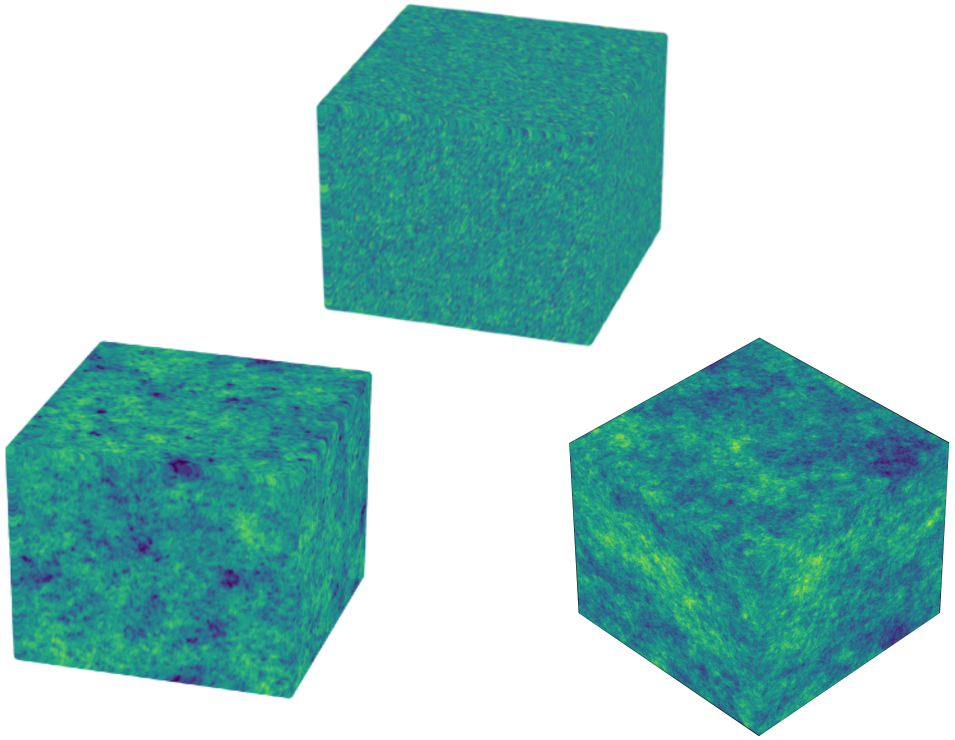
Lattice Simulations of Axion Inflation

Angelo Caravano (IPI fellow @ IAP, Paris)

Collaborators: E.Komatsu, K.D.Lozanov, J. Weller,…



@ Bernoulli Program 09/05/2024

Roadmap

0) Introduction and motivation

1) The method: lattice simulations of inflation

2) Lattice simulations of axion-U(1) inflation

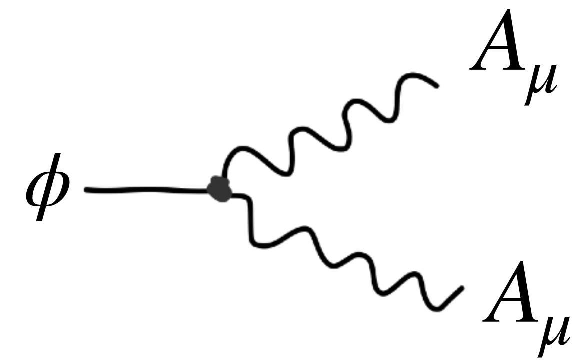
Axion-U(1) inflation

[**M. Anber, L. Sorbo** 0908.4089]
[**N. Barnaby, M. Peloso** 1011.1500]

Interaction between the inflation and a U(1) gauge field

$$A_\mu = (A_0, \vec{A})$$

$$\mathscr{L} \supset \frac{\alpha}{f} \phi \vec{E} \cdot \vec{B}$$



$$\vec{E} = -\nabla A_0 - \partial_t \vec{A}, \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

Axion-U(1) inflation

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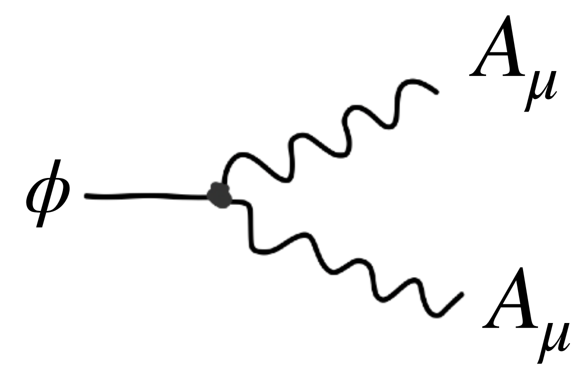

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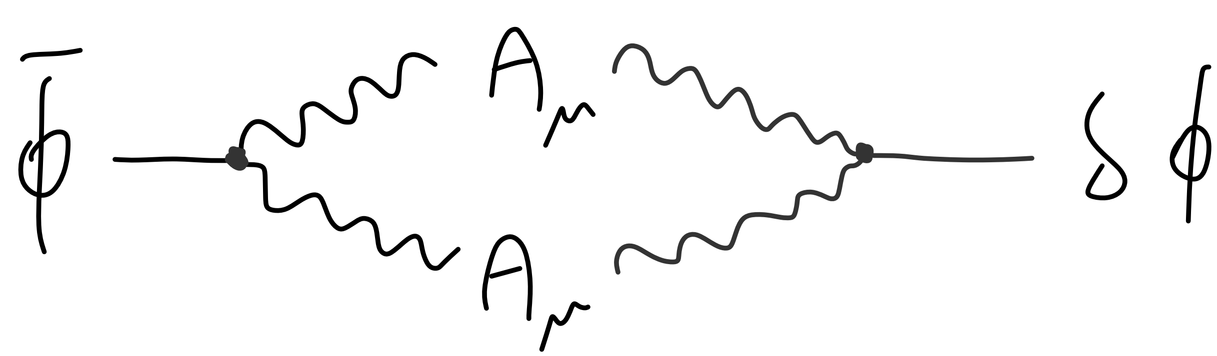
Observational consequences:

Production of electromagnetic field → decay into inflaton perturbations



Axion-U(1) inflation

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$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} M_{\rm Pl}^2 \mathscr{R} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\alpha}{4f} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

In math:

$$\vec{A} \quad \longrightarrow \quad A_\pm$$

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$$\partial_\tau^2 A_\pm + \left(k^2 \pm k\phi' \frac{\alpha}{f} \right) A_\pm = 0 \longrightarrow$$ Tachyonic growth of A_+ for $k < \phi' \frac{\alpha}{f}$

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Tachyonic growth of A_+ for $k < \phi'\dfrac{\alpha}{f}$

$$\left(\frac{\partial^2}{\partial\tau^2} + 2\mathscr{H}\frac{\partial}{\partial\tau} - \nabla^2 + a^2V''(\phi)\right)\delta\phi(\vec{x},\tau) = a^2\frac{\alpha}{f}\left(F_{\mu\nu}\tilde{F}^{\mu\nu} - \langle F_{\mu\nu}\tilde{F}^{\mu\nu}\rangle\right)$$

Axion-U(1) inflation

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$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Also source for gravitational waves:

$$\left(\frac{\partial^2}{\partial \tau^2} + 2\mathscr{H} \frac{\partial}{\partial \tau} - \nabla^2 \right) h_{ij}(\vec{x}, \tau) = 2a^2 \left(-E_i E_j - B_i B_j \right)^{TT}$$

Axion-U(1) inflation

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Known results

(Green function methods, in-in calculations)

- Power spectrum:

$$\mathscr{P}_\zeta(k) \simeq \mathscr{P}_{\rm vac} + \mathscr{P}_{\rm vac}^2 f_2(\xi) e^{4\pi\xi}$$

vacuum (free theory)

sourced

$$\mathscr{P}_{\rm vac} \simeq \frac{H^4}{(2\pi\dot{\phi})^2}$$

$$\xi = \frac{\alpha\dot{\phi}}{2fH}$$

Axion-U(1) inflation

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$$\xi = \frac{\alpha\dot{\phi}}{2fH}$$

- Bispectrum:

$$f_{\rm NL}^{\rm (equil.)}(\xi) \simeq \frac{f_3(\xi)\mathscr{P}_{\rm vac}^3 e^{6\pi\xi}}{\mathscr{P}_\zeta^2}$$

Axion inflation

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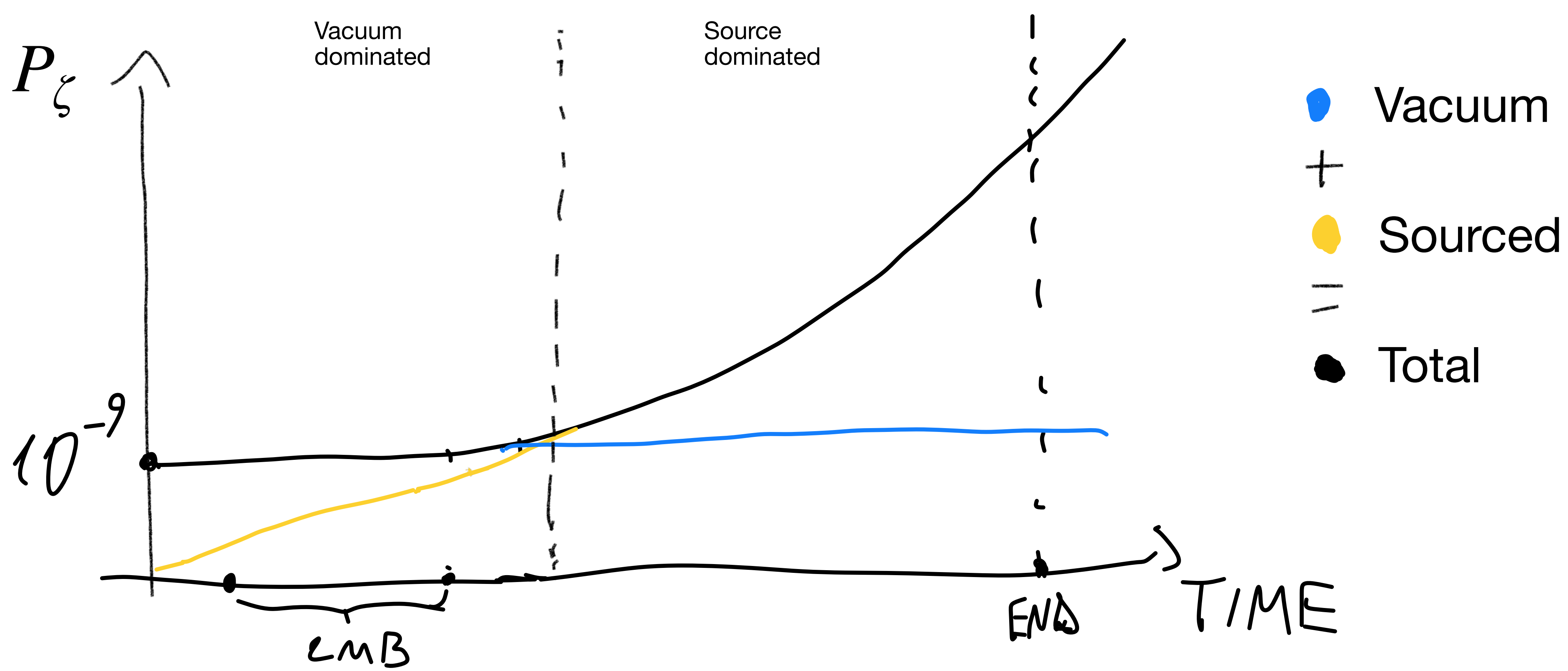
$$\xi = \frac{\alpha\dot{\phi}}{2fH}$$

Scalar perturbations naturally grow on small scales



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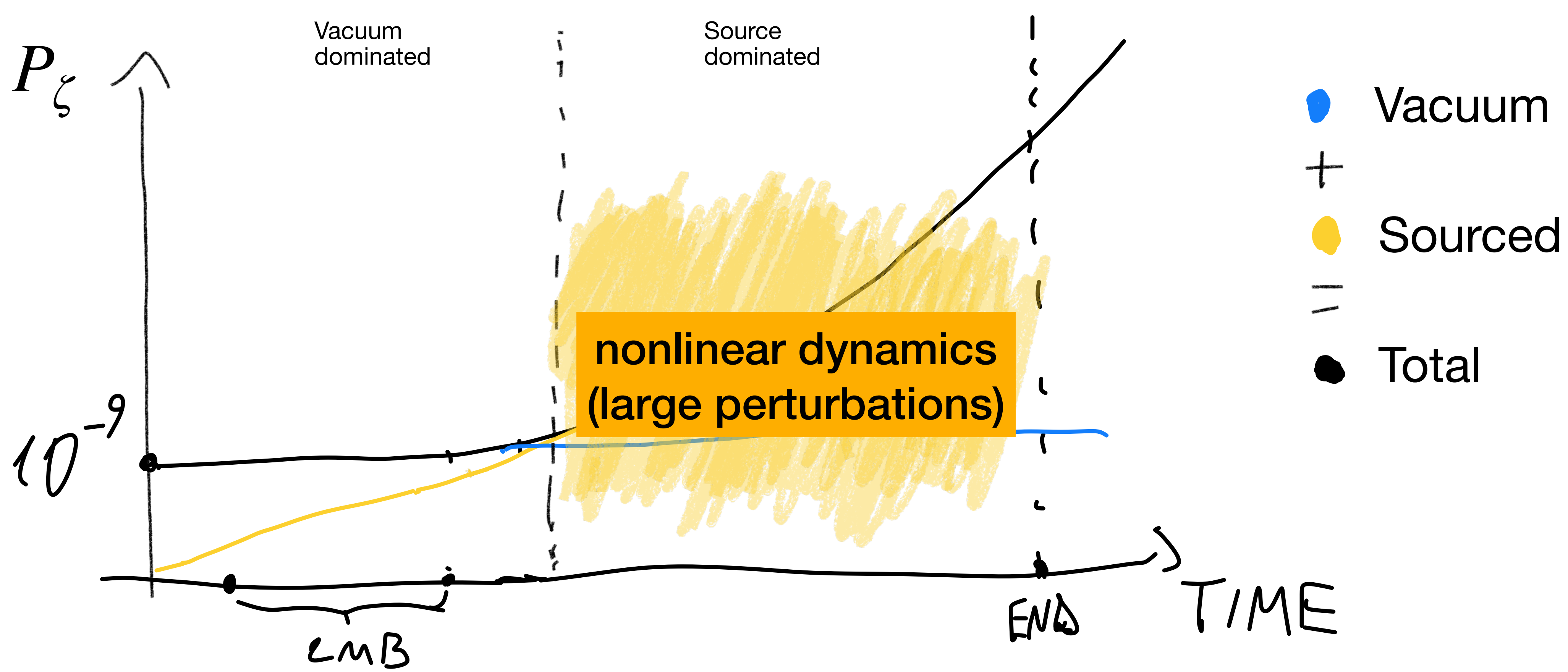

Axion inflation

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More precisely:

$$\partial_\tau^2 \bar\phi + 2\mathscr{H}\partial_\tau \bar\phi + a^2 V'(\bar\phi) = a^2 \frac{a}{f}\langle F_{\mu\nu}\tilde F^{\mu\nu}\rangle$$

If these terms become comparable ⟷ **backreaction**

↓

Sort of extra friction, but not so simple (as we will see)

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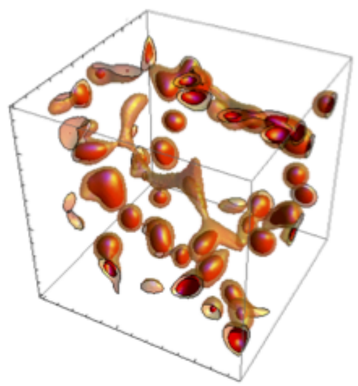
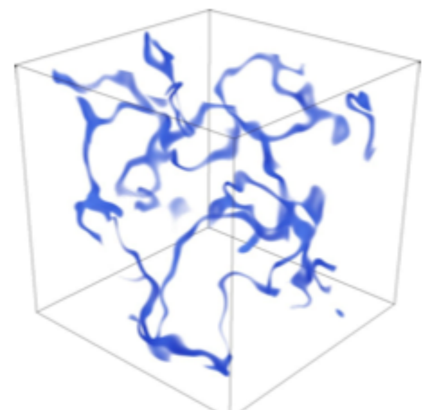
Lattice simulations

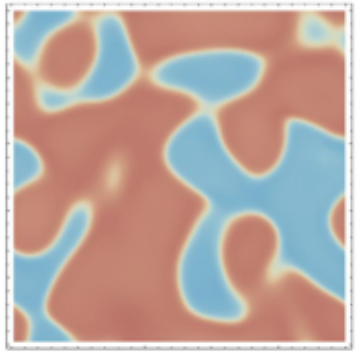
- Numerical tool to study non-perturbative cosmological phenomena.
- Examples: reheating phase after inflation, cosmological phase transitions.



