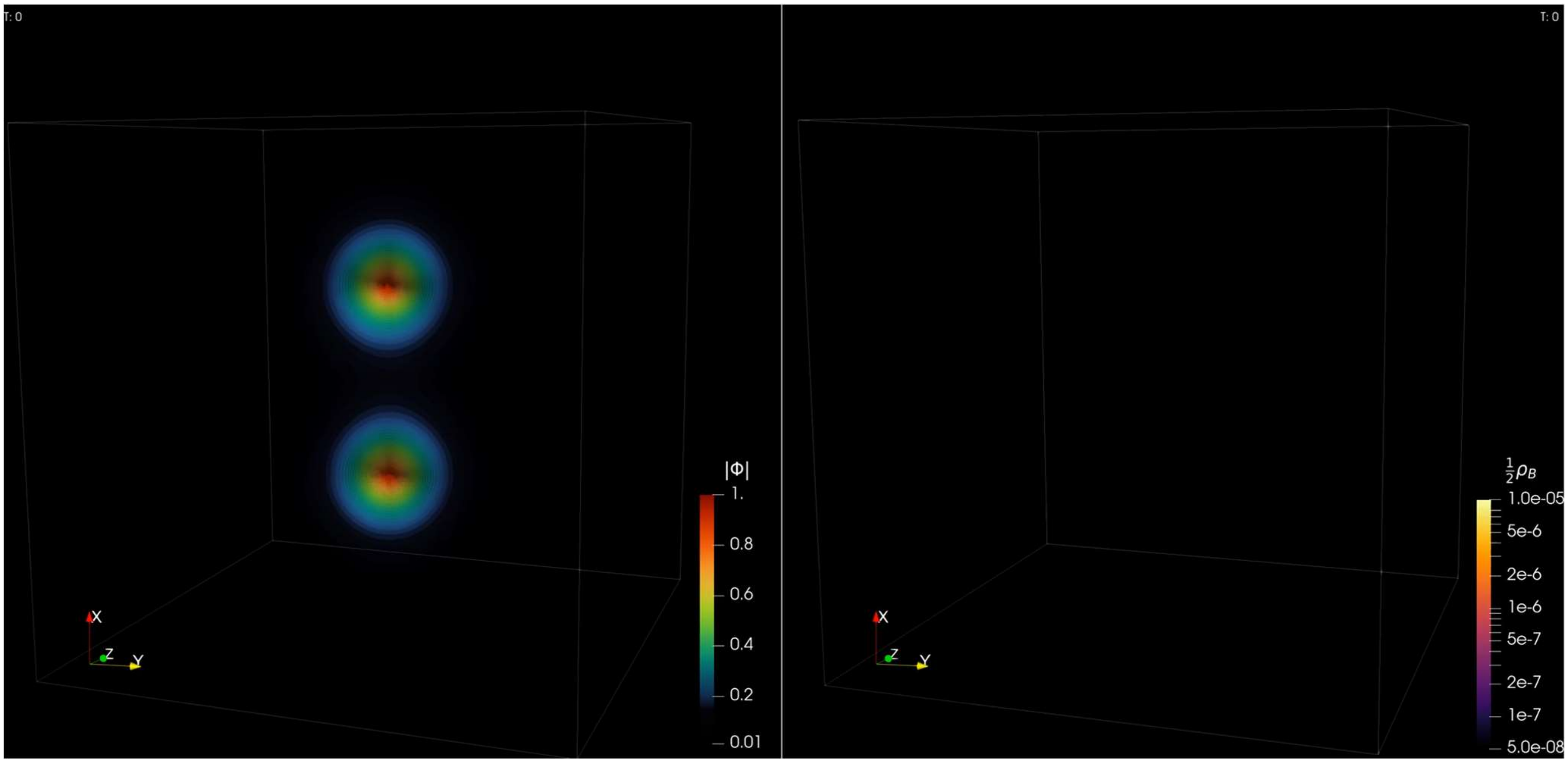
2 Bubbles - Higgs



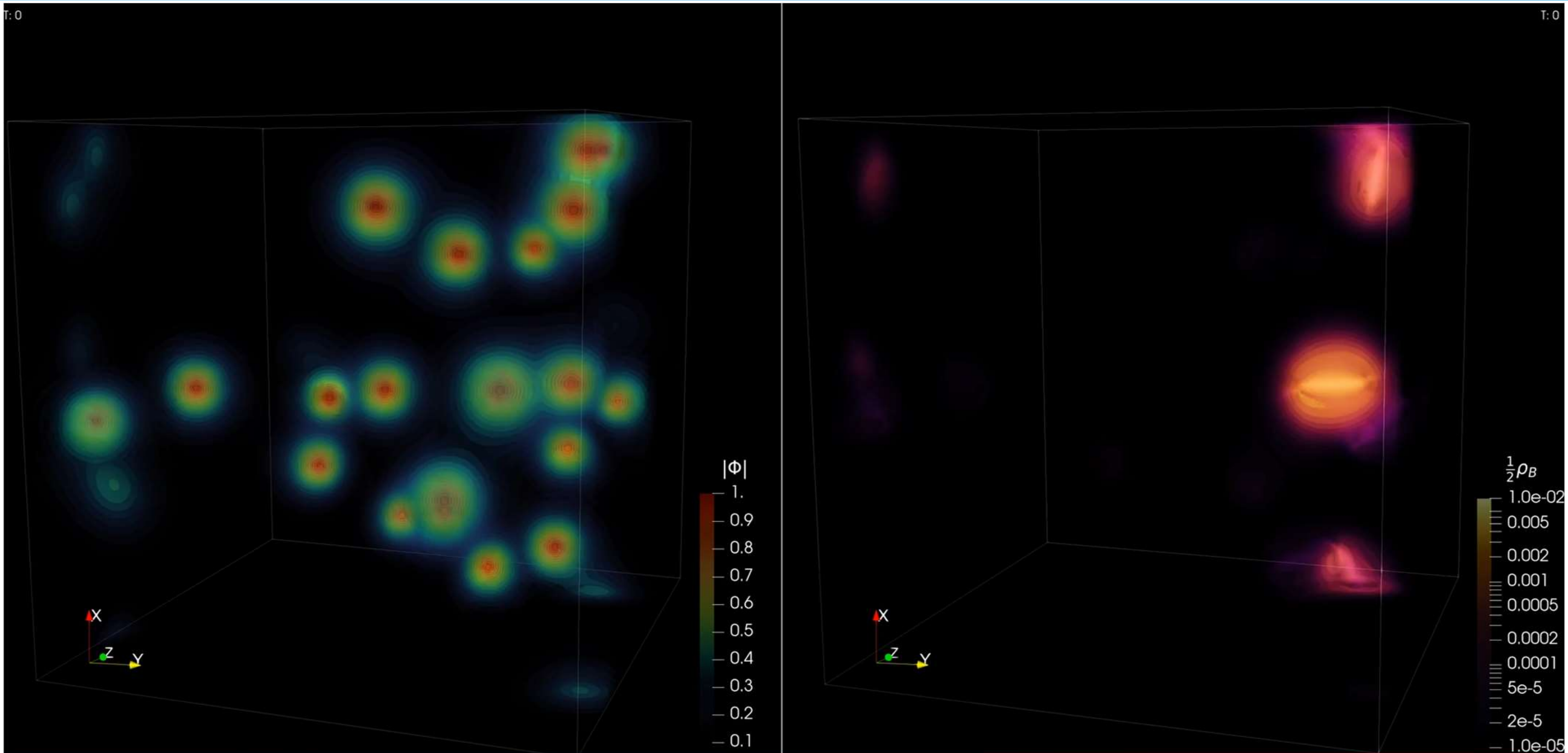
Credit: H Ritsch & M Renn



2 Bubbles – Higgs and B

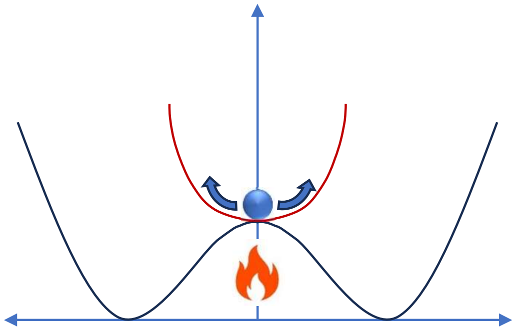


20 Bubbles – Higgs and B



Our setup

Initial conditions

