Magnetogenesis from axion-SU(2) inflation

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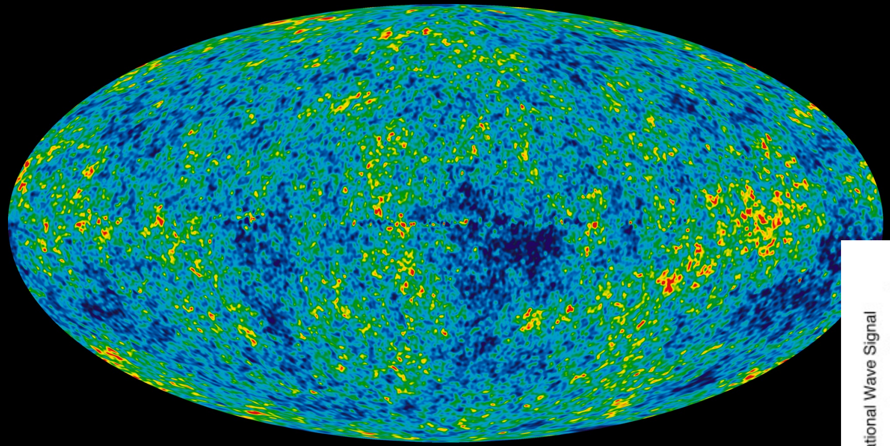
Outline:

- SU(2) gauge fields during inflation.
- Backreaction during axion-SU(2) inflation.
- Consequences for magentogenesis.

Preliminary!

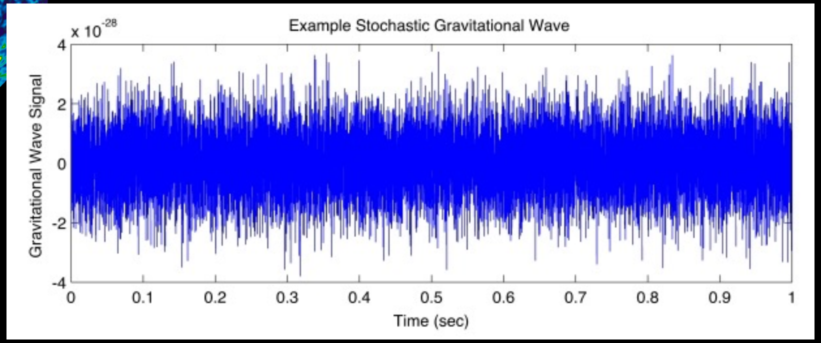
Trace multiple data points in a correlation

Scalar perturbations



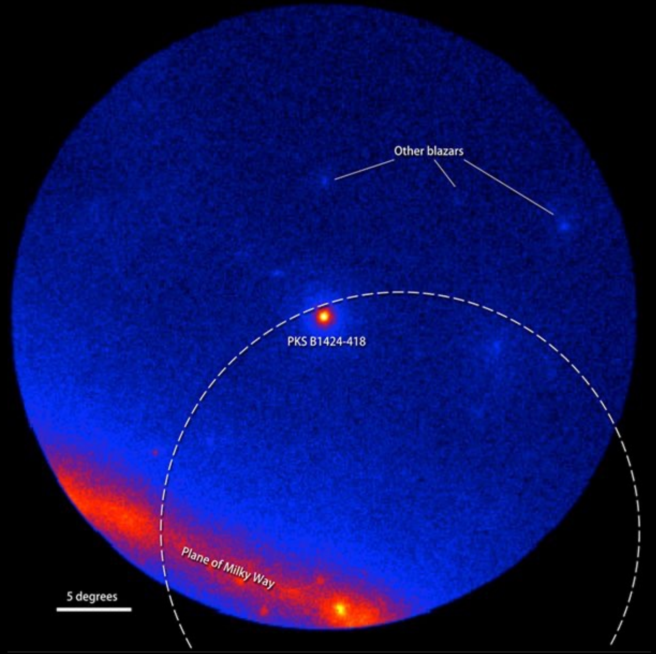
[Planck]

Graviational waves



[LIGO/VIRGO]

Primordial magnetic field



[NASA/DOE/LAT Collaboration]

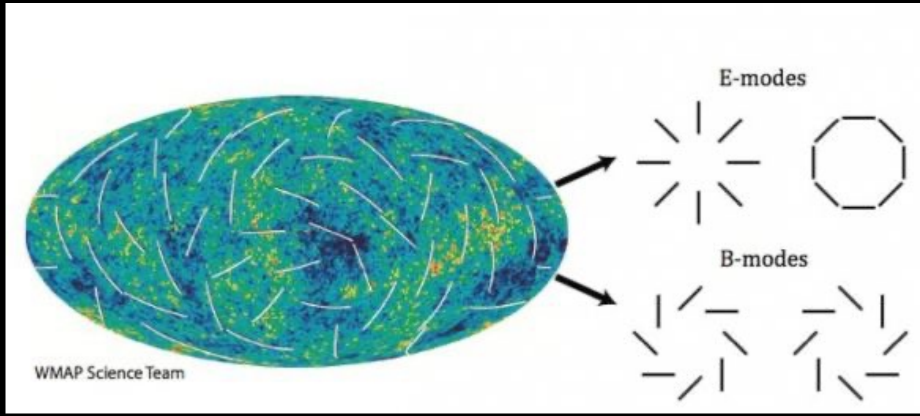
Signatures of GWs from gauge fields

[WMAP]

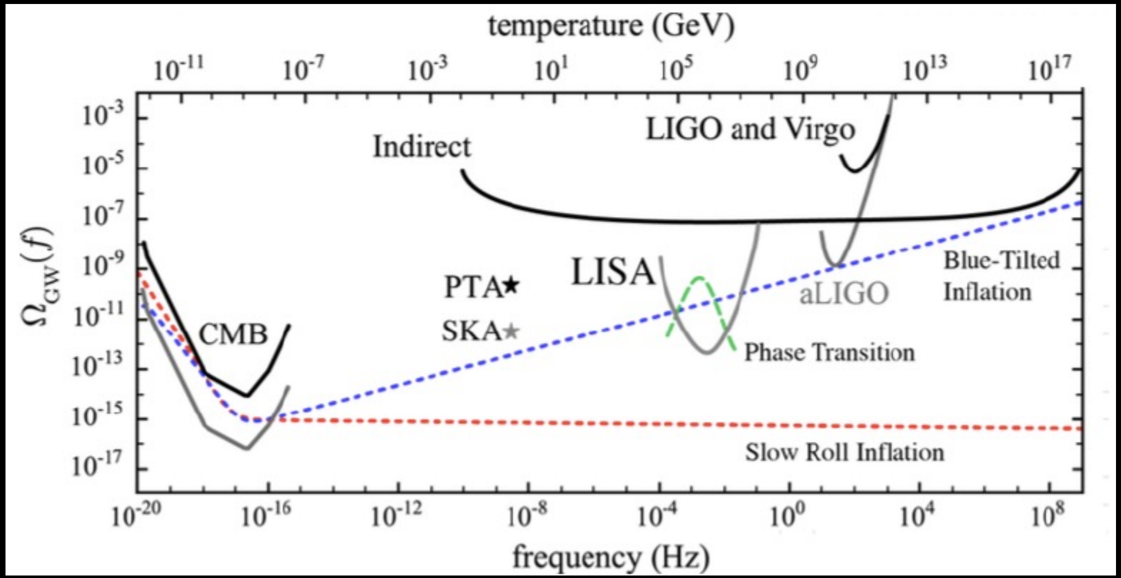
- Polarization: B-modes

+ parity odd CMB correlations

$$TB \neq 0, EB \neq 0$$

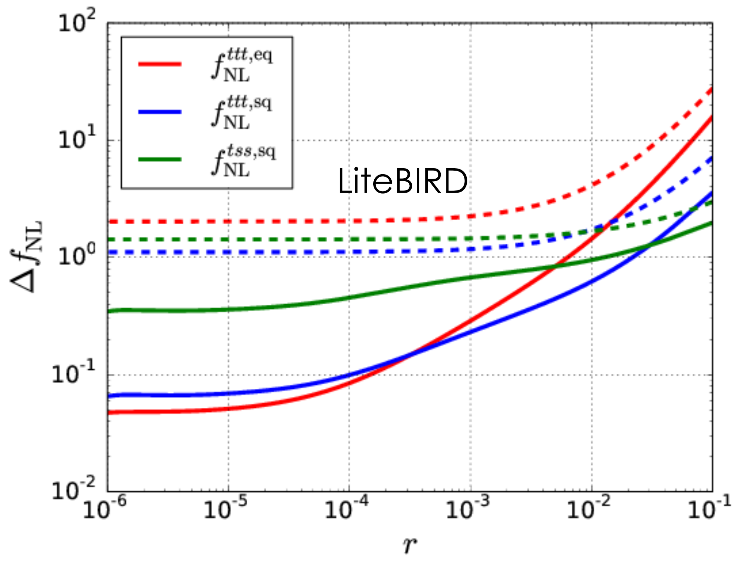


- Enhanced amplitude of GWs



[J. Garcia-Bellido]

- Non-zero tensor non-Gaussianity



[M. Shiraishi, 2019]

SU(2) gauge fields during inflation

Challenges ahead:

1) A homogeneous and isotropic background solution: Can be realised for SU(N) gauge fields.

[A.Maleknejad and M.M.Sheikh-Jabbari, 2011]

2) Dilution: $A_\mu \sim 1/a(t)$

New terms in gauge theory are required or a coupling with inflaton.