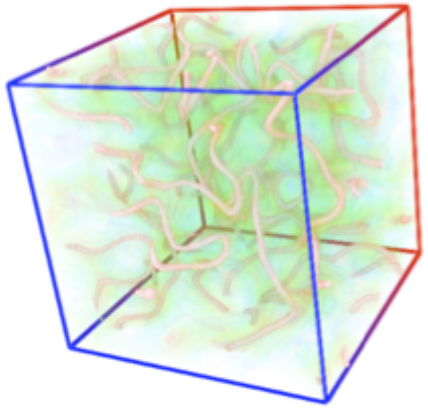
[M. A. Amin, R. Easther, H. Finkel, arXiv:1009.2505]



[A. V. Frolov, arXiv:1004.3559]



[M. A. Amin, J. Fan, K. D. Lozanov, M. Reece, arXiv:1802.00444]



[J. Dufaux, D.G. Figueroa, J. Garcia-Bellido, arXiv:1006.0217]

My goal:

Develop lattice techniques for inflation

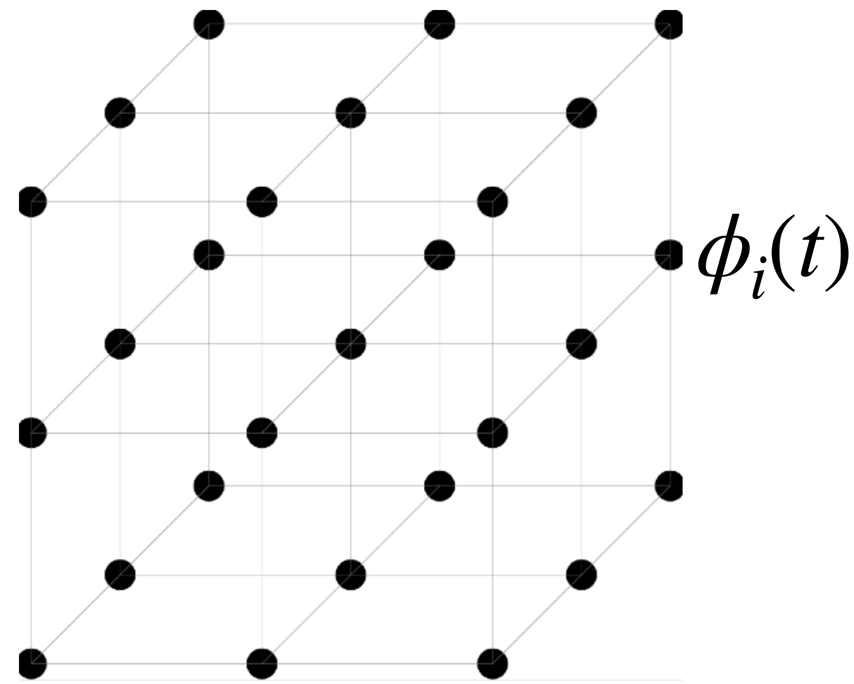
	arXiv
AC, E. Komatsu, K. D. Lozanov, J. Weller	2102.06378
	2110.10695
	2204.12874
AC	2209.13616
AC, S.Renaux-Petel, K. Inomata	2403.12811

AC, D. Jamieson, E. Komatsu [in preparation]

Lattice simulations

Put the continuous inflationary universe on a discrete cubic lattice:

$$\phi(\vec{x}, t)$$



$$\phi(\vec{x}, t) = \bar{\phi}(t) + \delta\phi(\vec{x}, t)$$

& perturbation theory on $\delta\phi$

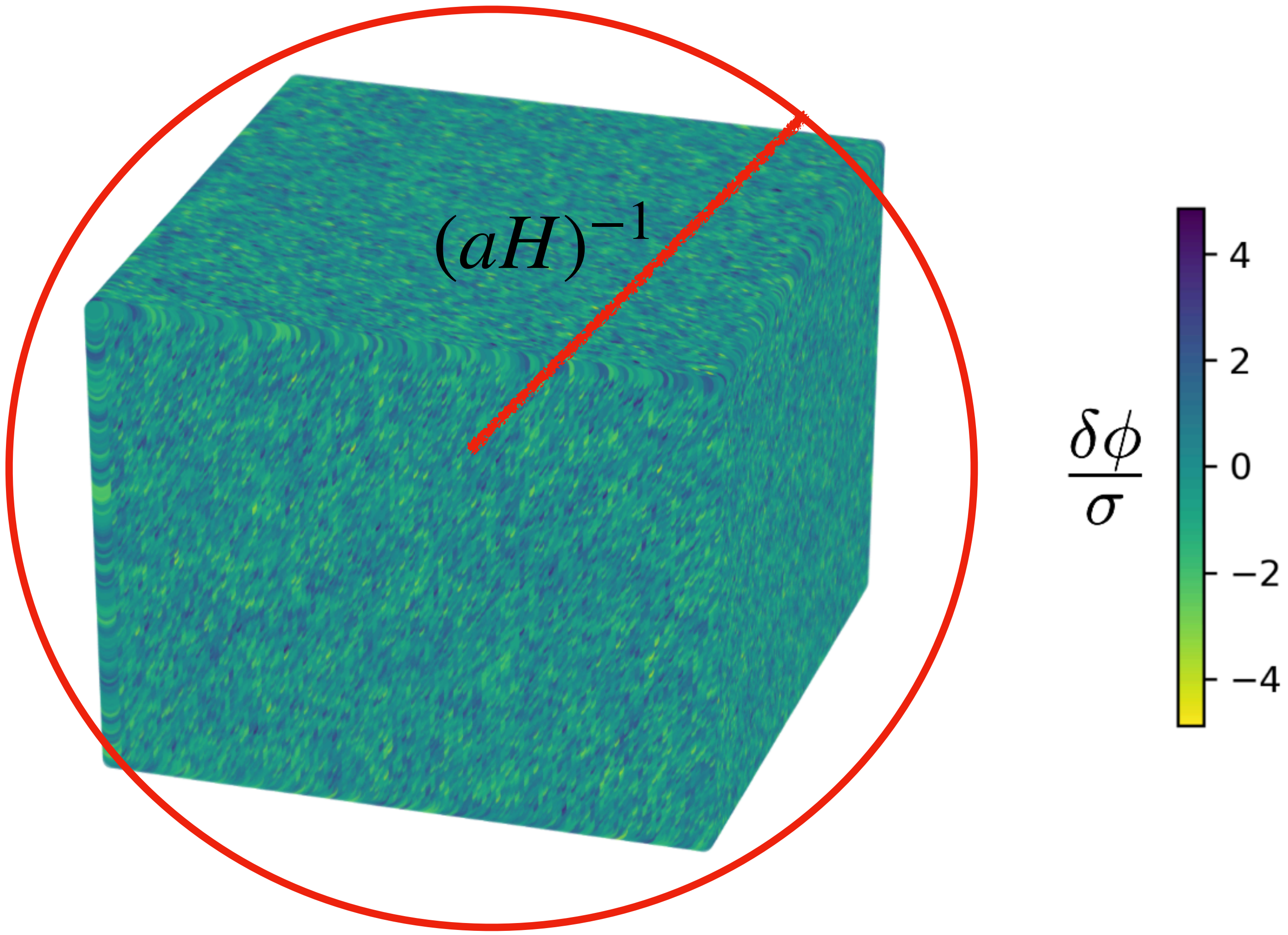
Non-linear evolution of ϕ_i

Numerically solve the classical eqs:

$$\frac{\partial \mathscr{L}}{\partial \phi_i} = \frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{\phi}_i}\right)$$

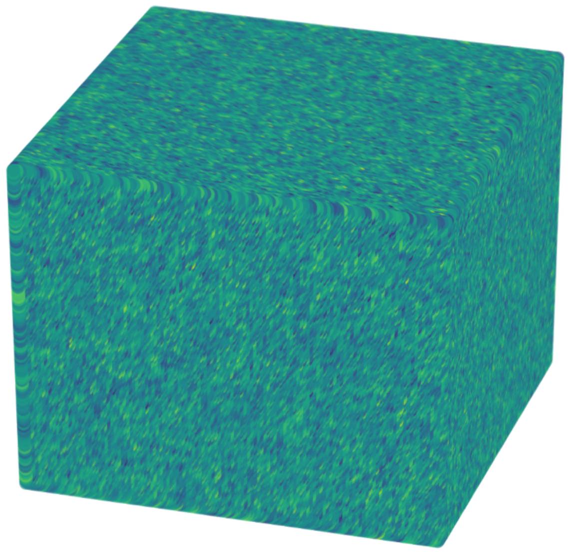
Lattice approach

Start with quantum fluctuations on sub-horizon box:



Lattice approach: initial conditions

[AC 2209.13616]



- $$\hat{\phi}(\vec{n}) = \sum_{\vec{m}} \left[\hat{a}_{\vec{m}}\, u(\vec{\kappa}_{\vec{m}})\, e^{i\frac{2\pi}{N}\vec{n}\cdot\vec{m}} + \hat{a}^{\dagger}_{\vec{m}}\, u^{\dagger}(\vec{\kappa}_{\vec{m}})\, e^{-i\frac{2\pi}{N}\vec{n}\cdot\vec{m}} \right]$$

  $\vec{n}$ = lattice site, $n_i, m_i \in 1,\ldots,N$. $\vec{\kappa}_{\vec{m}} = \frac{2\pi}{L}\vec{m}$

- Discrete Bunch-Davies spectrum: [AC+ 2102.06378]

  $$u(\vec{\kappa}) = \frac{L^{3/2}}{a\sqrt{2\omega_{\vec{\kappa}}}} e^{-i\omega_{\vec{\kappa}}\tau}, \qquad \omega^2_{\vec{\kappa}} = k^2_{\text{eff}}(\vec{\kappa}) + m^2$$

  (discrete dispersion relation)

- Stochastic approximation:

  $$\hat{a}_{\vec{m}} = e^{i2\pi\hat{Y}_{\vec{m}}}\sqrt{-\ln(\hat{X}_{\vec{m}})/2},$$

  $\hat{X}_{\vec{m}}, \hat{Y}_{\vec{m}}$ uniform randoms between 0 and 1

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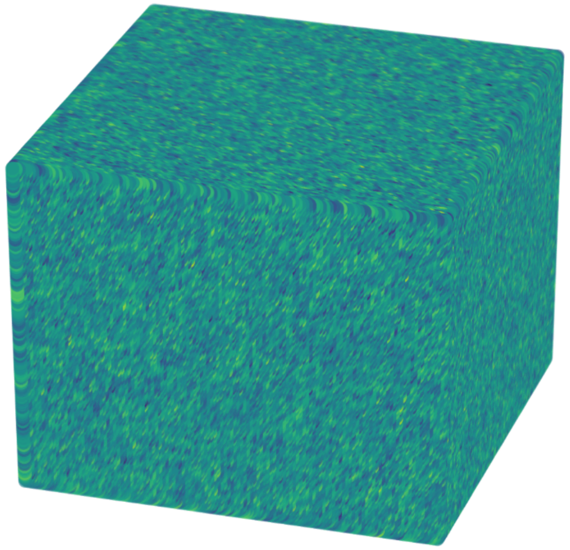

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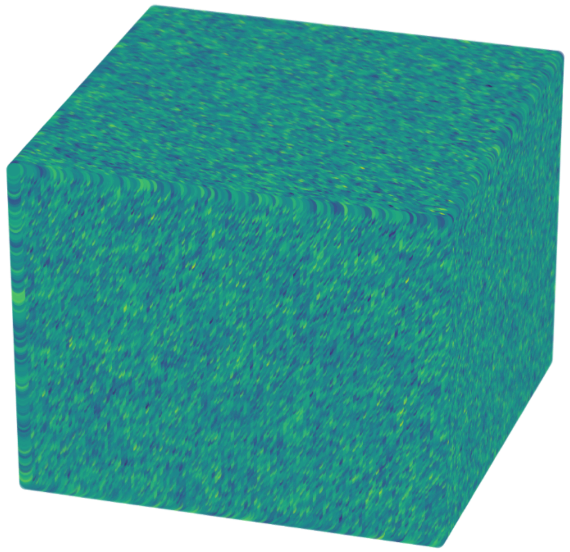

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(discrete dispersion relation)

$$k_{\text{eff}}^2(\vec{\kappa}_{\vec{m}}) = \frac{4}{(dx)^2}\left[\sin^2\left(\frac{\pi m_1}{N}\right) + \sin^2\left(\frac{\pi m_2}{N}\right) + \sin^2\left(\frac{\pi m_3}{N}\right)\right].$$

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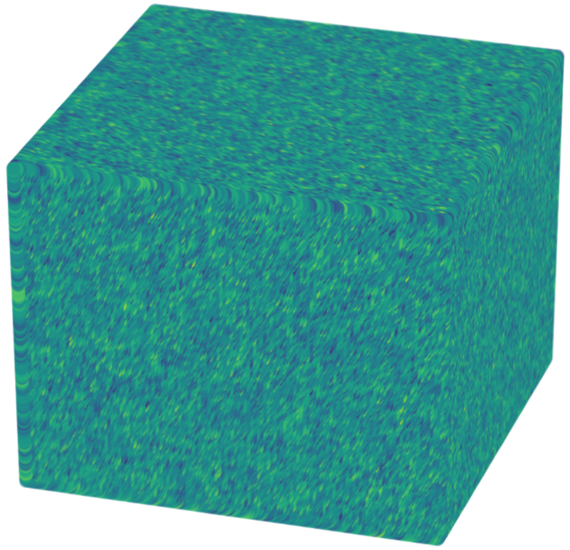
[**AC** 2209.13616]