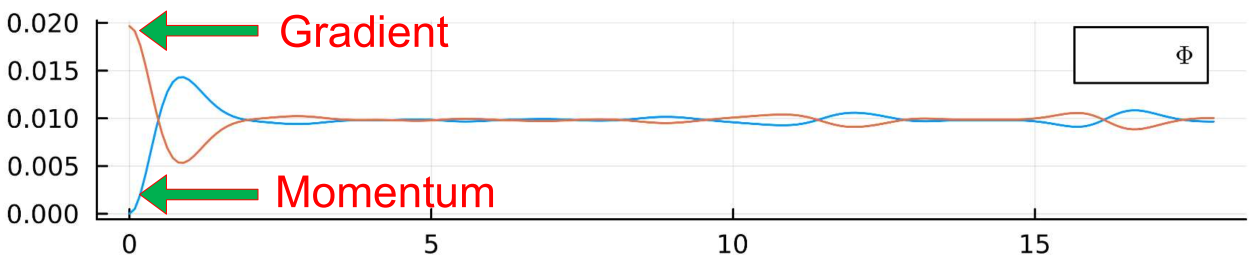


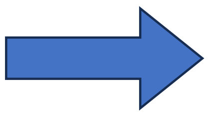
$$\dot{\Phi}(\vec{x}) = \dot{W}_i^a(\vec{x}) = \dot{Y}_i(\vec{x}) = 0$$

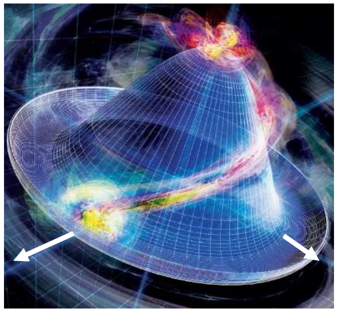
$$\Phi(\vec{x}) \quad W_i^a(\vec{x}) \quad Y_i^a(\vec{x})$$