3) Respect gauge symmetry:

$$f^2(\phi)F^a_{\mu\nu}F^{a\,\mu\nu}, \quad \chi F\tilde{F}, \quad \left(F\tilde{F}\right)^2$$

SU(2) gauge fields during inflation

Gauge-flation: [A.Maleknejad and M.M.Sheikh-Jabbari, 2011]

$$S = \int d^4x \sqrt{-\det(g_{\mu\nu})}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \frac{\kappa}{96}\left(F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu}\right)^2\right]$$

Chromo-natural inflation: [P. Adshead , M. Wyman, 2012]

$$S = \int d^4x \sqrt{-\det(g_{\mu\nu})}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}\left((\partial\chi)^2 - \mu^4\left(1+\cos(\frac{\chi}{f})\right)\right) - \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \frac{\chi}{8f}\lambda F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu}\right]$$

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$$S=\int d^4x\sqrt{-\det(g_{\mu\nu})}\left[\frac{M_{\rm Pl}^2}{2}R-\underbrace{\frac{1}{2}\left((\partial\chi)^2-\mu^4\left(1+\cos(\frac{\chi}{f})\right)\right)}_{\text{Natural inflation}}-\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu}+\frac{\chi}{8f}\lambda F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu}\right]$$

Natural inflation

[K. Freese, J. A. Frieman and A. V. Olinto, 1990]

SU(2) gauge fields during inflation

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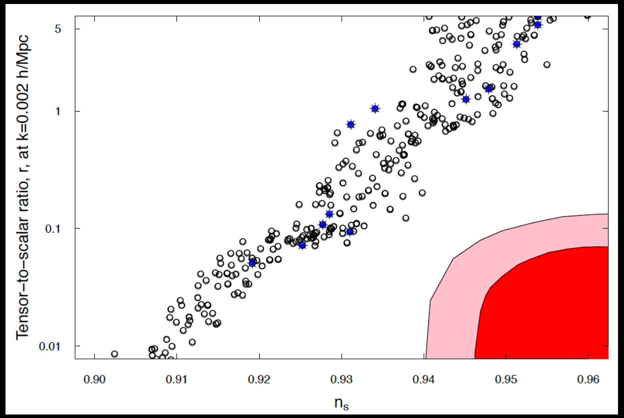
Ruled out by observations!



[P. Adshead, E. Martinec, and M. Wyman, 2013]

SU(2) gauge fields during inflation

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However, both models may be realised as a spectator sector.

[E. Dimastrogiovanni, M. Fasiello and T. Fujita, 2017]

[O.I. and E. I. Sfakianakis, 2021]

Spectator Chromo-natural inflation

[E. Dimastrogiovanni, M. Fasiello, T. Fujita, 2017]

$$S = \int d^4x \sqrt{-\det(g_{\mu\nu})} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right.$$
$$\left. - \frac{1}{2}\left((\partial\chi)^2 - \mu^4 \left(1 + \cos(\frac{\chi}{f}) \right) \right) - \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \frac{\chi}{8f} \lambda F^a_{\mu\nu} \tilde{F}^{a\,\mu\nu} \right]$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\epsilon^{abc} A^b_\mu A^c_\nu$$

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Isotropic solution for the background:

$$A^a_0 = 0,$$

$$A^a_i = \delta^a_i a(t) Q(t)$$

Spectator Chromo-natural inflation

[E. Dimastrogiovanni, M. Fasiello, T. Fujita, 2017]

$$S = \int d^4x \sqrt{-\det(g_{\mu\nu})} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{2}\left((\partial\chi)^2 - \mu^4 \left(1 + \cos(\frac{\chi}{f}) \right) \right) - \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \frac{\chi}{8f} \lambda F^a_{\mu\nu} \tilde{F}^{a\,\mu\nu} \right]$$

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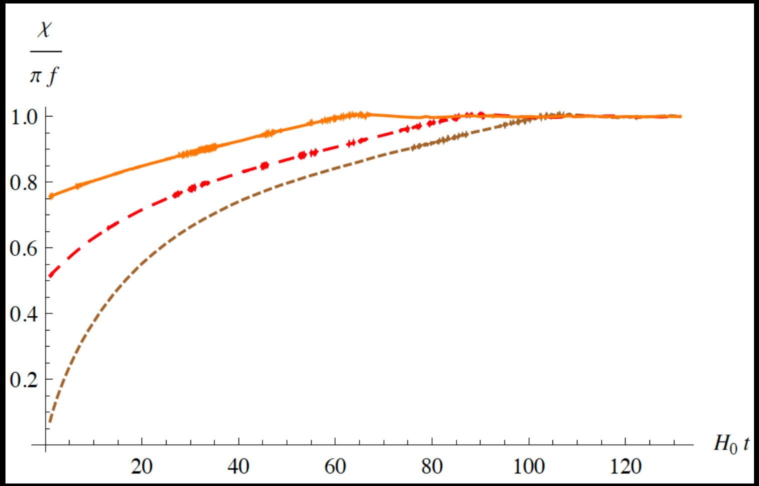

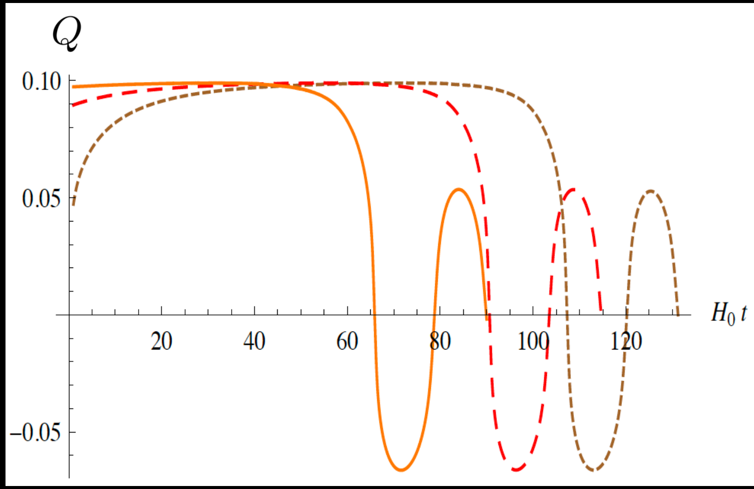

[A. Maleknejad, M.M. Sheikh-Jabbari, J. Soda]

Spectator Chromo-natural inflation

[E. Dimastrogiovanni, M. Fasiello, T. Fujita, 2017]

$$S = \int d^4x \sqrt{-\det(g_{\mu\nu})}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{2}\left((\partial\chi)^2 - \mu^4\left(1+\cos(\frac{\chi}{f})\right)\right) - \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \frac{\chi}{8f}\lambda F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu}\right]$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\epsilon^{abc}A^b_\mu A^c_\nu$$

Chromo-Natural inflation attractor:

[P. Adshead , M. Wyman, 2012]

$$\frac{\lambda}{f}\dot{\chi} = 2gQ + \frac{2H^2}{gQ}, \qquad \dot{Q} = -HQ + \frac{f}{3g\lambda}\frac{U_\chi}{Q^2}$$

The axion effective potential is minimized when:

$$Q \simeq \left(\frac{-fU_\chi(\chi)}{3g\lambda H}\right)^{1/3}$$

Tensor perturbations

$$\delta A^1_\mu = a(0, t_+, t_\times, 0), \quad \delta A^2_\mu = a(0, t_\times, -t_+, 0), \quad \delta g_{11} = -\delta g_{22} = a^2 h_+, \quad \delta g_{12} = a^2 h_\times$$

$$h_+ = \frac{h_L + h_R}{\sqrt{2}}, \quad h_\times = \frac{h_L - h_R}{i\sqrt{2}}, \quad t_+ = \frac{t_L + t_R}{\sqrt{2}}, \quad t_\times = \frac{t_L - t_R}{i\sqrt{2}}$$

Tensor perturbations

$$\psi''_{R,L} + \left(k^2 - \frac{2}{\eta^2}\right)\psi_{R,L} = f_1(T_{R,L}, m_Q, \dot{Q}, k, \eta)$$

$$T''_{R,L} + \left\{k^2 + \frac{2}{\eta^2}\left[m_Q\xi \pm k\eta(m_Q + \xi)\right]\right\}T_{R,L} = f_2(\psi_{R,L}, m_Q, \dot{Q}, k, \eta)$$

$$h_{L,R} = \frac{\sqrt{2}}{M_p a}\psi_{L,R}, \qquad t_{L,R} = \frac{1}{\sqrt{2}a}T_{L,R}$$

Tensor perturbations

$$\psi''_{R,L} + \left(k^2 - \frac{2}{\eta^2}\right)\psi_{R,L} = f_1(T_{R,L}, m_Q, \dot{Q}, k, \eta)$$

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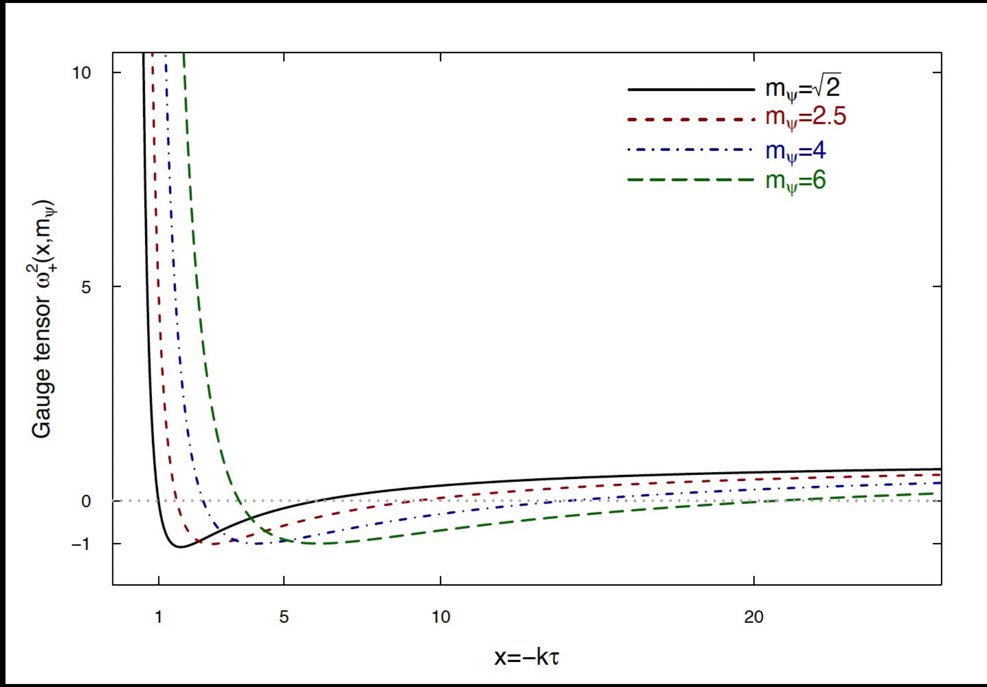