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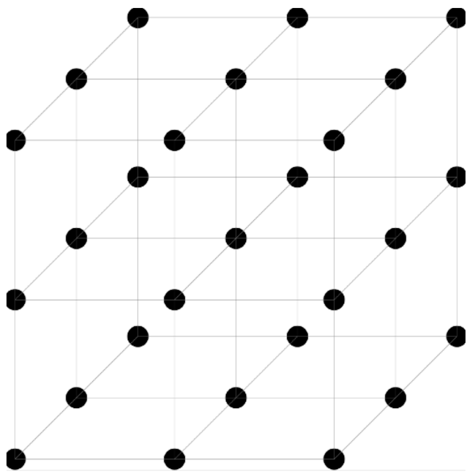
Lattice approach: evolution

[**AC** 2209.13616]

Solve numerically for all lattice points:



$$\phi''(\vec{n}) + 2H\phi'(\vec{n}) - \nabla^2\phi(\vec{n}) + a^2\frac{\partial V}{\partial \phi}(\vec{n}) = 0$$

+ Friedmann equation for scale factor $\dfrac{d^2a}{d\tau^2} = \dfrac{1}{6}\left(\langle\rho\rangle - 3\langle p\rangle\right)a^3$

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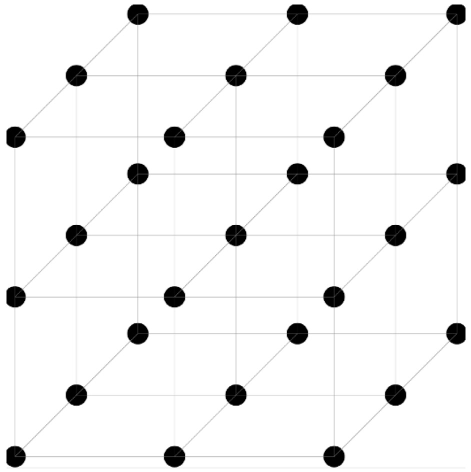

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+ Friedmann equation for scale factor $\quad \dfrac{d^2 a}{d\tau^2} = \dfrac{1}{6}\left(\langle\rho\rangle - 3\langle p\rangle\right) a^3$

Assuming **unperturbed metric** $\quad ds^2 = a^2(-d\tau^2 + d\vec{x}^2)\quad$ because:

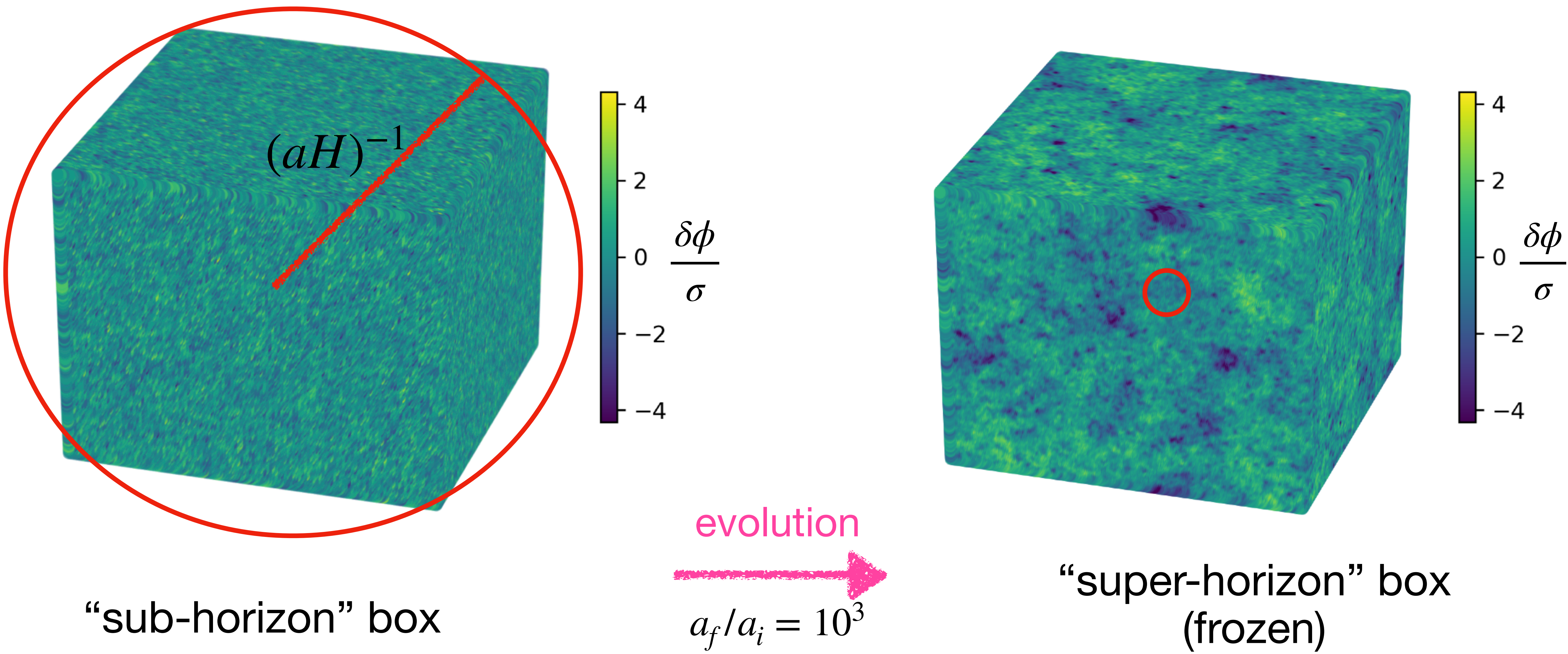
- $\delta g_{ij} \equiv 0$ (gauge freedom)
- $\delta g_{0\mu} \propto \epsilon = \dfrac{\dot{H}}{H^2} = \dfrac{\frac{1}{2}\dot{\phi}^2}{M_{\rm Pl}^2 H^2} \to 0$, known as "decoupling limit" of gravity $M_{\rm Pl} \to \infty$

See e.g.

C. Cheung et al. [0709.0293]
S. R. Behbahani et al. [1111.3373]
P. Creminelli et al. [2401.10212]

Lattice simulations of inflation

[AC 2209.13616]



Lattice simulations of inflation

For single-field simulations, see:

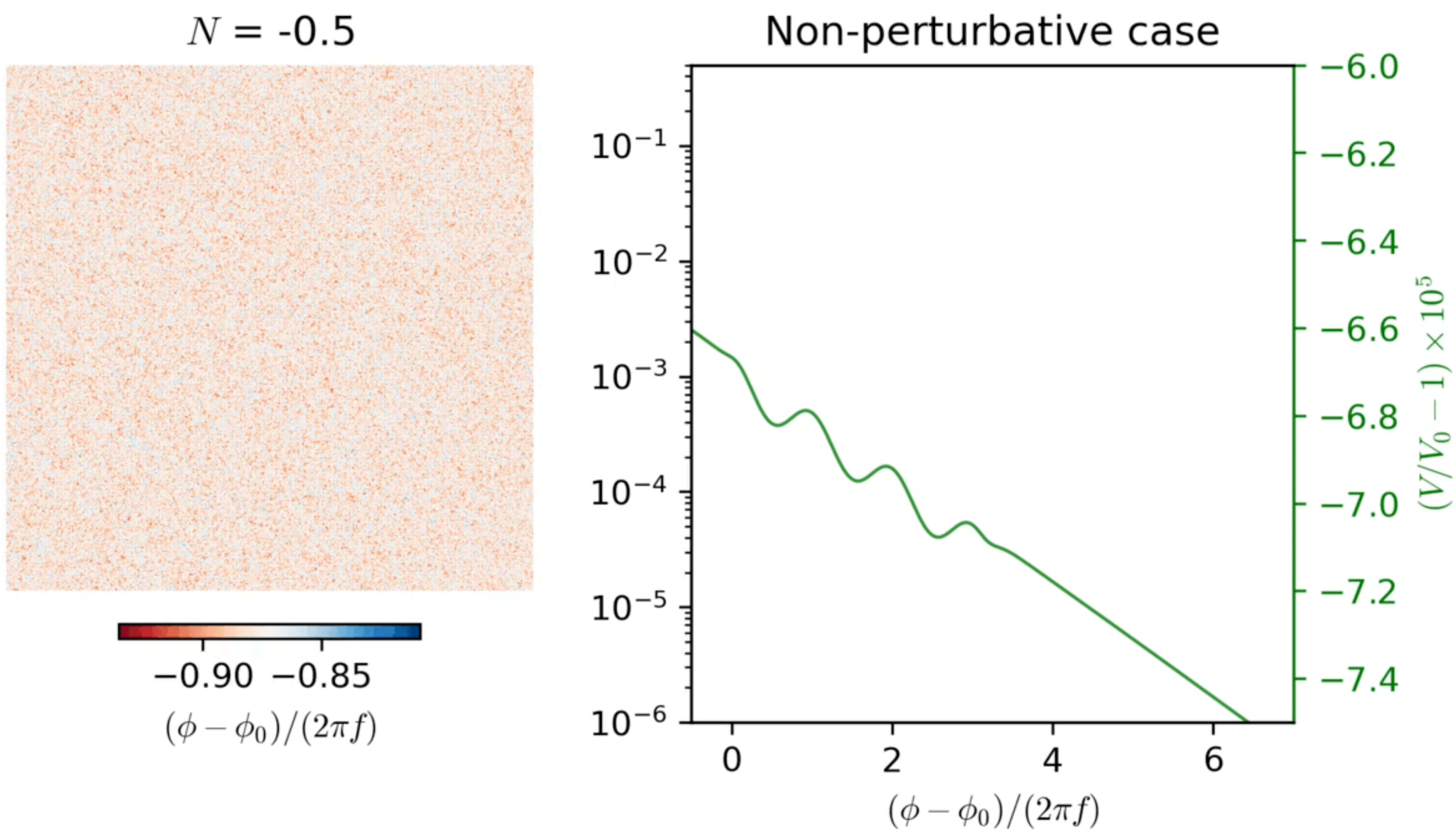
arXiv > astro-ph > arXiv:2403.12811

Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 19 Mar 2024]

The Inflationary Butterfly Effect: Non-Perturbative Dynamics From Small-Scale Features

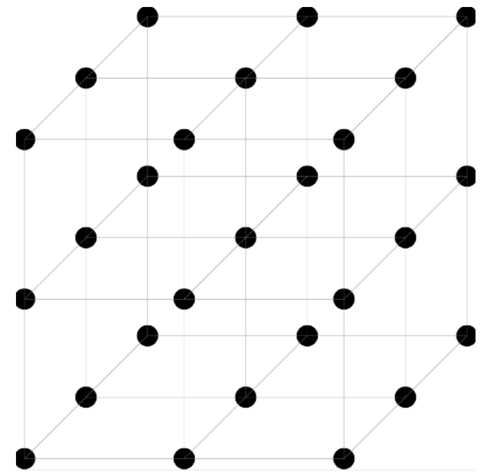
Angelo Caravano, Keisuke Inomata, Sébastien Renaux-Petel



Lattice simulation: axion-U(1)

[AC 2209.13616]

Using the "PDE" approach for the Gauge field.



In the Lorenz Gauge $\partial^\mu A_\mu = 0$:

$$\phi'' + 2H\phi' - \partial_j\partial_j\phi + a^2\frac{\partial V}{\partial\phi} = -\,a^2\frac{\alpha}{4f}F_{\mu\nu}\tilde{F}^{\mu\nu},$$

$$A_0'' - \partial_j\partial_j A_0 = \frac{\alpha}{f}\epsilon_{ijk}\partial_k\phi\partial_i A_j,$$

$$A_i'' - \partial_j\partial_j A_i = \frac{\alpha}{f}\epsilon_{ijk}\phi'\partial_j A_k - \frac{\alpha}{f}\epsilon_{ijk}\partial_j\phi(A_k' - \partial_k A_0)$$

+ Friedmann equation for scale factor $\frac{d^2a}{d\tau^2} = \frac{1}{6}\left(\langle\rho\rangle - 3\langle p\rangle\right)a^3$

Note that $\partial^\mu A_\mu = 0$ is not automatically satisfied, needs to be checked!

This approach was used in preheating sims, e.g. P. Adshead et al. [1909.12842] P. Adshead et al. [1909.12843]

Lattice simulation: axion-U(1)

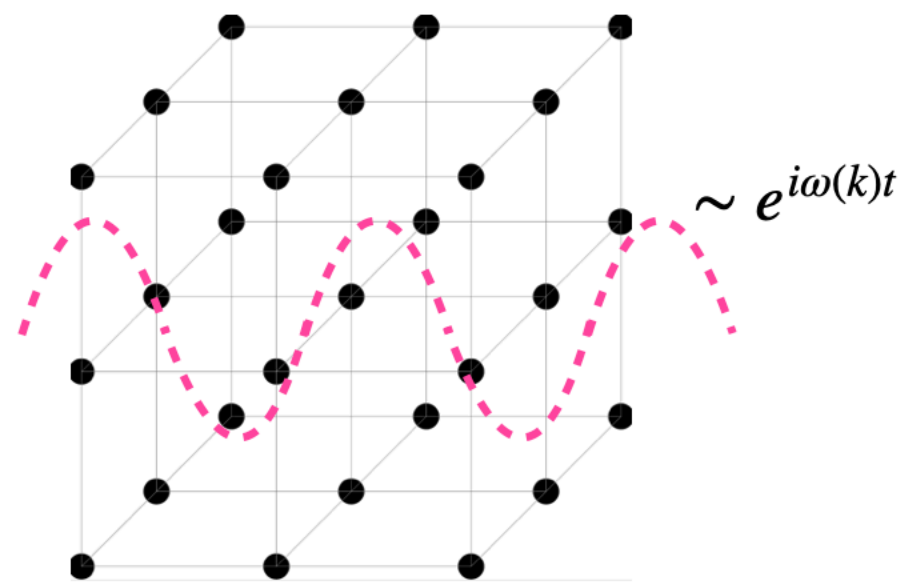
AC+ 2102.06378
AC+ 2110.10695

In 2102.06378 and 2110.10695, we studied the consequences of discretization

Continuous space:

$$\omega^2(k) = k^2$$

Lattice:



$$\omega^2(k) \neq k^2 \quad \left[= \frac{sin^2(k\ \Delta x/2)}{(\Delta x/2)^2} \right]$$