Bose-Einstein Distribution of Fourier modes

Evolve until equilibrium

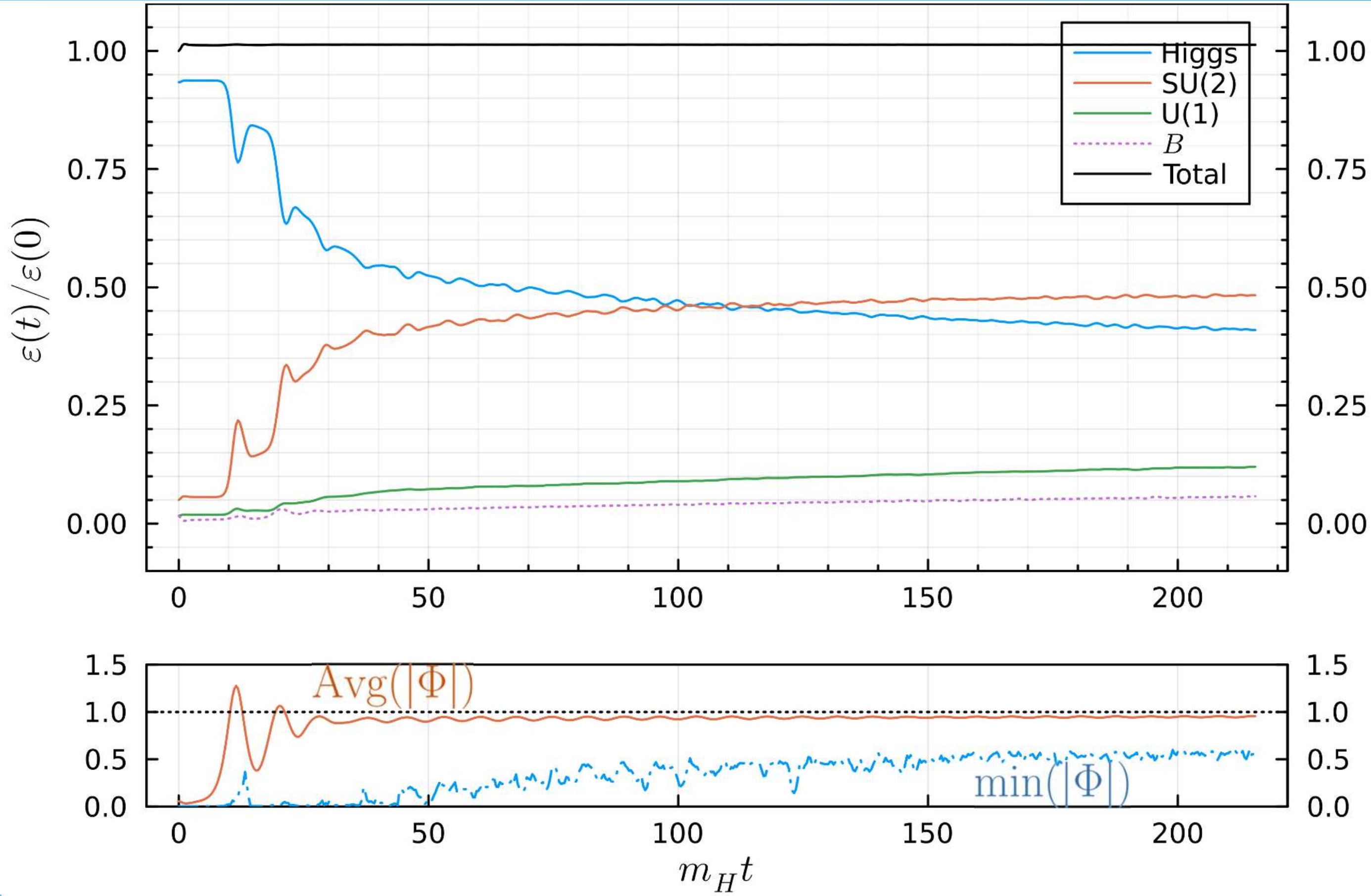




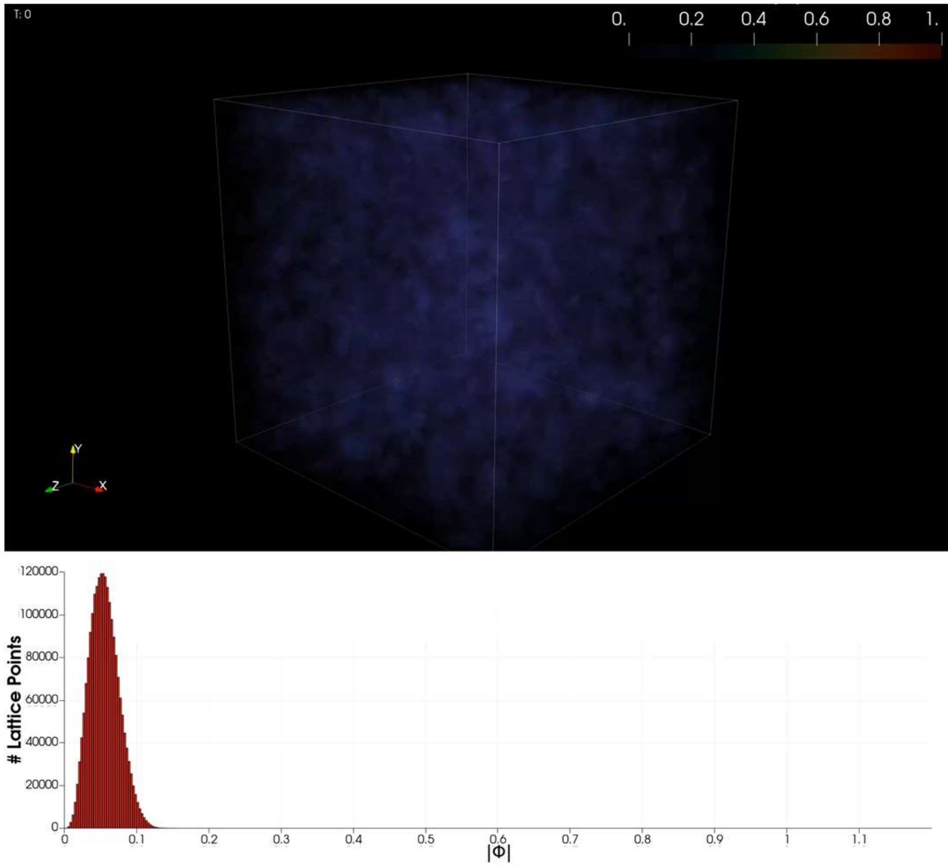