[P. Adshead, E. Martinec, M. Wyman, 2013]

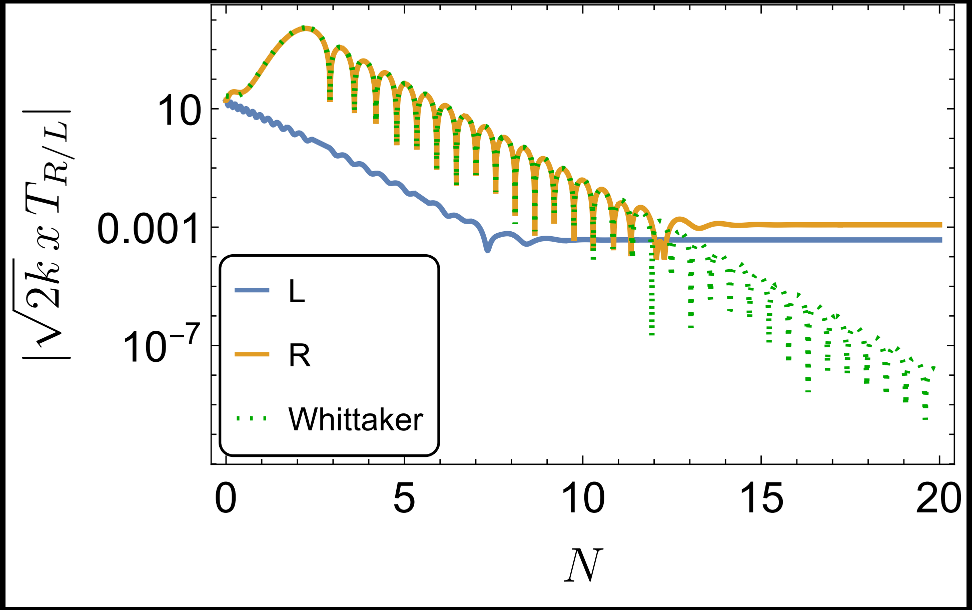
$$\xi = \frac{\lambda}{2fH}\dot{\chi} \qquad m_Q = \frac{gQ}{H}$$

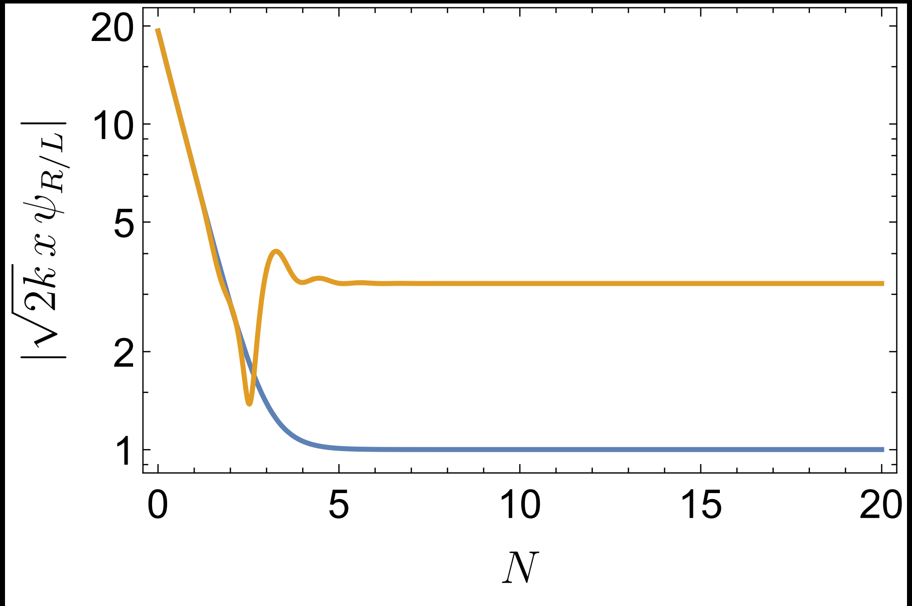
$$w^2_{R,L} = \left\{k^2 + \frac{2}{\eta^2}\left[m_Q\xi \pm k\eta(m_Q + \xi)\right]\right\}$$

$$\rightarrow k_{\min}, k_{\max}$$

Tensor perturbations

$$m_Q = 3.95, \quad g = 10^{-2}$$



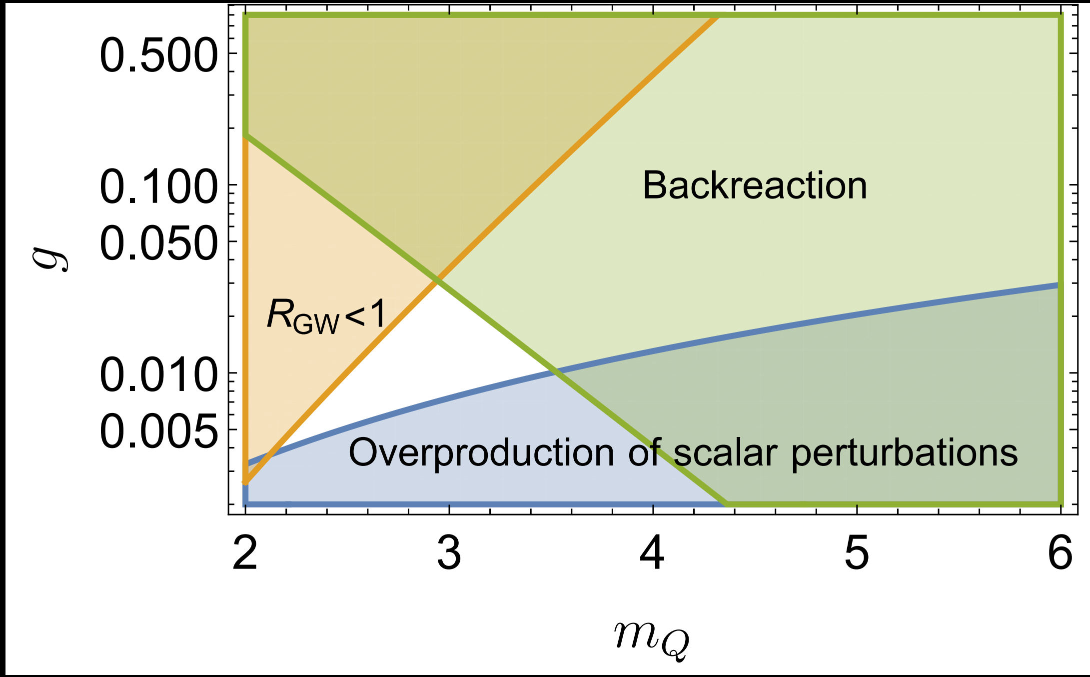


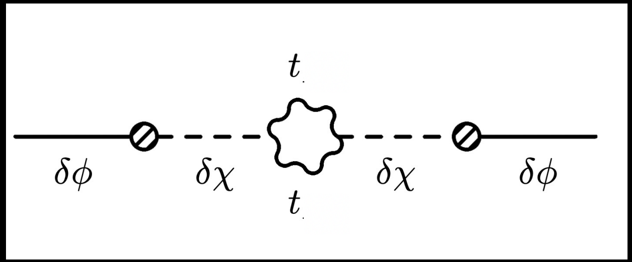
$$\mathcal{P}_h^{(s)}(k) = \frac{H^2}{\pi^2 M_{\rm pl}^2} \left| \sqrt{2k} \left(-\frac{k}{aH} \right) \lim_{\eta \to 0} \psi_R^{(s)}(k, \eta) \right|^2$$

$$\mathcal{P}_h^{\rm tot}(k) = \mathcal{P}_h^{(s)}(k) + \mathcal{P}_h^{(v)}(k)$$

Viable parameter space in spectator axion-SU(2) inflation

$$\mathcal{R}_{\rm GW} = \frac{\mathcal{P}_h^{(s)}}{\mathcal{P}_h^{(v)}} \qquad m_Q = \frac{gQ}{H}$$