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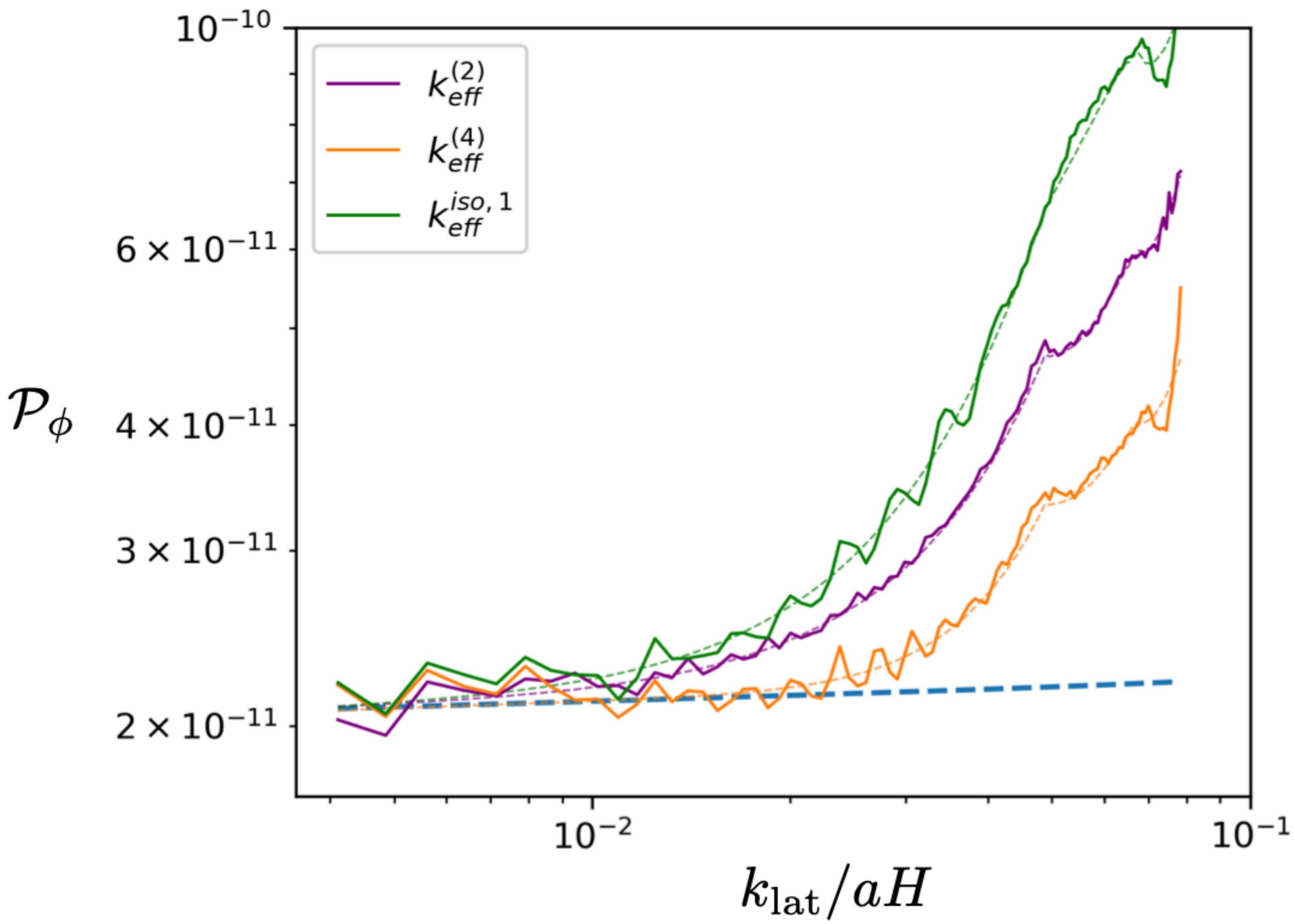

Lattice simulation: axion-U(1)

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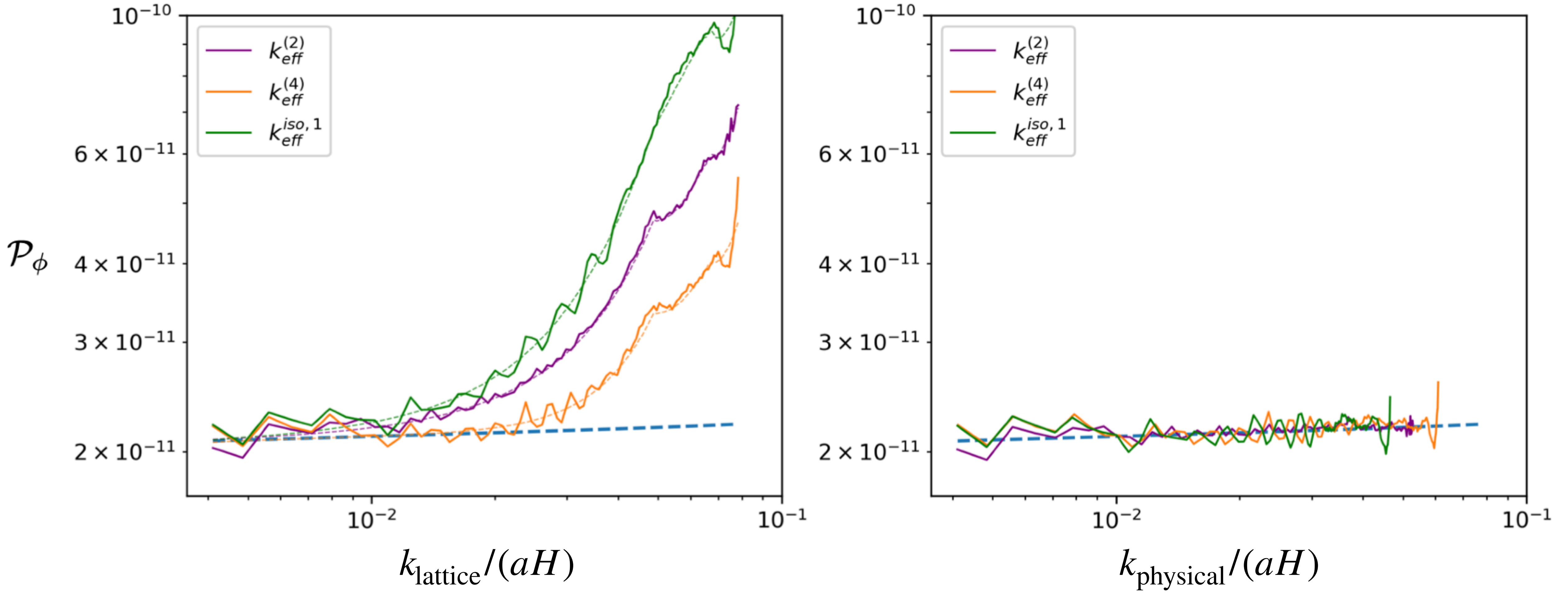
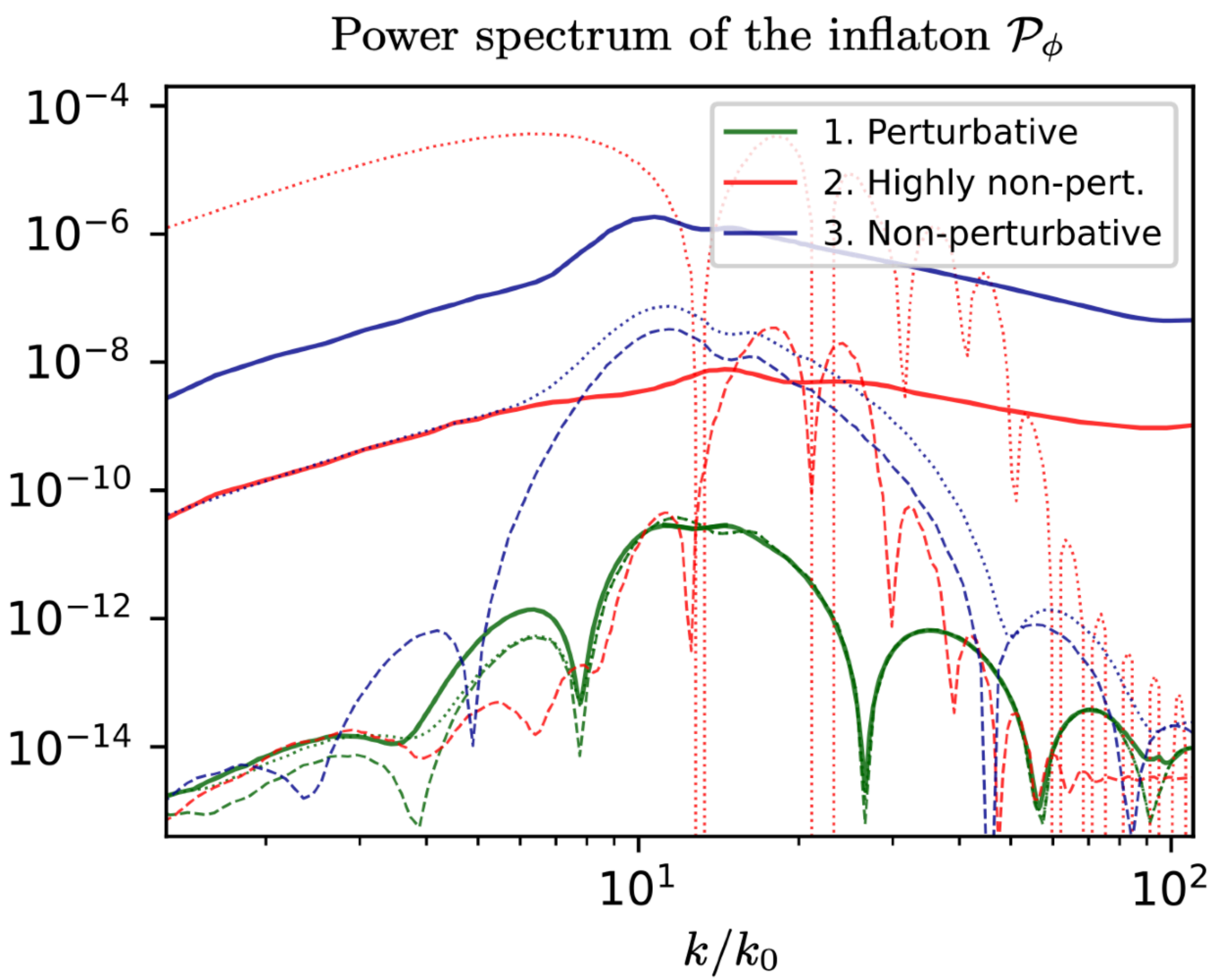
Trick: identify $k_{\rm physical} = \omega(k_{\rm lattice})$



Lattice simulation: loop effects

[AC+ 2403.12811]

Off-topic: this is what allowed precise comparison with perturbation theory at 1-loop



Lattice simulation: axion-U(1)

AC+ 2102.06378

AC+ 2110.10695

For tachyonic enhancement of gauge fields, discretization is more important

Continuous space:

$$A''_{\pm} + \left(k^2 \pm k\bar{\phi}'\frac{\alpha}{f}\right)A_{\pm} = 0.$$

Lattice:

$$A''_{\pm} + \left(k^2_{\rm lapl} \pm \frac{\alpha}{f}\phi'\vec{k}_{\rm sd} \cdot \frac{\vec{\kappa}}{|\vec{\kappa}|}\right)A_{\pm} = 0.$$

$$k_{\rm sd} \neq k_{\rm lapl}$$

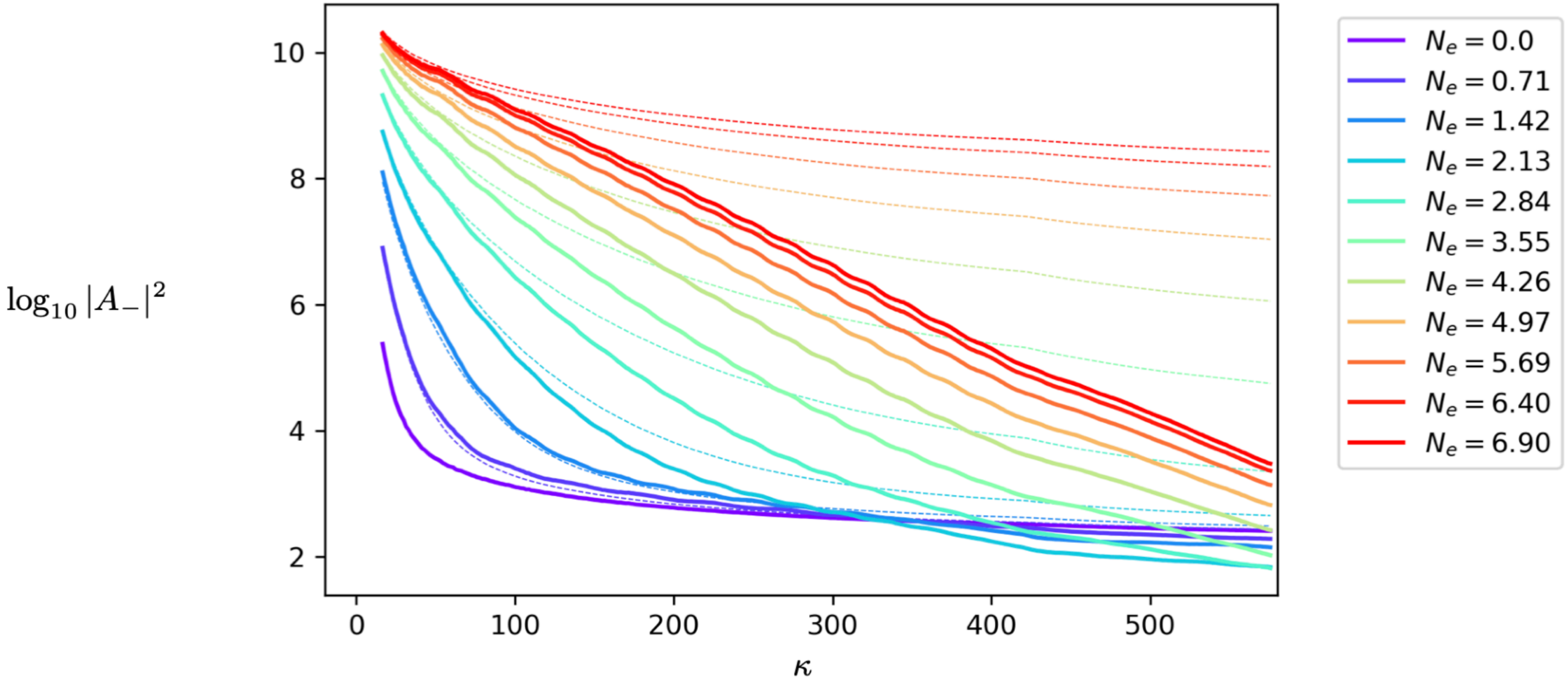
Lattice simulation: axion-U(1)

AC+ 2102.06378

AC+ 2110.10695

For tachyonic enhancement of gauge fields, discretization is more important

For a second-order scheme:



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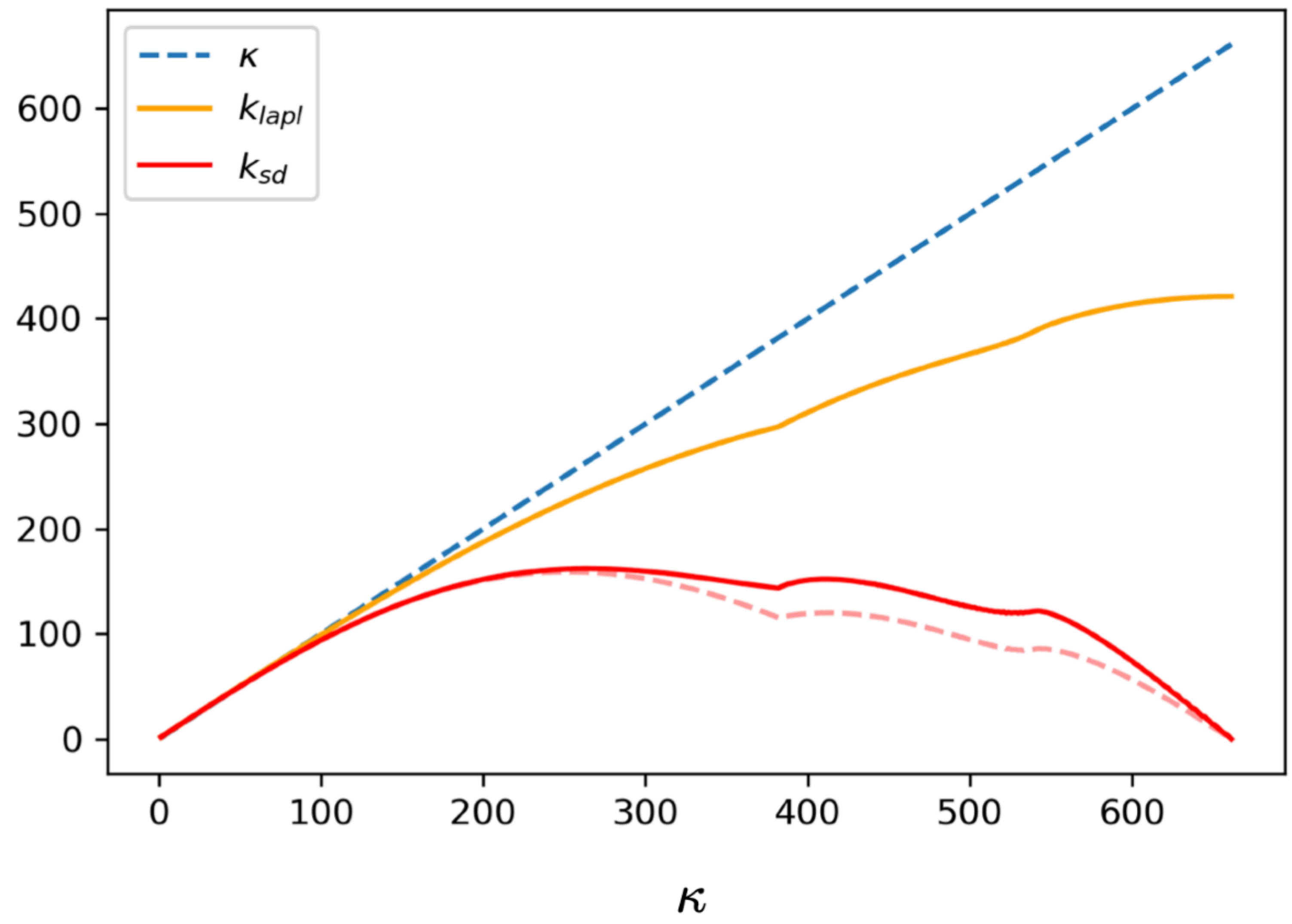


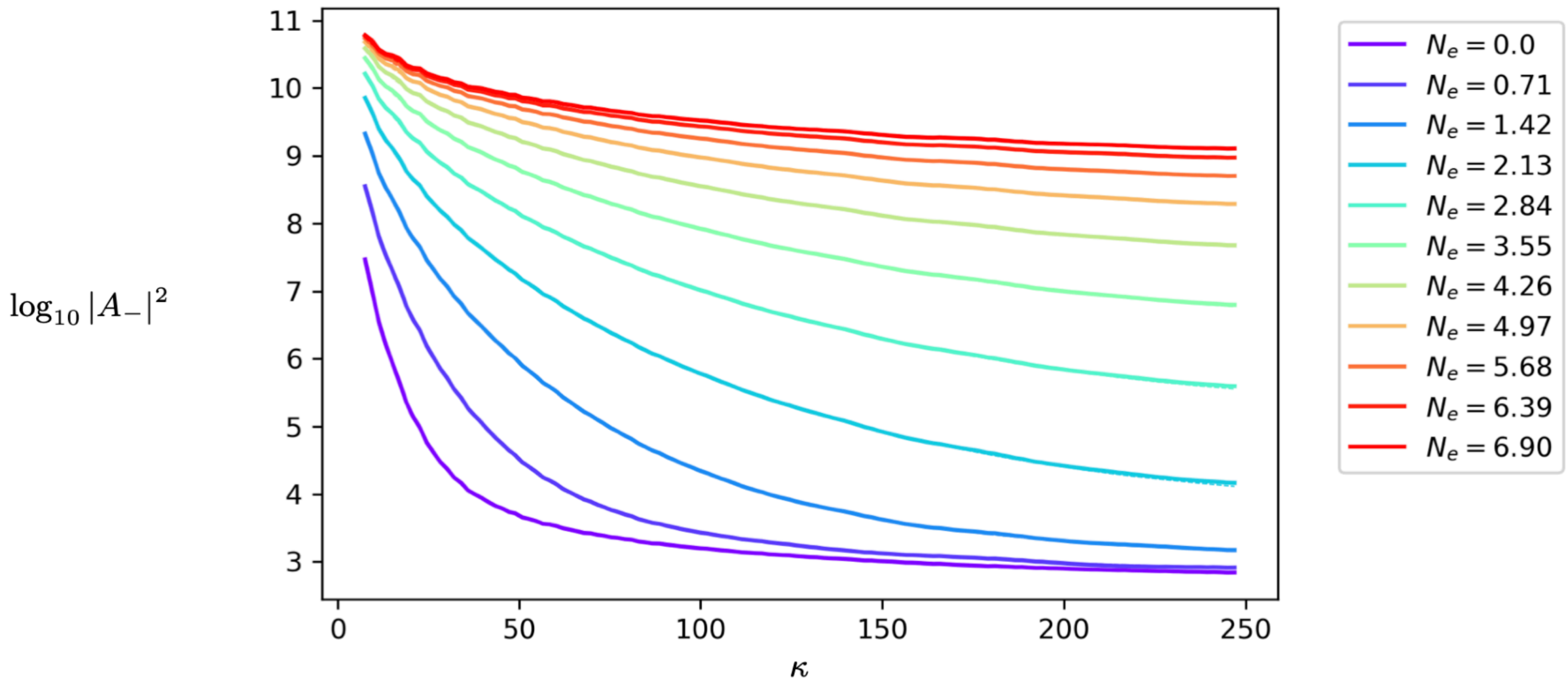
Lattice simulation: axion-U(1)

AC+ 2102.06378

AC+ 2110.10695

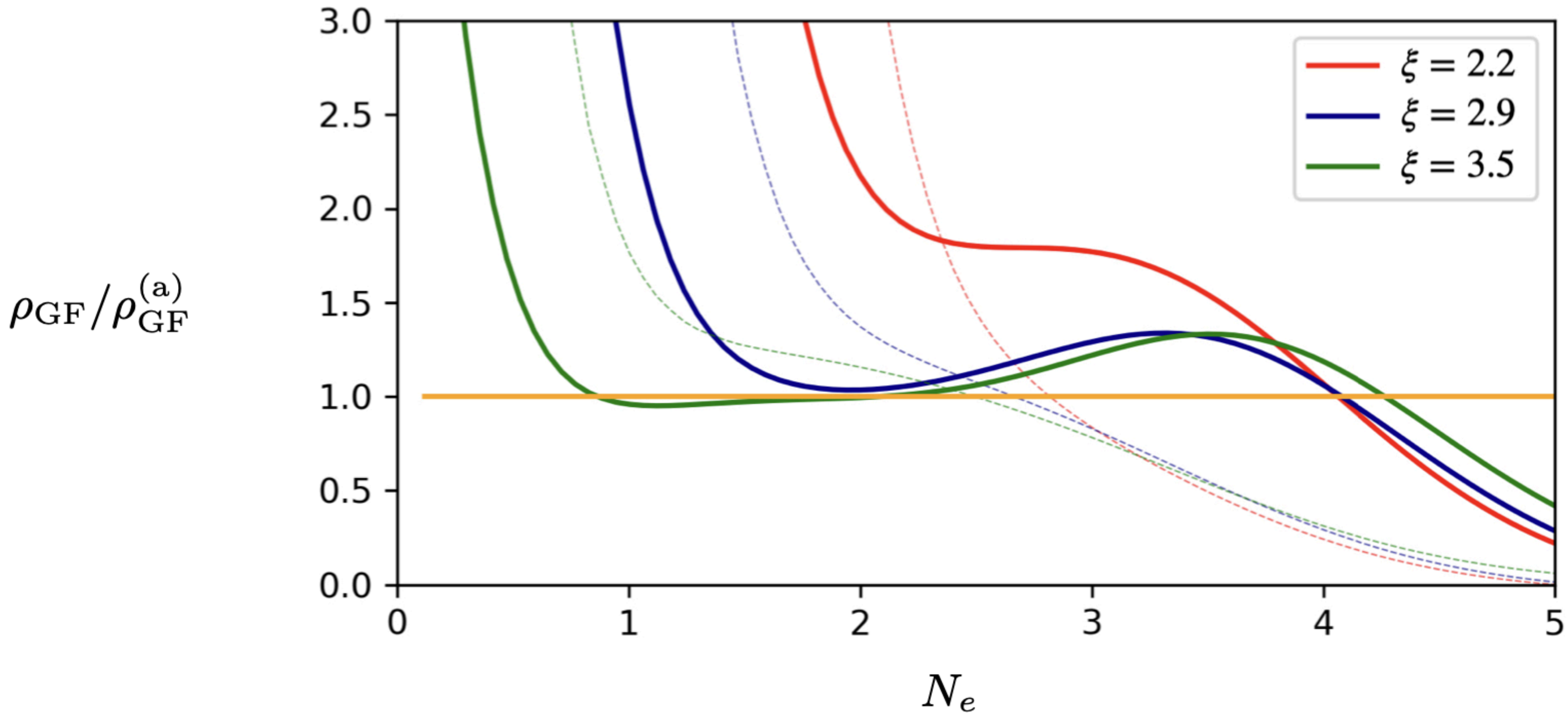
For tachyonic enhancement of gauge fields, discretization is more important

Solution: find a scheme with $k_{\rm sd} \simeq k_{\rm lapl}$, for example 4th order



Lattice simulation: axion-U(1)

We also studied the vacuum contribution to gauge field energy-density



Analytical prediction with cut-off regularisation:

$$\rho_{\rm GF}^{(a)} = \frac{1}{4\pi^2 a^2} \int_{(8\xi)^{-1}}^{2\xi} \left[\; A'_- {}^{2} + k^2 \; A_- {}^{2} \right] = \frac{6!}{2^{19}\pi^2} \frac{H^4}{\xi^3} e^{e\pi\xi}$$

Results of the simulation:

1. Large scales: small $\xi = \frac{\alpha\dot{\phi}}{2fH}$ perturbative regime (no backreaction)

2. Small scales: large $\xi = \frac{\alpha\dot{\phi}}{2fH}$ non-perturbative regime (with backreaction)

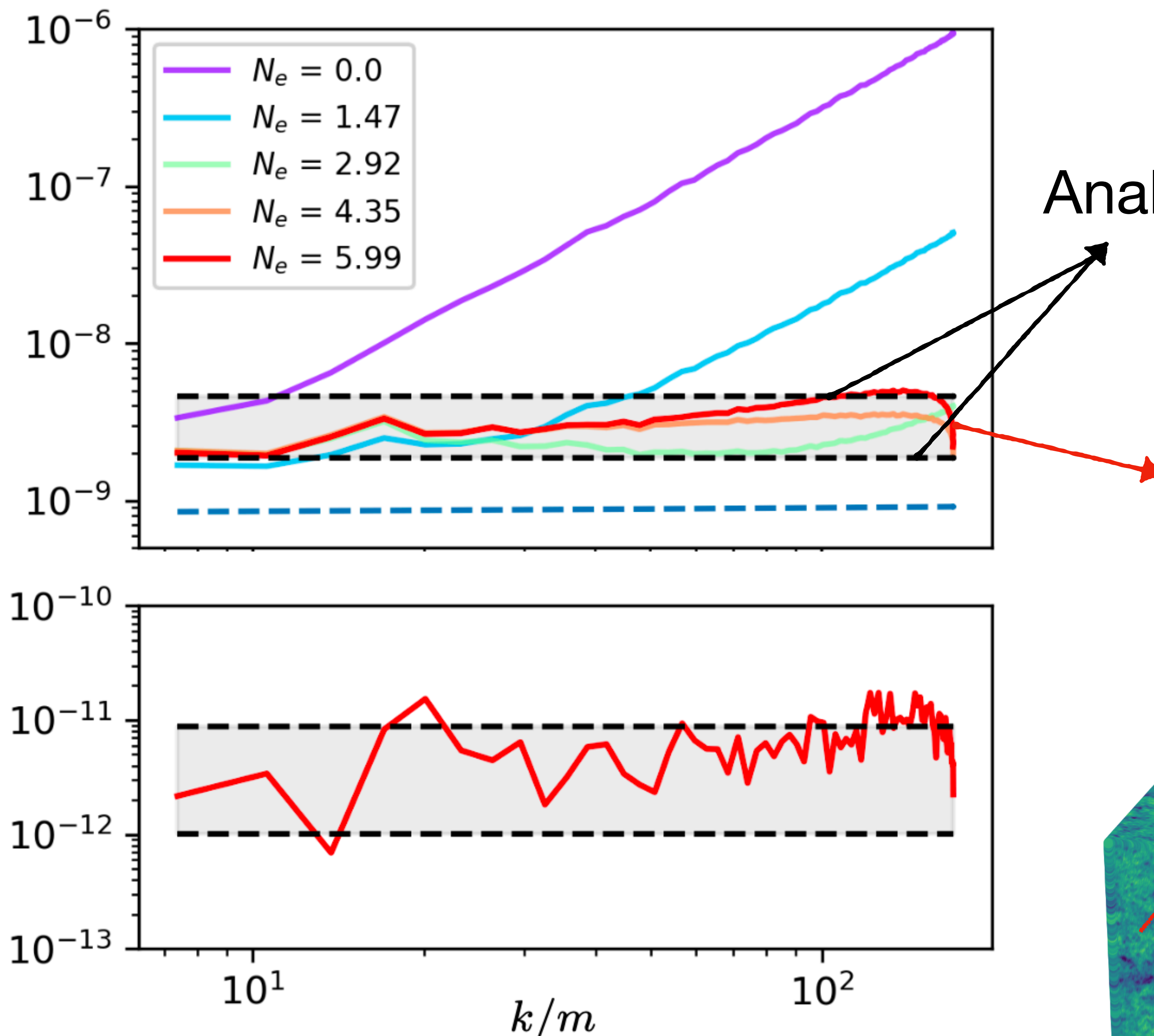
Perturbative regime (no backreaction)

AC+ 2204.12874

Simulation confirms analytical results (**<u>very nontrivial)</u>**

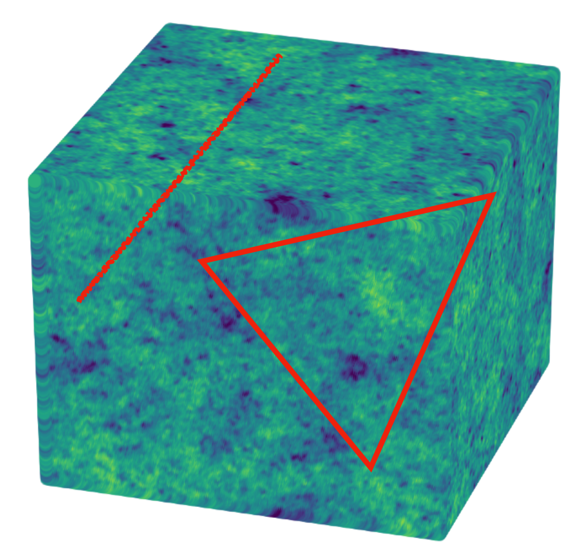
Power spectrum: $\mathcal{P}_\zeta$

Equilateral bispectrum: $k^6 \mathcal{B}_\zeta$



Analytical result

Lattice



Perturbative regime (no backreaction)