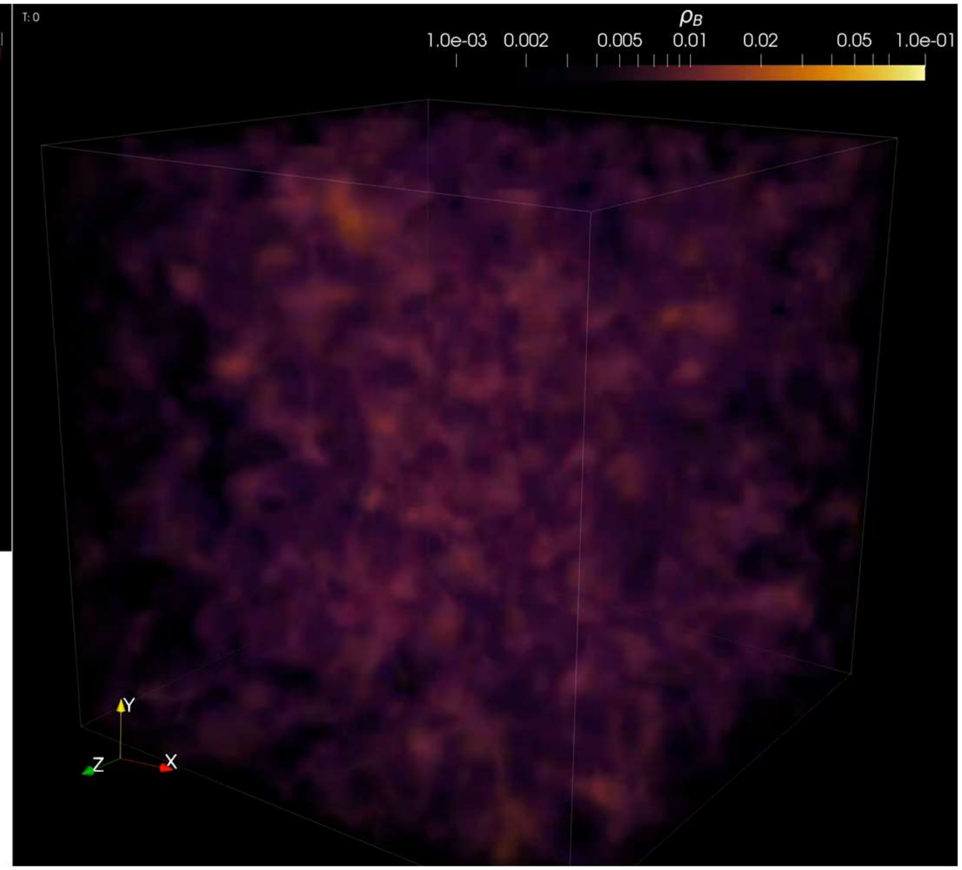
Preliminary results

Periodic Boundary Conditions
Temporal Gauge
Time Evolution : RK4

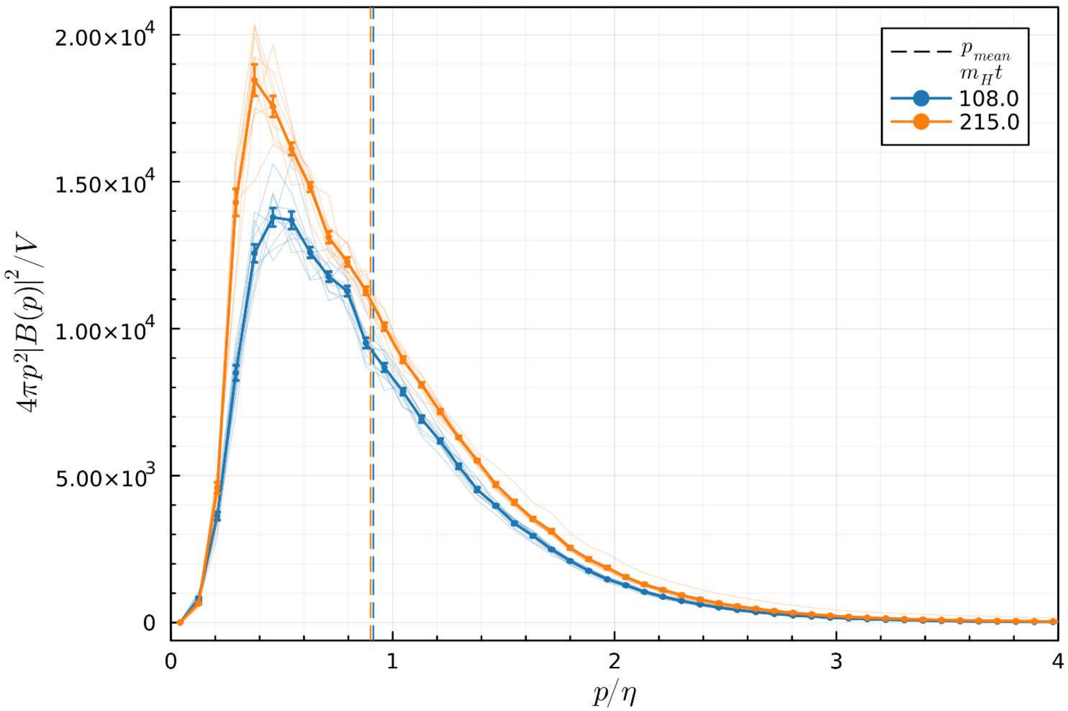