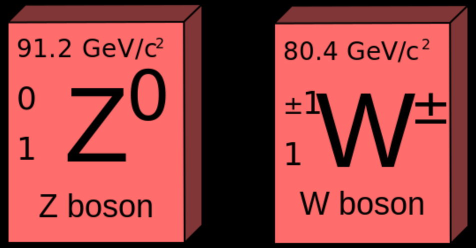


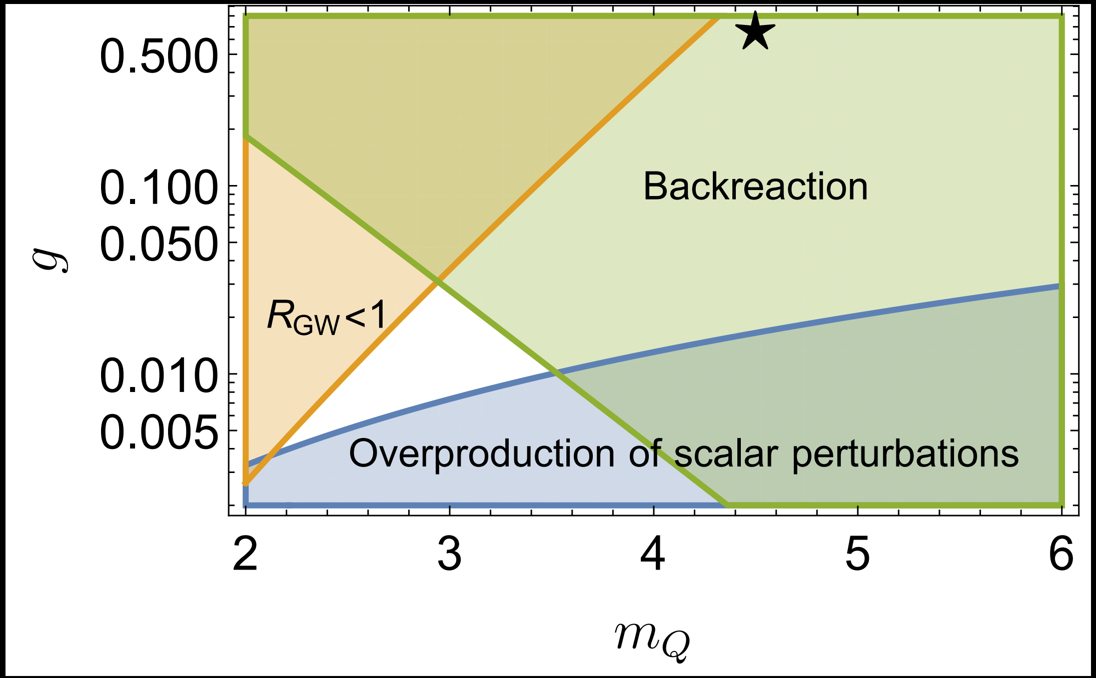
[E. Dimastrogiovanni, M. Fasiello and T. Fujita, 2017]

[A.Papageorgiou, M. Peloso, C. Unal, 2019]

Identifying Standard Model weak bosons as the SU(2) sector

$SU(2) \longleftrightarrow$

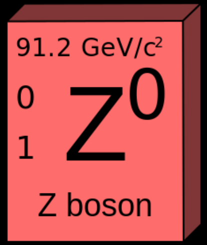


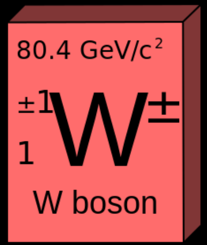


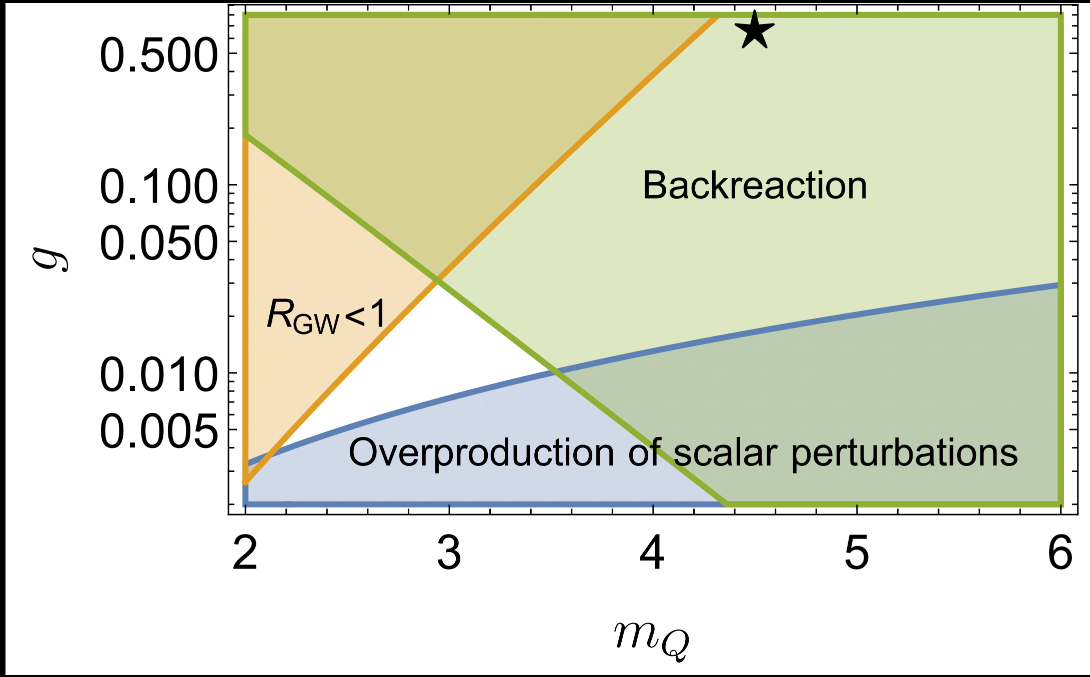
$$g \propto \mathcal{O}(0.1)$$

Identifying Standard Model weak bosons as the SU(2) sector

$SU(2) \longleftrightarrow$







To study magnetogenesis

we need to know dynamics in

the backreaction regime!

$$g \propto \mathcal{O}(0.1)$$

Backreaction in axion-U(1) inflation

[M. Anber, L. Sorbo, 2009]

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm pl}^2}{2} R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} - \frac{\alpha_\Lambda}{4} \frac{\phi}{M_{\rm pl}} F^a_{\mu\nu} \tilde{F}^{a\,\mu\nu} \right]$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Backreaction in axion-U(1) inflation

[M. Anber, L. Sorbo, 2009]

$$\begin{cases} \ddot{\phi} = -3H\dot{\phi} + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{\alpha_\Lambda}{a^3 m_p}\vec{E}\cdot\vec{B} \\ \dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{\alpha_\Lambda}{a m_p}\left(\dot{\phi}\vec{B} - \vec{\nabla}\phi\times\vec{E}\right) \\ \ddot{a} = -\frac{a}{3m_p^2}\left(2\rho_K - \rho_V + \rho_{EM}\right) \end{cases}$$

$$\begin{cases} \vec{\nabla}\cdot\vec{E} = -\frac{\alpha_\Lambda}{a m_p}\vec{\nabla}\phi\cdot\vec{B} \\ H^2 = \frac{1}{3m_p^2}\left(\rho_K + \rho_G + \rho_V + \rho_{EM}\right) \end{cases}$$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$
$$\vec{E} = \partial_t\vec{A}$$

$$V(\phi) = \frac{1}{2}m^2\phi^2$$ ← axion field

Backreaction in axion-U(1) inflation

$$\begin{cases} \ddot{\phi} = -3H\dot{\phi} + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{\alpha_\Lambda}{a^3 m_p}\vec{E}\cdot\vec{B} \\ \dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{\alpha_\Lambda}{a m_p}\left(\dot{\phi}\vec{B} - \vec{\nabla}\phi\times\vec{E}\right) \\ \ddot{a} = -\frac{a}{3m_p^2}\left(2\rho_K - \rho_V + \rho_{EM}\right) \end{cases}$$

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Homogeneous backreaction:

Backreaction in axion-U(1) inflation

$$
\begin{cases}
\ddot{\phi} = -3H\dot{\phi} + \frac{1}{a^2}\cancel{\vec{\nabla}^2\phi} - m^2\phi + \frac{\alpha_\Lambda}{a^3 m_p}\vec{E}\cdot\vec{B} \\
\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{\alpha_\Lambda}{a m_p}\left(\dot{\phi}\vec{B} - \cancel{\vec{\nabla}\phi\times\vec{E}}\right) \\
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\end{cases}
$$

$$
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\end{cases}
$$

Homogeneous backreaction:

neglect: $\vec{\nabla}^2\phi,\ \vec{\nabla}\phi\times\vec{E},\ \langle(\vec{\nabla}\phi)^2\rangle$

Backreaction in axion-U(1) inflation

$$\begin{cases} \ddot{\phi} = -3H\dot{\phi} + \frac{1}{a^2}\cancel{\vec{\nabla}^2\phi} - m^2\phi + \frac{\alpha_\Lambda}{a^3 m_p}\langle \vec{E}\cdot\vec{B}\rangle \\ \dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\langle \vec{\nabla}\times\vec{B}\rangle - \frac{\alpha_\Lambda}{a m_p}\left(\dot{\phi}\vec{B} - \cancel{\vec{\nabla}\phi\times\vec{E}}\right) \\ \ddot{a} = -\frac{a}{3m_p^2}\left(2\rho_K - \rho_V + \rho_{EM}\right) \end{cases}$$

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Homogeneous backreaction:

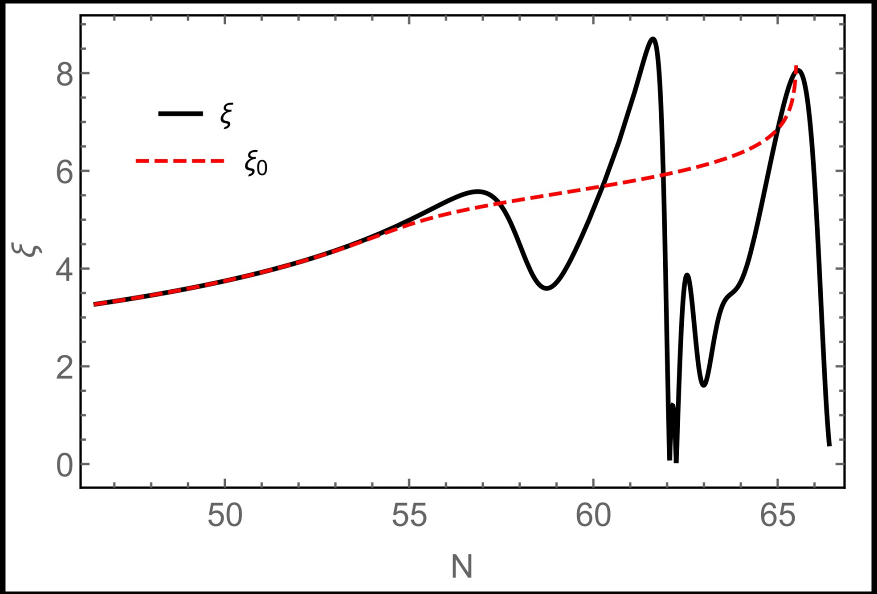
neglect: $\vec{\nabla}^2\phi,\ \vec{\nabla}\phi\times\vec{E},\ \langle(\vec{\nabla}\phi)^2\rangle$