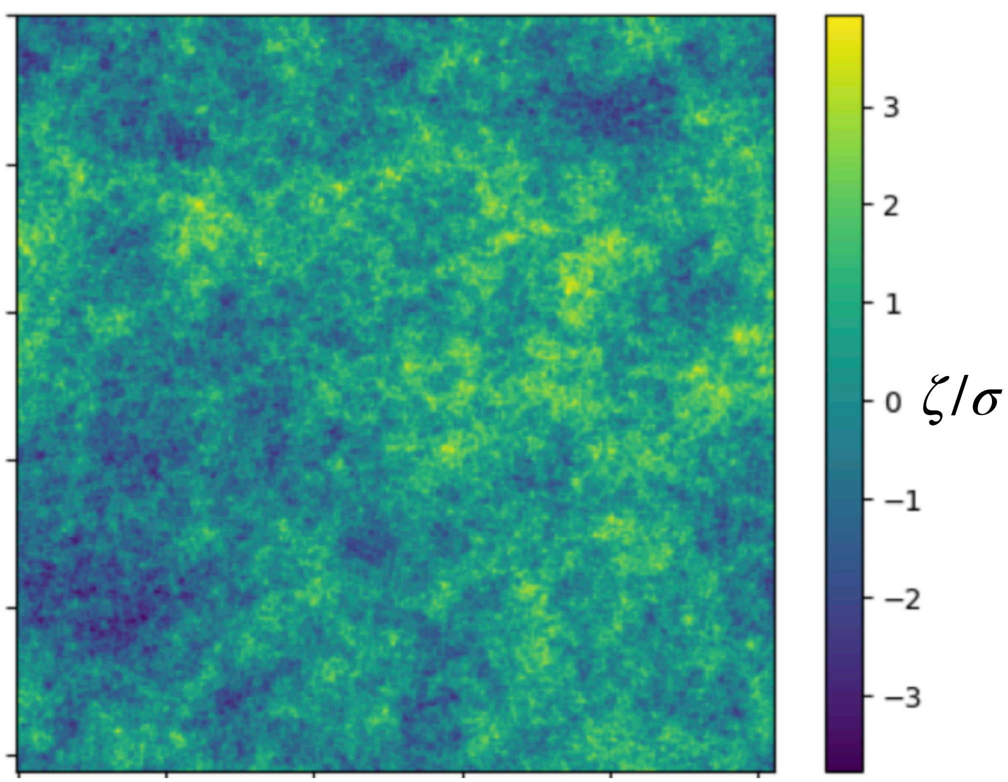
AC+ 2204.12874

Thanks to the lattice, we know $\zeta(\mathbf{x}, t)$!

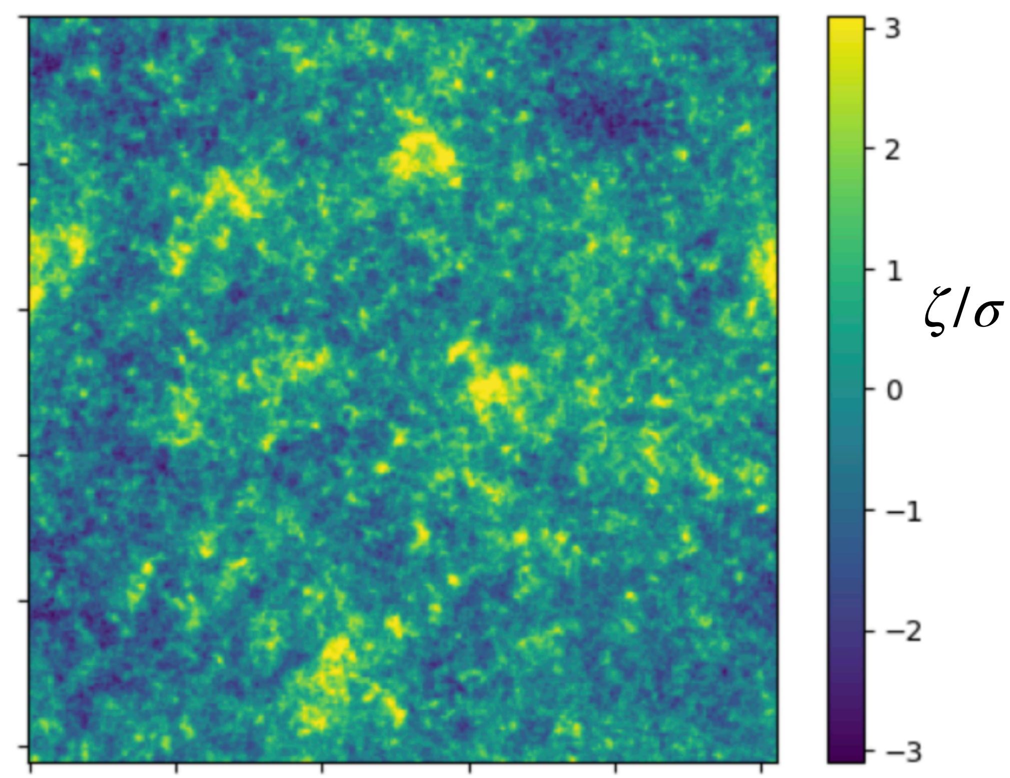
The first computation of nonlinear $\zeta(\mathbf{x}, t)$!

Beyond simplifying assumptions
$\zeta \simeq \zeta_G + f_{\rm NL} K[\zeta_G, \zeta_G]$

Gaussian (ζ_G)



Non-Gaussian (ζ_{NG})



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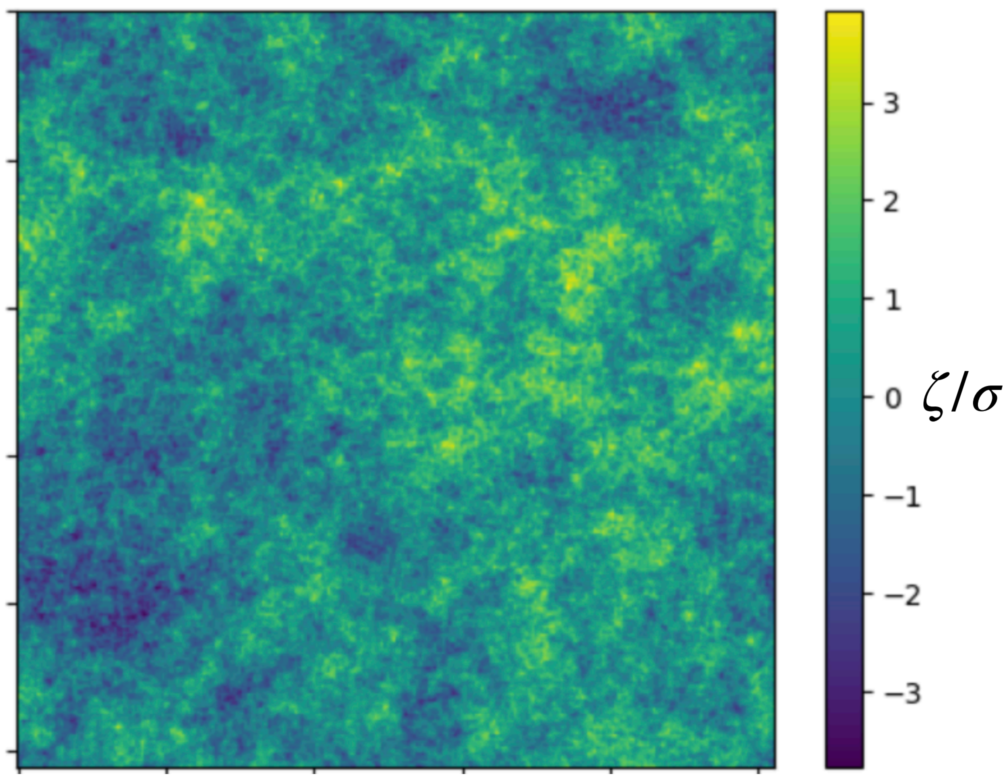

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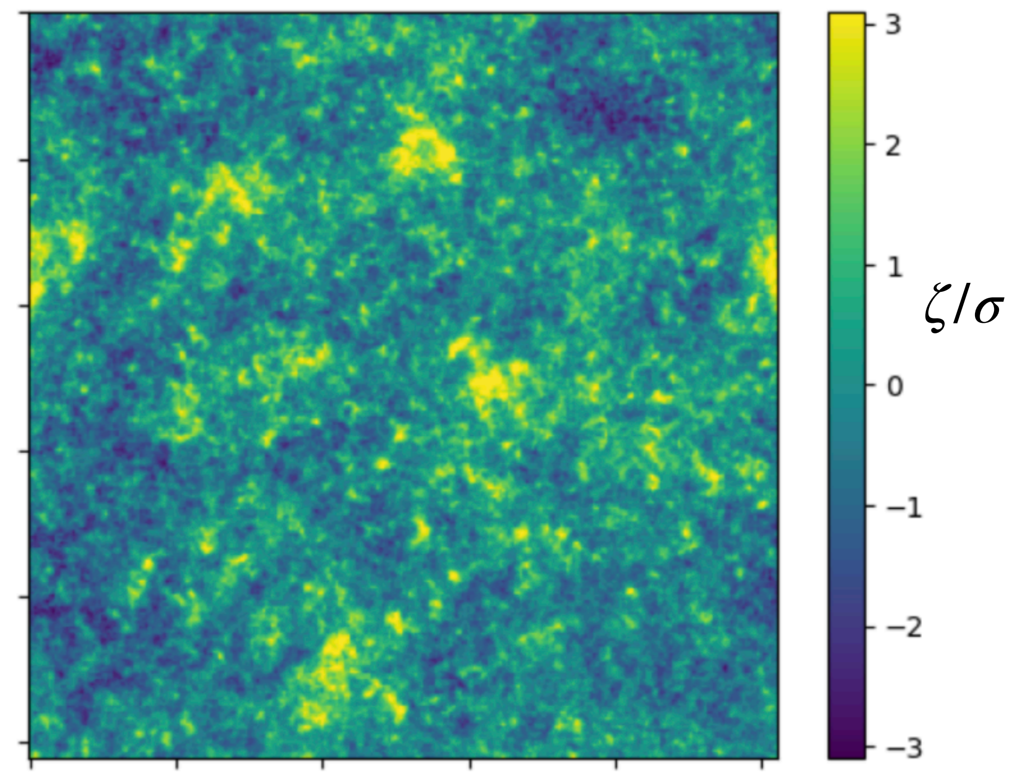
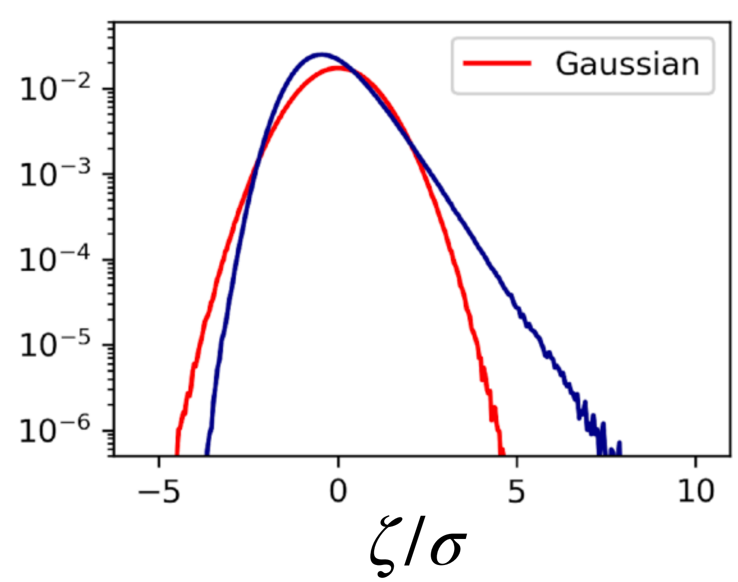




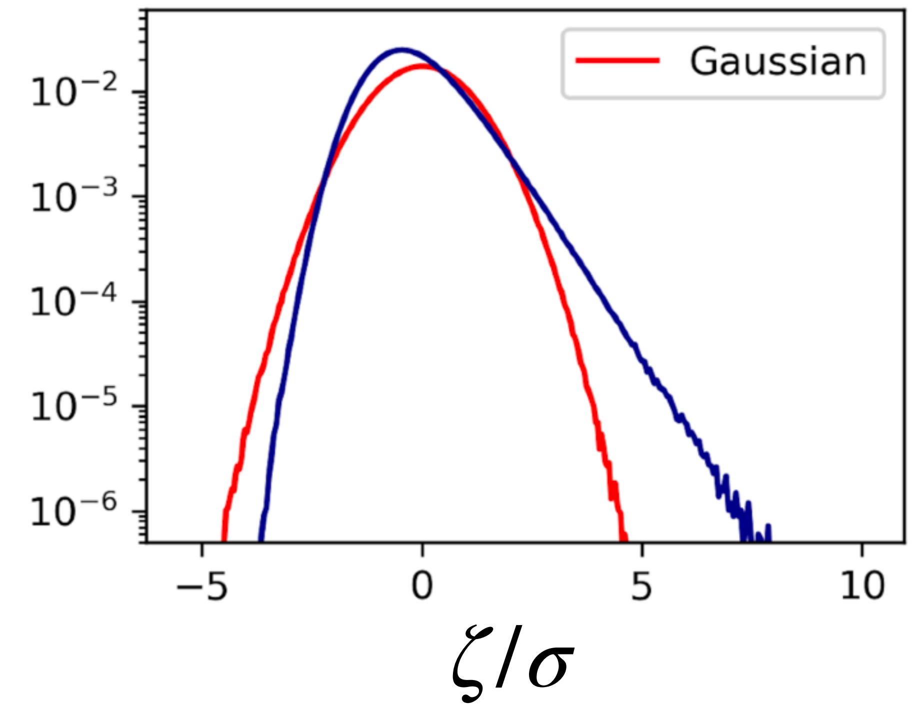
Perturbative regime (no backreaction)

AC+ 2204.12874

Define cumulants:

$$\kappa_n = \frac{\langle \zeta^n \rangle_c}{\sigma^n}$$

κ_3 "skewness", κ_4 "kurtosis", etc.