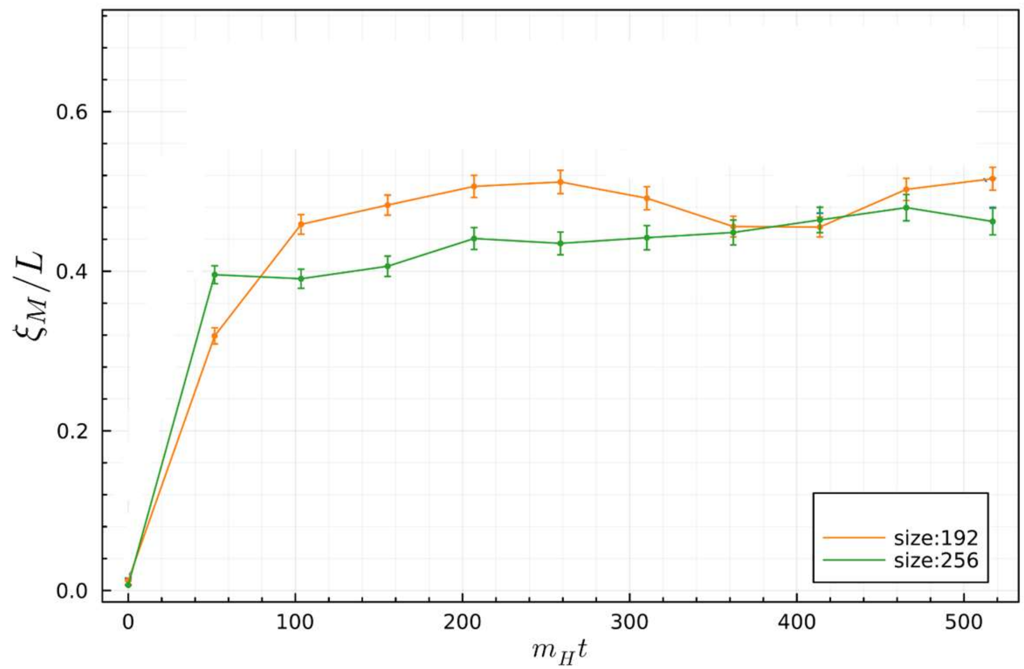
Preliminary results





Preliminary results





$$\xi_M = \frac{\int (2\pi p^{-1}) E_M dp}{\int E_M dp}$$

Part II: Summary

- GPU code developed to simulate EWSB

TO DO

- Peak scale growth (For large lattices)
- Small-k scaling
- CP violating terms

More to come soon!

Thank You