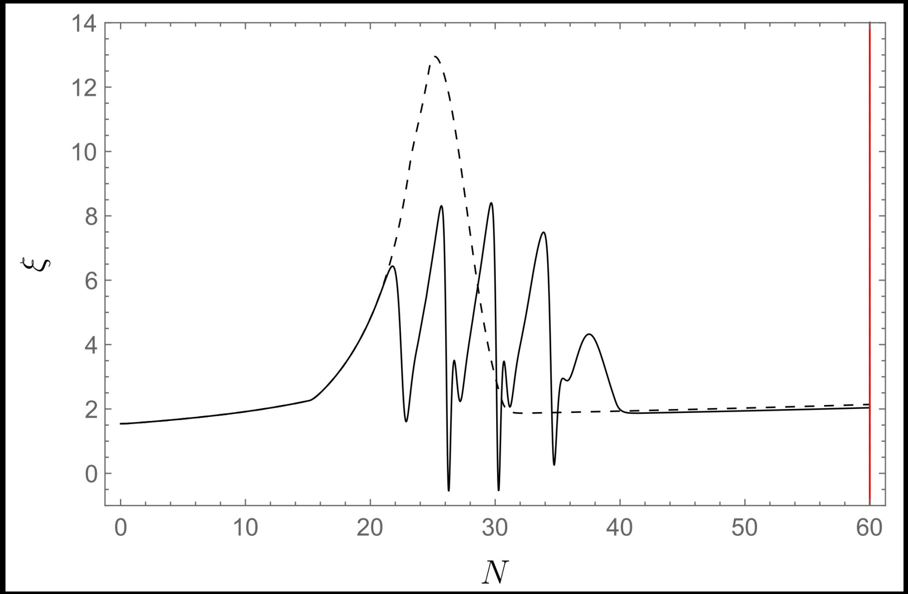
volume average: $\vec{E}\cdot\vec{B}\to\langle\vec{E}\cdot\vec{B}\rangle$

Homogeneous backreaction in axion-U(1) inflation

See talk by V. Domcke

$$\xi = \frac{\dot{\phi}}{2fH}$$





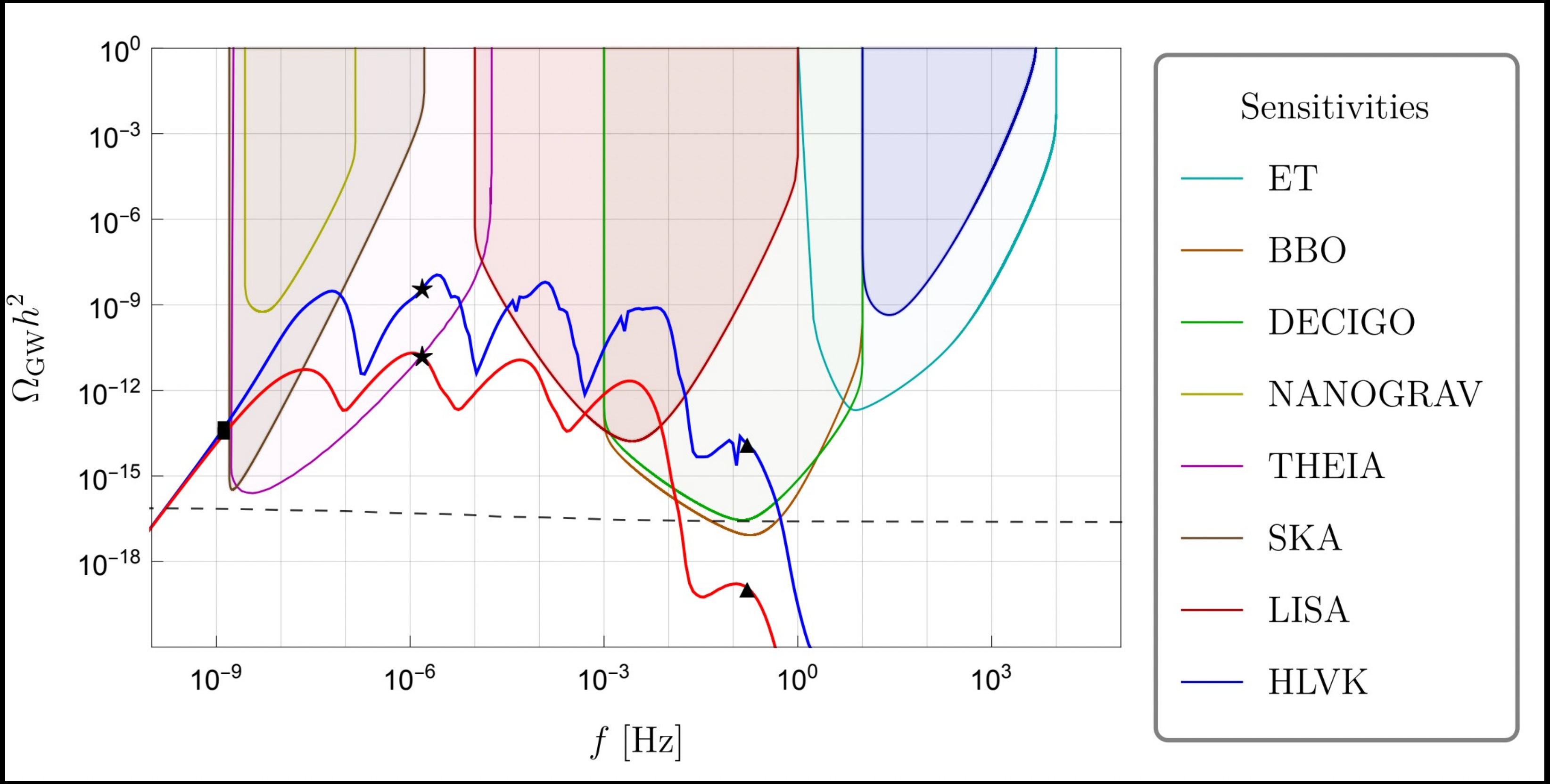
[S.-L. Cheng , W. Lee, K.-W. Ng, 2016]

...

[V. Domcke ,V. Guidetti, Y. Welling, A. Westphal, 2020]

[J. Garcia-Bellido, A. Papageorgiou, M. Peloso and L. Sorbo, 2023]

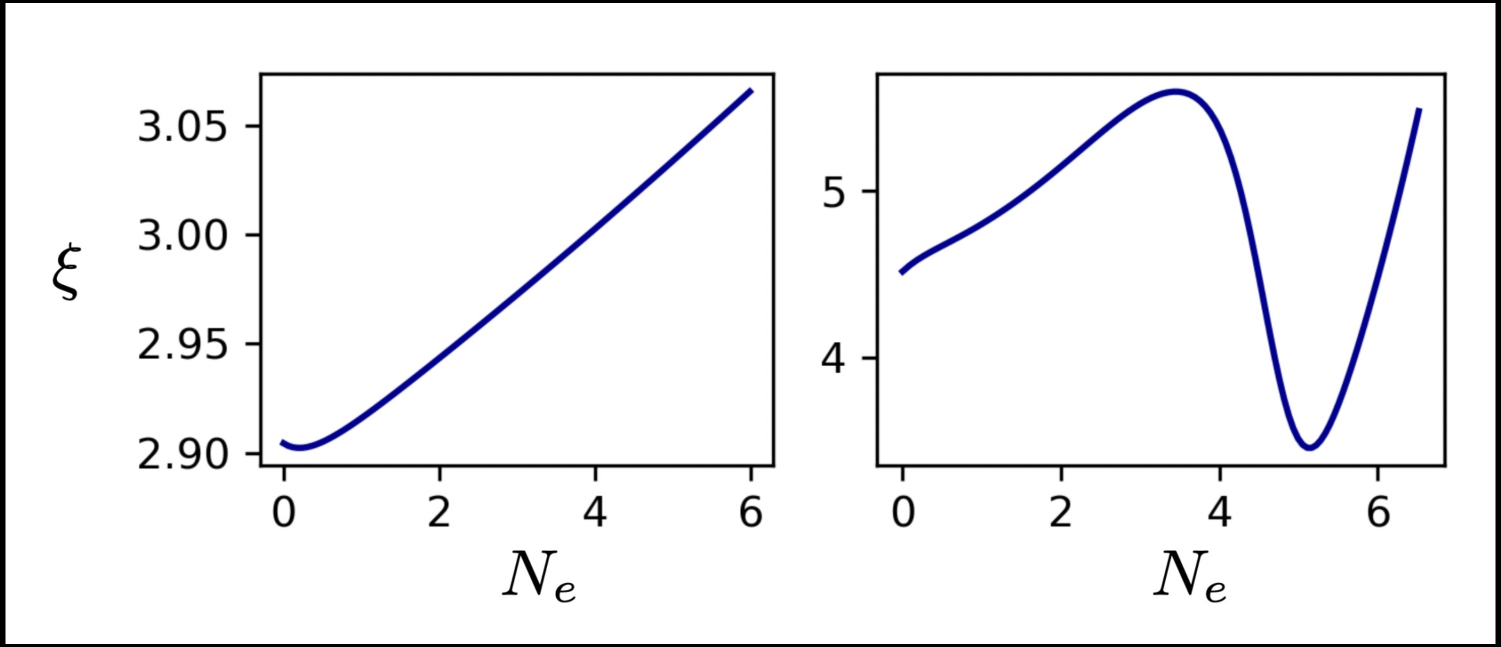
Homogeneous backreaction in axion-U(1) inflation