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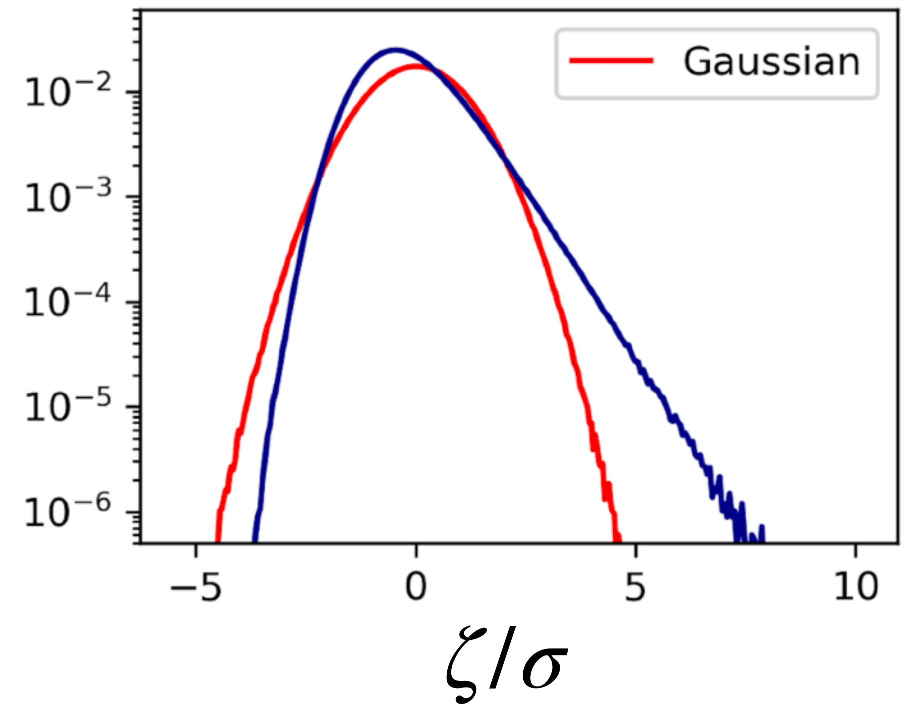

$$\ldots > \kappa_6 > \kappa_5 > \kappa_4 > \kappa_3$$

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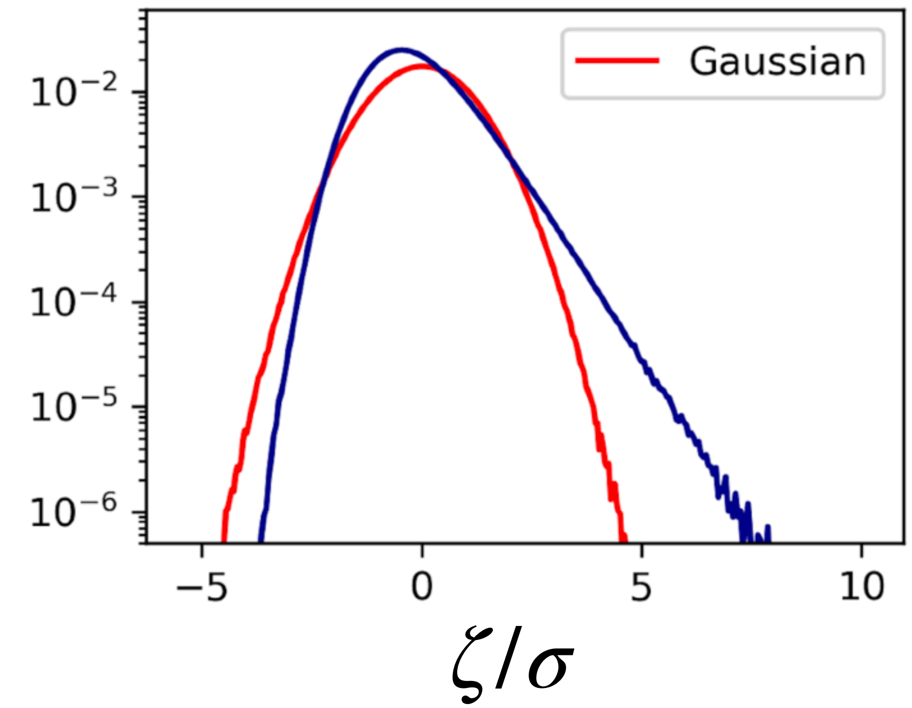

$$\ldots > \kappa_6 > \kappa_5 > \kappa_4 > \kappa_3$$

Higher order polyspectra

Trispectrum

Bispectrum

Perturbative regime (no backreaction)

AC+ 2204.12874

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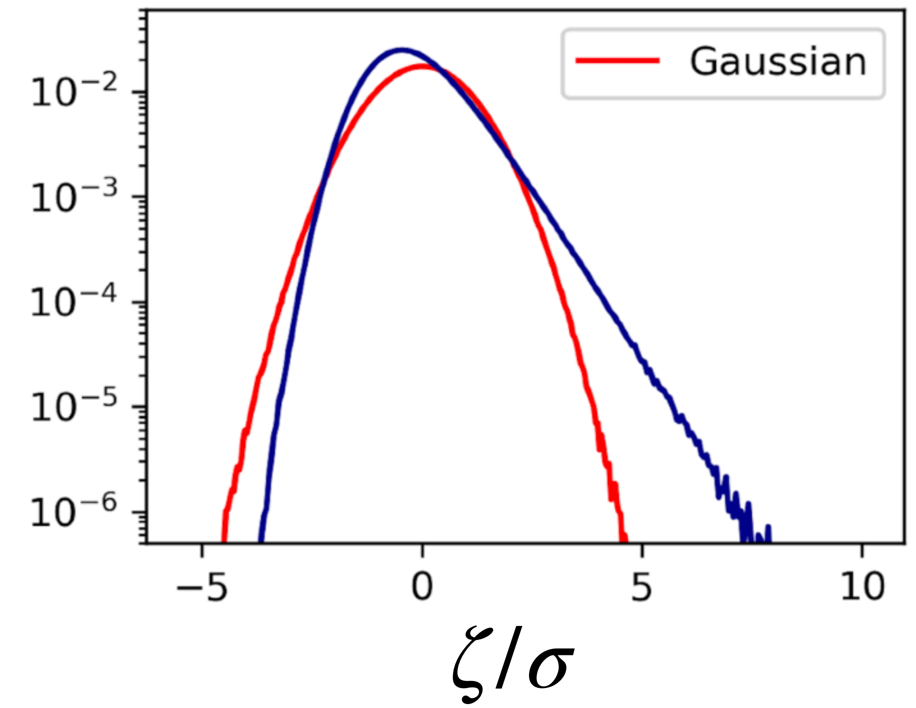


$$\ldots > \kappa_6 > \kappa_5 > \kappa_4 > \kappa_3$$

$$\zeta \neq \zeta_G + f_{\rm NL} K[\zeta_G, \zeta_G]$$

Backreaction

AC+ 2204.12874

Study transition linear $\longrightarrow$ nonlinear

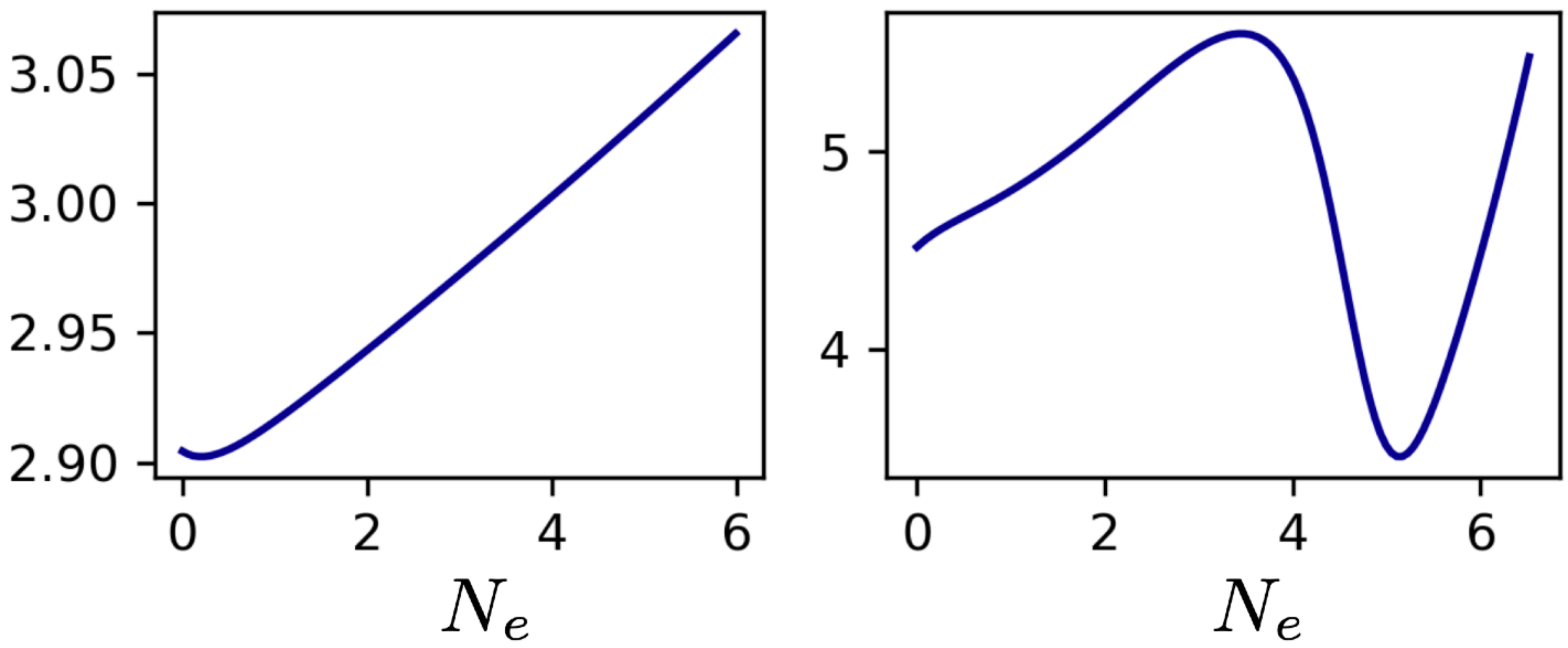
$$\xi = \frac{\alpha \dot{\phi}}{2fH}$$

Linear

(no backreaction)

Linear-nonlinear transition

(strong backreaction)



e-folds number (time)

Backreaction

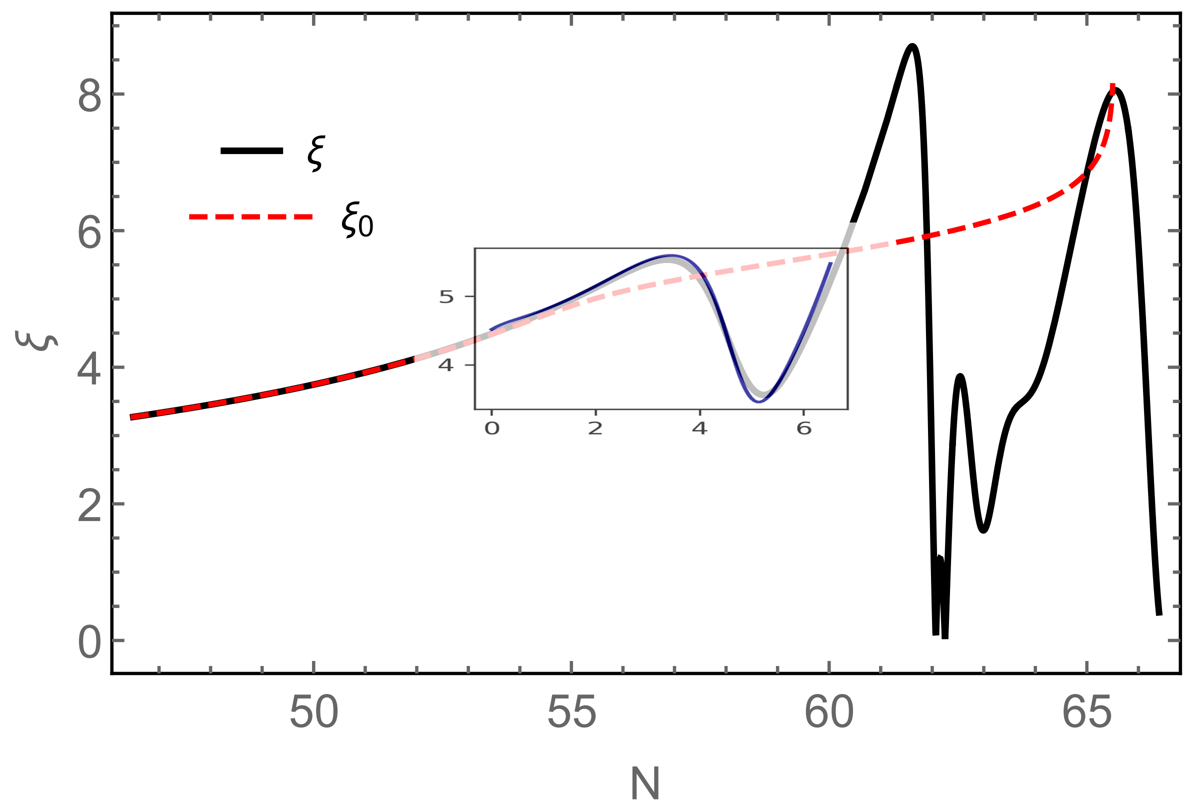


Figure from 2002.02952

The first lattice confirmation of what is found in:

[**V. Domcke, V. Guidetti, Y. Welling, A. Westphal** arXiv:2002.02952]

[**E.V. Gorbar, K. Schmitz, O. O. Sobol, S. I. Vilchinskii** arXiv:2109.01651]

Backreaction

AC+ 2204.12874

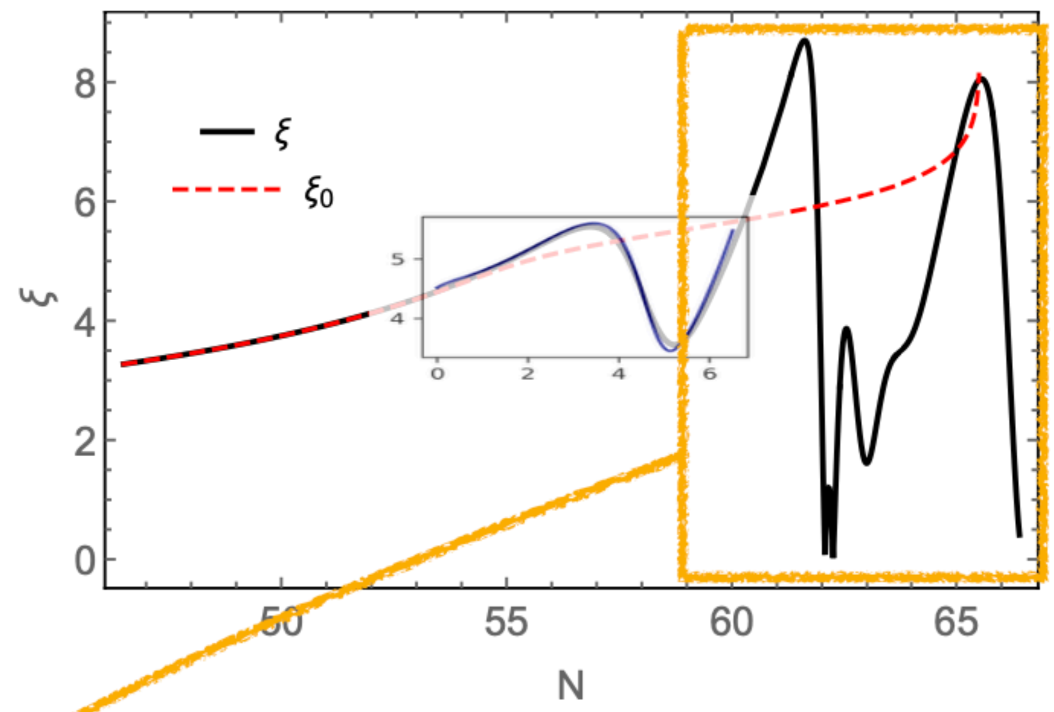


Figure from 2002.02952
[courtesy of V. Domcke]