[J. Garcia-Bellido, A. Papageorgiou, M. Peloso and L. Sorbo, 2023]

"A flashing beacon in axion inflation"

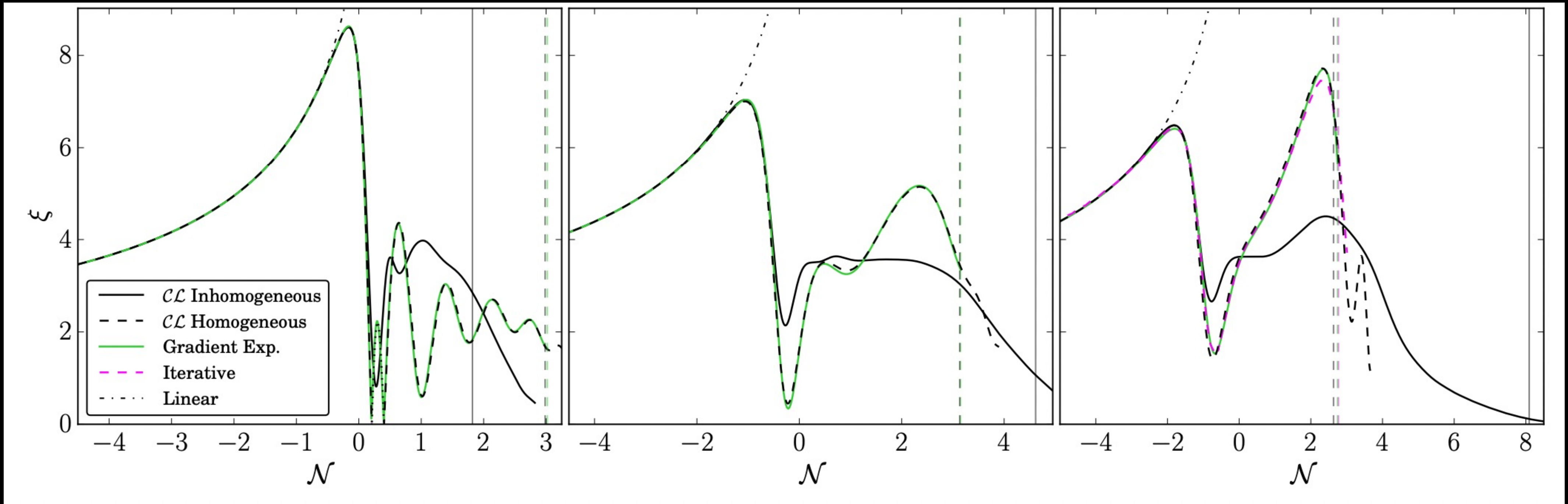
Inhomgeneous backreaction in axion-U(1) inflation. Lattice simulations.



[A. Caravano, E. Komatsu, K.D. Lozanov and J. Weller, 2022]

See talks by
D. Figueroa
A .Caravano

Inhomgeneous backreaction in axion-U(1) inflation. Lattice simulations.



[D.G. Figueroa, J. Lizarraga, A. Urio and J. Urrestilla, 2023]

CosmoLattice

See talks by
D. Figueroa
A .Caravano

Backreaction of axion-SU(2) dynamics during inflation

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Ramkishor Sharma,[a,b] **and Axel Brandenburg**[a,e,f,g]

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CASE WESTERN RESERVE UNIVERSITY

Stockholm University

NORDITA

Homogeneous backreaction in axion-SU(2) inflation

[E. Dimastrogiovanni, M. Fasiello and T. Fujita, 2017]
[T. Fujita, R. Namba and Y. Tada, 2018]

$$Q'' + 2\mathcal{H}Q' + \left(\mathcal{H}' + \mathcal{H}^2\right) Q + 2g^2a^2Q^3 - \frac{g\lambda}{f} a\chi' Q^2 + a^2\mathcal{T}_{\rm BR}^{Q} = 0,$$

$$\chi'' + 2\mathcal{H}\chi' + a^2U_\chi(\chi) + \frac{3g\lambda}{f} aQ^2\left(Q' + \mathcal{H}Q\right) + a^2\mathcal{T}_{\rm BR}^{\chi} = 0,$$

$$\mathcal{T}_{\rm BR}^{Q} = \frac{g}{3a^2}\int \frac{d^3k}{(2\pi)^3}\left(\xi H - \frac{k}{a}\right)|T_R|^2,$$

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The Pencil Code

a high-order finite-difference code for compressible MHD

[A. Brandenburg, A. Johansen, P. Bourdin, W. Dobler, W. Lyra et al.]

Homogeneous backreaction in axion-SU(2) inflation

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The Pencil Code

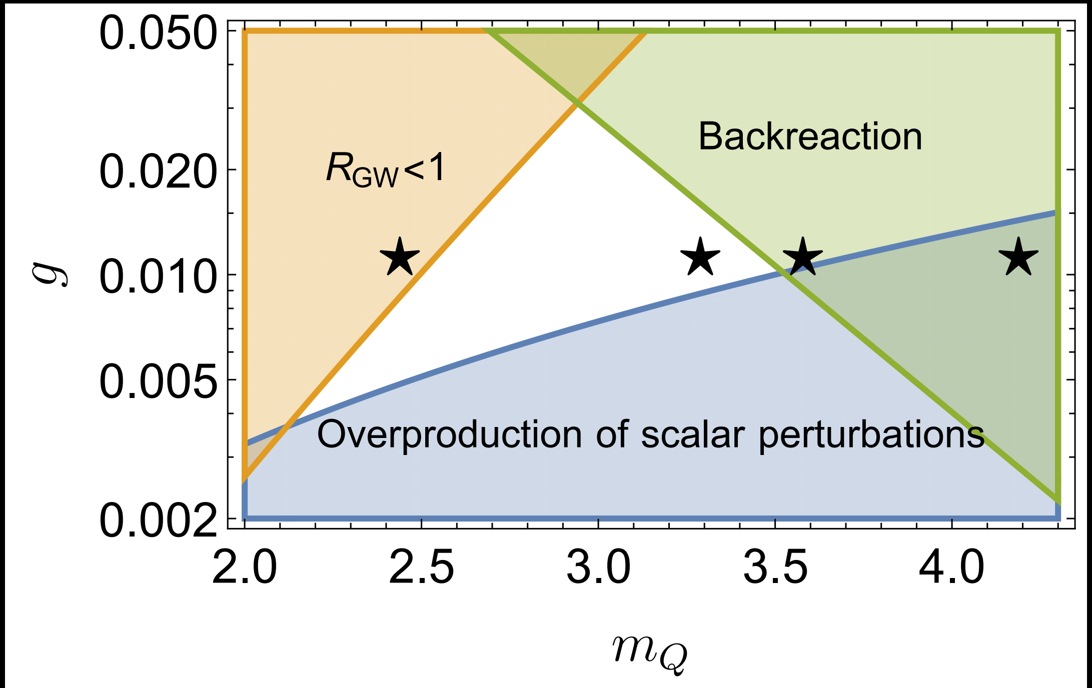
a high-order finite-difference code for c

[A. Brandenburg, A. Johansen, P. Bourdi

- Homogeneous backreaction
- Assumption $\dot{H} = 0$

Fiducial parameters for numerical simulations

[OI, E. I. Sfakianakis, R. Sharma, A. Brandenburg, 2023]



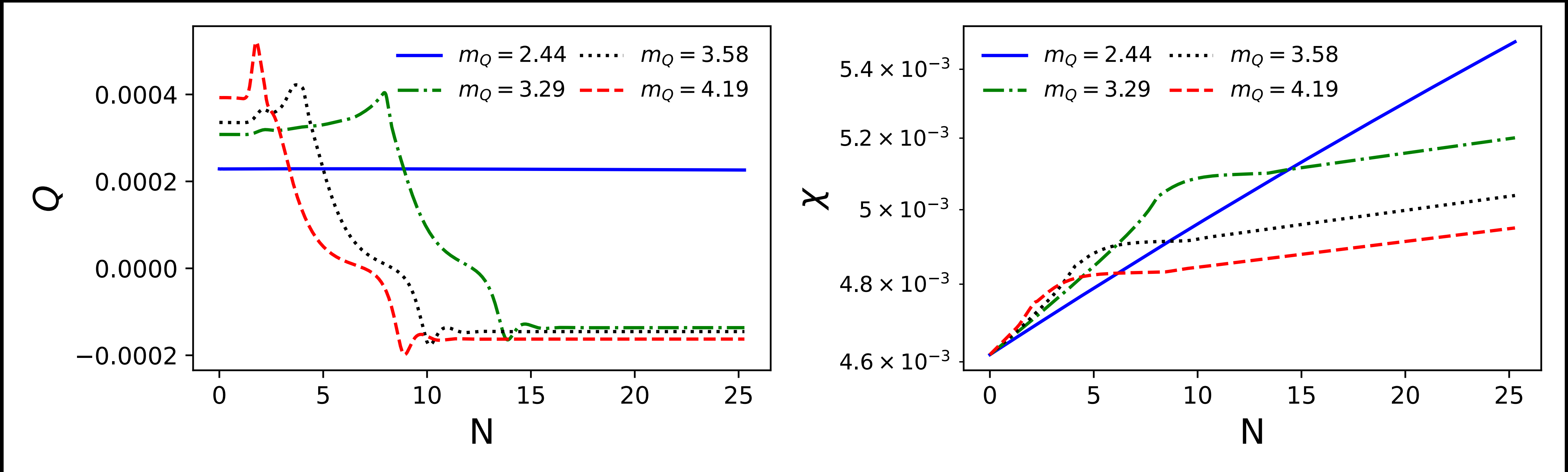
$$m_Q = 2.44,\ 3.29,\ 3.58,\ 4.19$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\epsilon^{abc} A^b_\mu A^c_\nu$$

$$m_Q = \frac{gQ}{H}$$

Background solution with backreaction

[OI, E. I. Sfakianakis, R. Sharma, A. Brandenburg, 2023]



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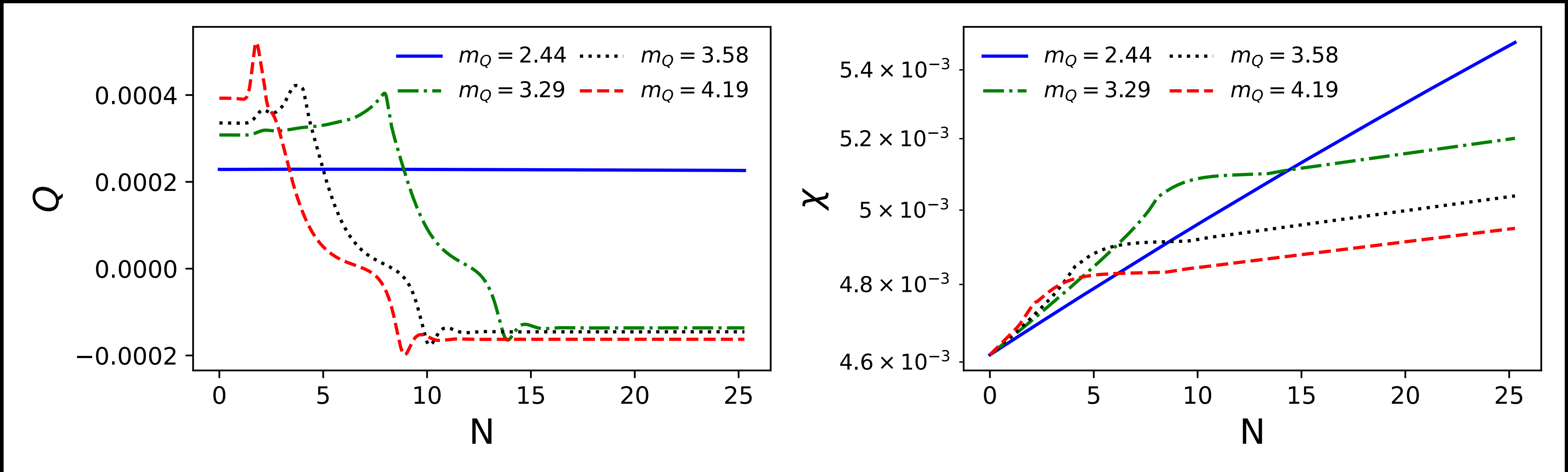
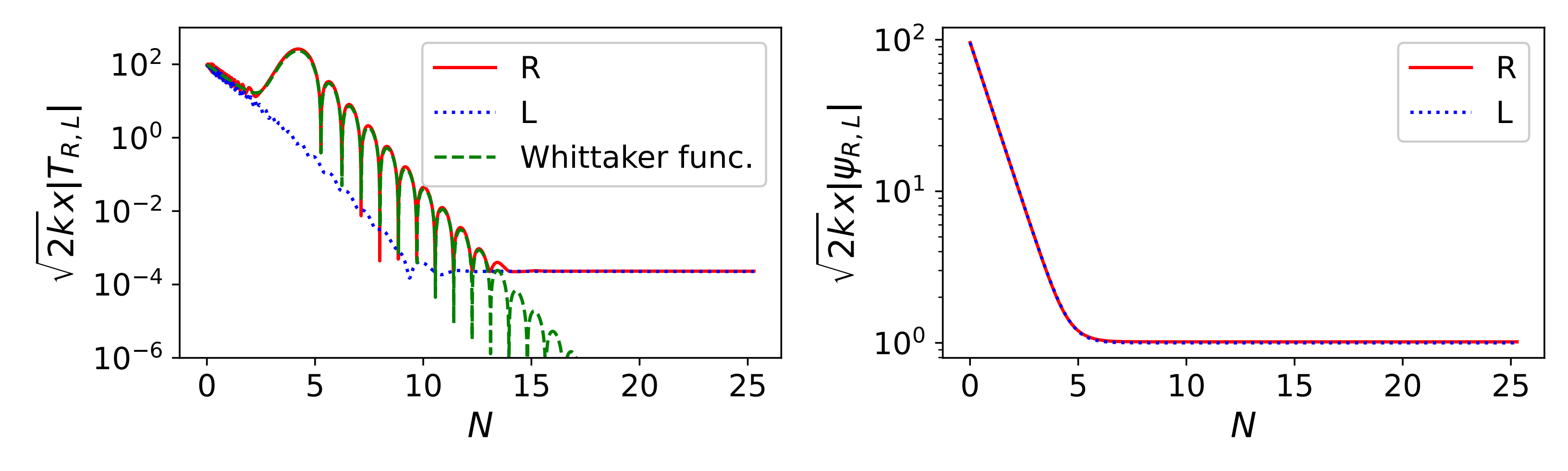
[OI, E. I. Sfakianakis, R. Sharma, A. Brandenburg, 2023]



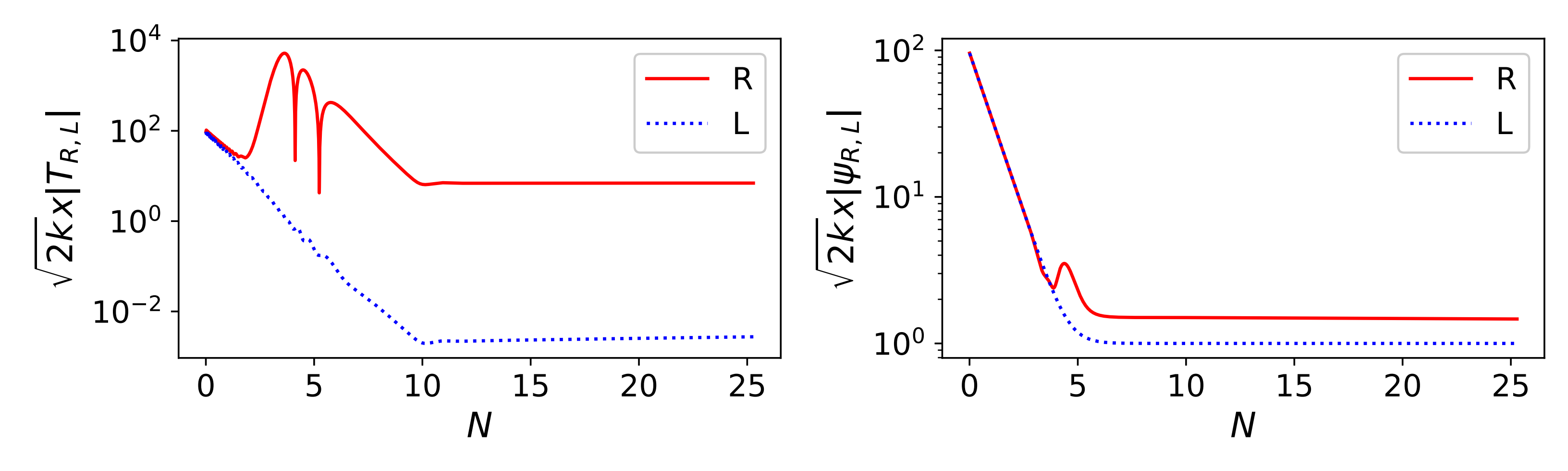
$$\frac{\lambda}{af}\chi' \simeq -\frac{2H^2}{gQ}, \qquad Q \simeq \text{const}$$

Tensor perturbations

[OI, E. I. Sfakianakis, R. Sharma, A. Brandenburg, 2023]