What happens here is still under investigation. See e.g.:

D. Figueroa, J. Lizarraga, A. Urio, J. Urrestilla
2303.17436

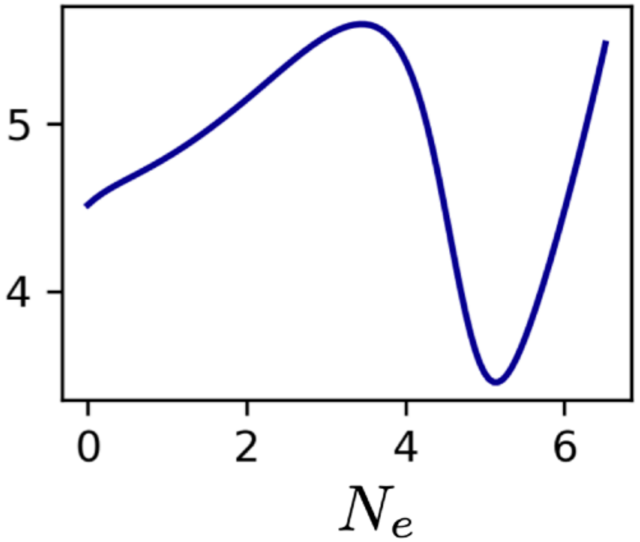
Lattice simulation

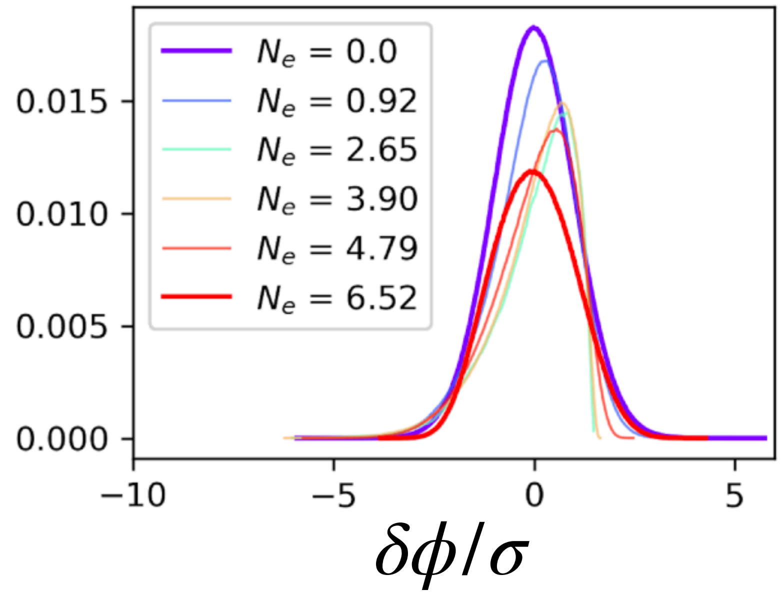
M. Peloso, L. Sorbo
2209.08131

Analytical

Backreaction: scalar statistics

AC+ 2204.12874

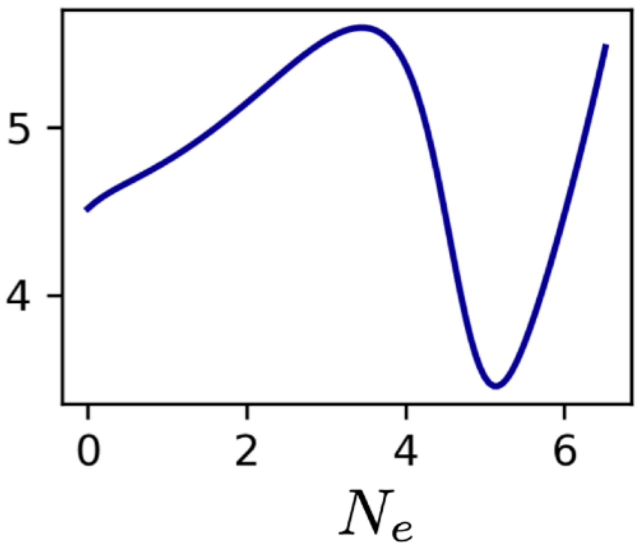


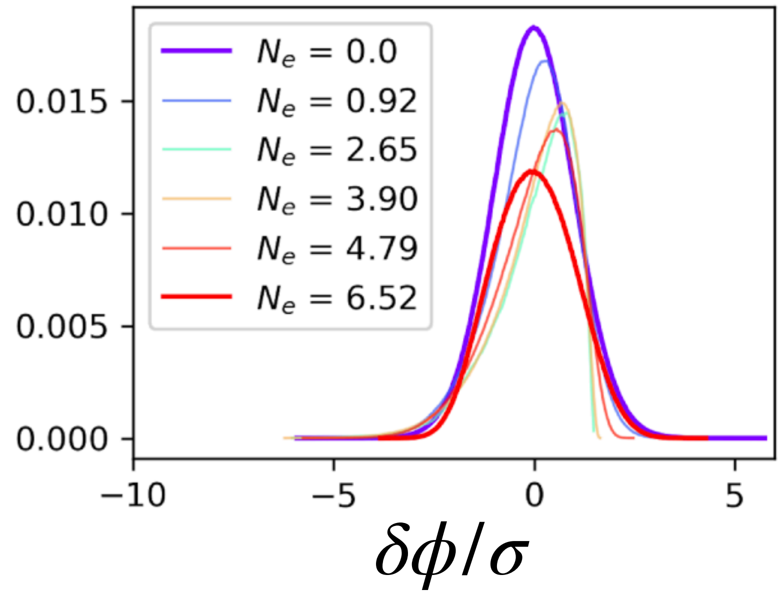


Non-Gaussianity is **suppressed** in the nonlinear regime!

Backreaction: scalar statistics

AC+ 2204.12874





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This invalidates PBH bounds, allowing for a sizeable GW signal at PTA

THE ASTROPHYSICAL JOURNAL LETTERS

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The NANOGrav 15 yr Data Set: Evidence for a Gravitational-wave Background

Gabriella Agazie[1], Akash Anumarlapudi[1], Anne M. Archibald[2], Zaven Arzoumanian[3], Paul T. Baker[4], Bence Bécsy[5], Laura Blecha[6], Adam Brazier[7,8], Paul R. Brook[9], Sarah Burke-Spolaor[10,11] + Show full author list

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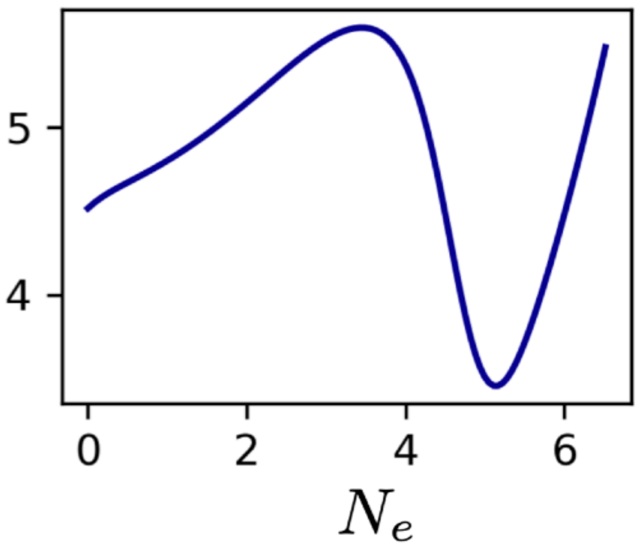
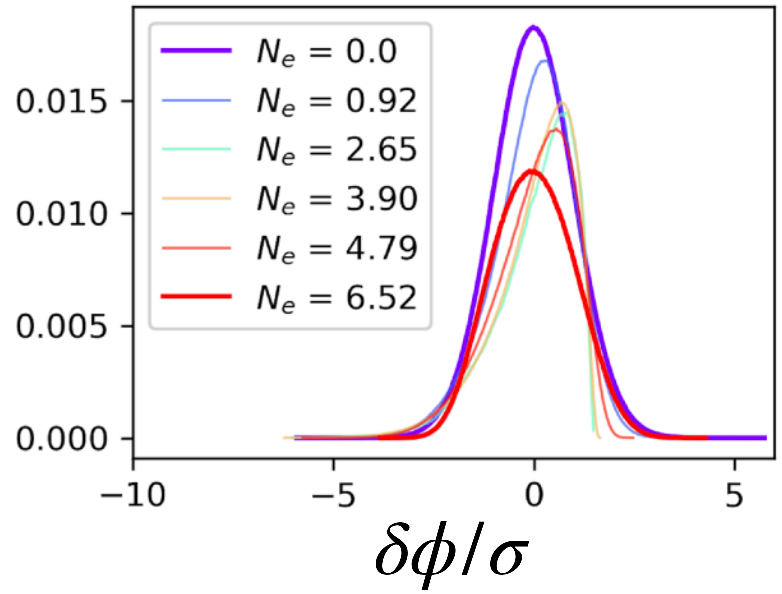
The Astrophysical Journal Letters, Volume 951, Number 1
Focus on NANOGrav's 15 yr Data Set and the Gravitational Wave Background
Citation Gabriella Agazie *et al* 2023 *ApJL* **951** L8
DOI 10.3847/2041-8213/acdac6

Backreaction: scalar statistics

AC+ 2204.12874





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NANOGrav signal from axion inflation

Xuce Niu,[a] and Moinul Hossain Rahat[b]

[a]Institute for Fundamental Theory, Department of Physic
Gainesville, FL 32611, USA
[b]School of Physics & Astronomy, University of Southampt

E-mail: xuce.niu@ufl.edu, M.H.Rahat@soton.ac.uk

Axion-Gauge Dynamics During Inflation as the Origin of Pulsar Timing Array Signals and Primordial Black Holes

Caner Ünal,[1,2,*] Alexandros Papageorgiou,[3,†] and Ippei Obata[4,‡]

[1]*Department of Physics, Ben-Gurion University of the Negev, Be'er Sheva 84105, Israel*
[2]*Feza Gursey Institute, Bogazici University, Cengelkoy, Istanbul, Turkey*
[3]*Particle Theory and Cosmology Group, Center for Theoretical Physics of the Universe, Institute for Basic Science (IBS), 34126 Daejeon, Korea*
[4]*Kavli Institute for the Physics and Mathematics of the Universe (Kavli IPMU, WPI), UTIAS, The University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba, 277-8583, Japan*

We demonstrate that the recently announced signal for a stochastic gravitational wave background (SGWB) from pulsar timing array (PTA) observations, if attributed to new physics, is compatible with primordial GW production due to axion-gauge dynamics during inflation. More specifically we find that axion-$U(1)$ models may lead to sufficient particle production to explain the signal while simultaneously source some fraction of sub-solar mass primordial black holes (PBHs) as a signature. Moreover there is a parity violation in GW sector, hence the model suggests chiral GW search as a concrete target for future. We further analyze the axion-$SU(2)$ coupling signatures and find that in the low/mild backreaction regime, it is incapable of producing PTA evidence and the tensor-to-scalar ratio is low at the peak, hence it overproduces scalar perturbations and PBHs.

Backreaction: scalar statistics

How does this compare with known results?

- Power spectrum: $$\mathcal{P}_\zeta(k) \simeq \mathcal{P}_{\rm vac} + \mathcal{P}_{\rm vac}^2 f_2(\xi) e^{4\pi\xi}$$

$$\mathcal{P}_{\rm vac} \simeq \frac{H^4}{(2\pi\dot{\phi})^2}$$

$$\xi = \frac{\alpha\dot{\phi}}{2fH}$$

- $$f_{\rm NL}^{\rm (equil.)}(\xi) \simeq \frac{f_3(\xi)\mathcal{P}_{\rm vac}^3 e^{6\pi\xi}}{\mathcal{P}_\zeta^2} \xrightarrow{\xi \gg 1} 0$$

$$\mathcal{P}_\zeta^2 \sim e^{8\pi\xi}$$

Backreaction: scalar statistics

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$$\langle \zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle \sim f_3(\xi)\mathscr{P}_{\rm vac}^3 e^{6\pi\xi}$$

$$\langle \zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3}) \overset{\xi \gg 1}{\gg} 1$$

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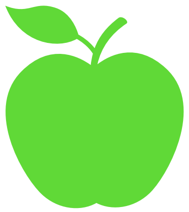

Backreaction: scalar statistics

AC+ 2204.12874

More carefully (still schematically):

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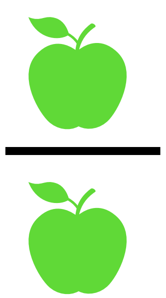

$$\frac{\langle \zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle}{\big(\langle \zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\rangle\big)^{3/2}} \xrightarrow{\xi \gg 1} \sim \frac{f_3(\xi)}{f_2^{3/2}(\xi)}$$

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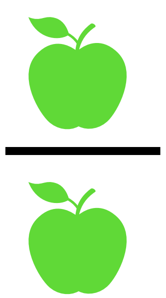

$$\kappa_3 \sim \frac{\langle \zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle}{\left(\langle \zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\rangle\right)^{3/2}} \xrightarrow{\xi \gg 1} \sim \frac{f_3(\xi)}{f_2^{3/2}(\xi)}$$

Backreaction: scalar statistics

What is the physical interpretation? **Central limit theorem!**

Look at the source term:

$$\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)(k) = \sum_{k'} F_{\mu\nu}(k')\ \tilde{F}^{\mu\nu}(k-k')\,.$$

Backreaction: scalar statistics

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$$\frac{1}{8\xi} < \frac{k'}{aH} < 2\xi \longrightarrow$$ Many terms for large ξ

Backreaction: scalar statistics

AC+ 2204.12874

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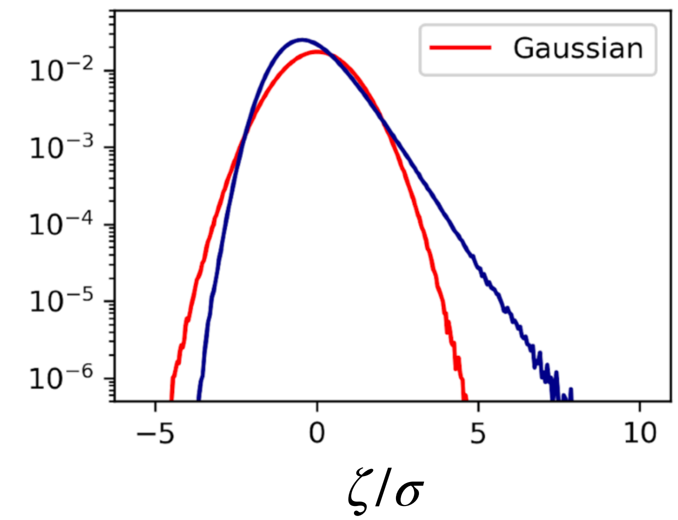
$$\frac{1}{8\xi} < \frac{k'}{aH} < 2\xi$$ → Many terms for large ξ

Analogous to fermion production: [**P. Adshead, L. Pearce, M. Peloso, M. A. Roberts, L. Sorbo** arXiv:1803.04501]

Summary

- First lattice simulation of an axion-gauge system during inflation



Results:

- Understanding non-Gaussinity beyond f_{NL}

$$\ldots > \kappa_6 > \kappa_5 > \kappa_4 > \kappa_3$$

- First step towards fully nonlinear understanding of backreaction in axion inflation