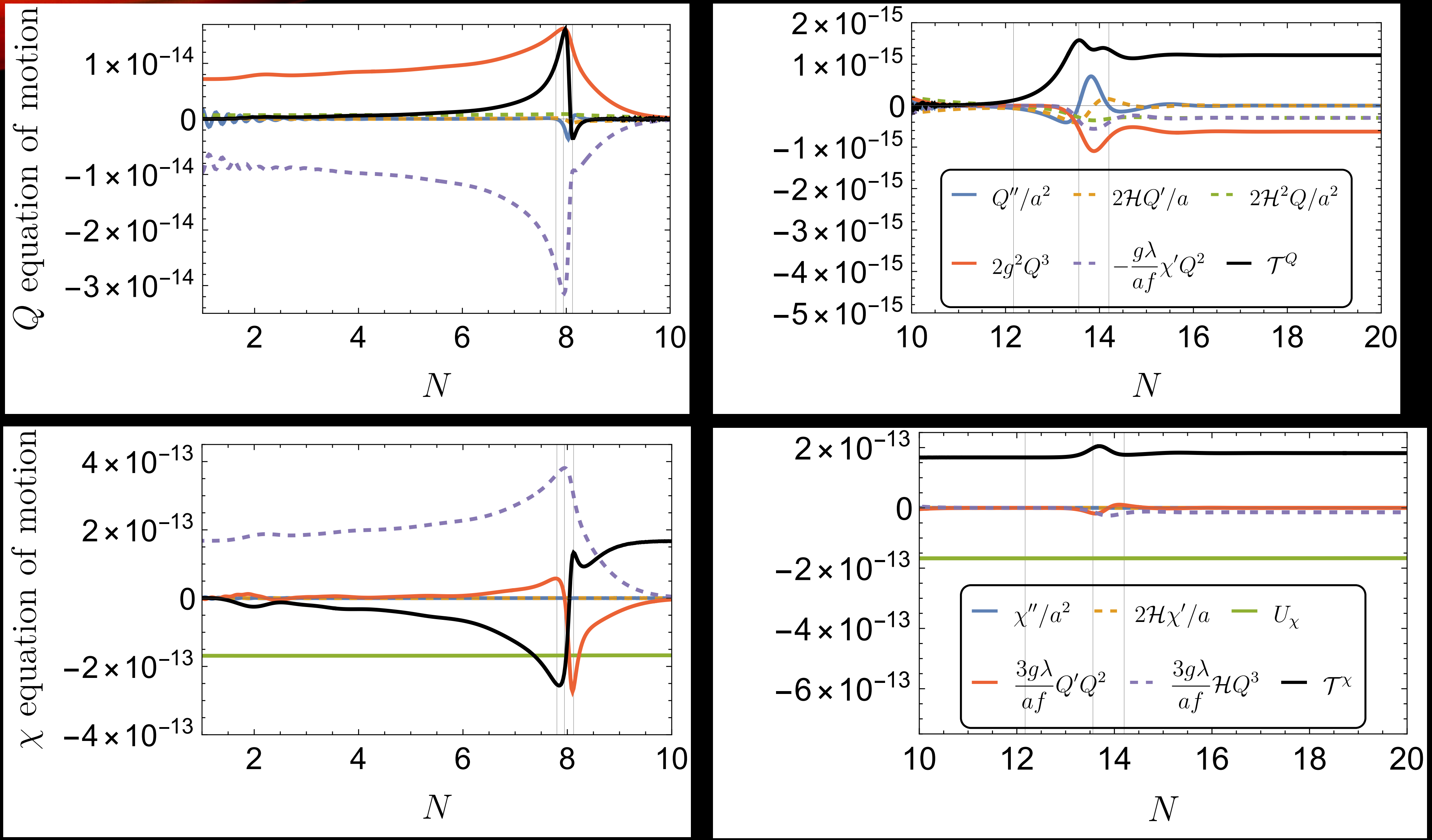


$m_Q = 2.44$

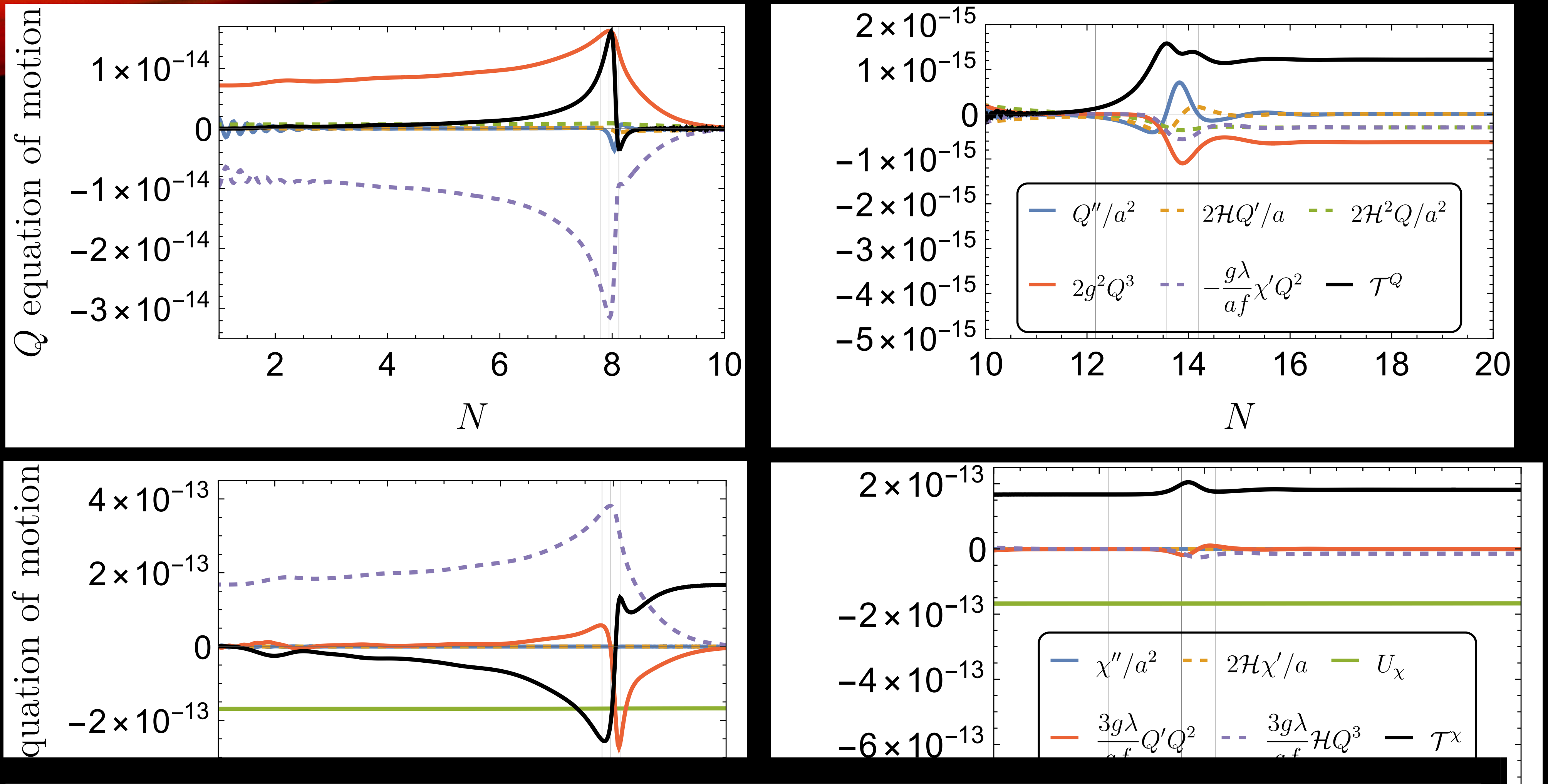


$m_Q = 3.58$

Homogeneous backreaction in axion-SU(2) inflation



Homogeneous backreaction in axion-SU(2) inflation

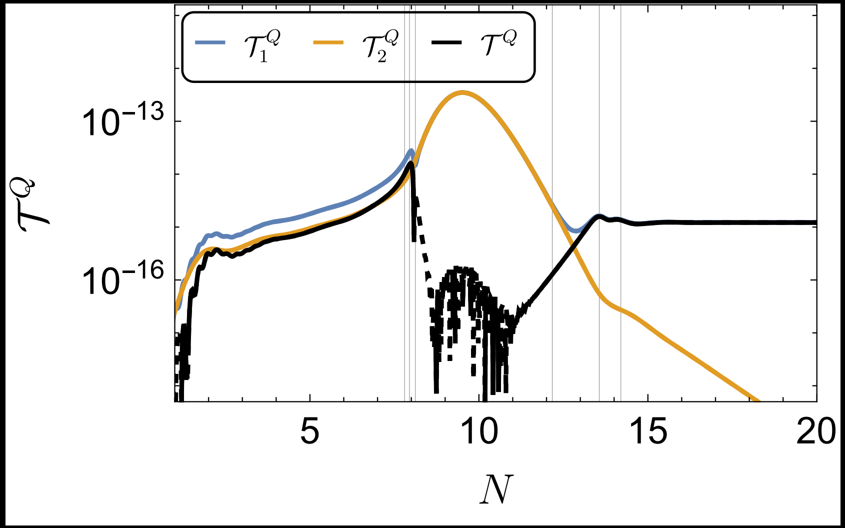


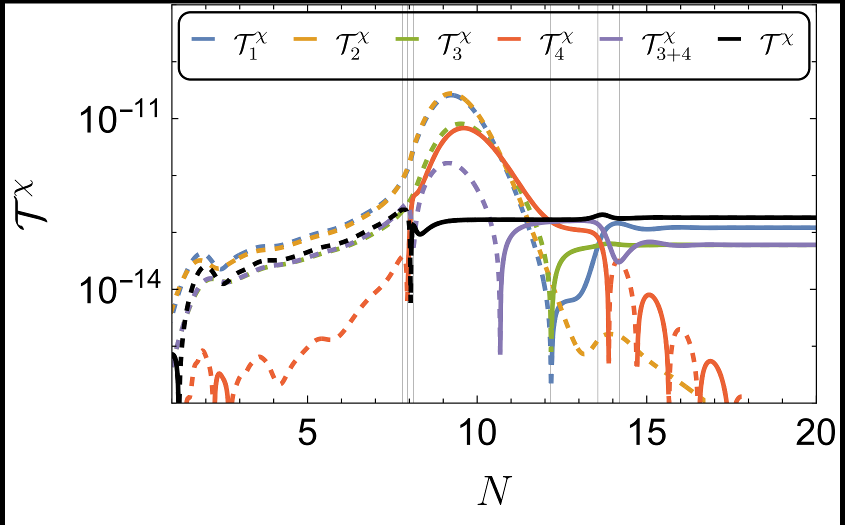
Stage I	Stage II	Stage III
$\mathcal{T}^Q_{\rm BR} \propto \exp\left(\mathcal{O}(1)N\right)$ $\lvert\mathcal{T}^\chi_{\rm BR}\rvert \propto \exp\left(\mathcal{O}(1)N\right)$	$\mathcal{T}^Q_{\rm BR} \approx 0$ $U_\chi + (3g\lambda/f)HQ^3 + \mathcal{T}^\chi_{\rm BR} \simeq 0$	$4H^2Q + 2g^2Q^3 + \mathcal{T}^Q_{\rm BR} \simeq 0$ $U_\chi + (3g\lambda/f)HQ^3 + \mathcal{T}^\chi_{\rm BR} \simeq 0$

New dynamical attractor solution

Stage III
$4H^2Q + 2g^2Q^3 + \mathcal{T}^Q_{\rm BR} \simeq 0$ $U_\chi + (3g\lambda/f)HQ^3 + \mathcal{T}^\chi_{\rm BR} \simeq 0$

$$\rightarrow \quad \frac{\mathcal{T}^Q_{\rm BR}}{\mathcal{T}^\chi_{\rm BR}} \equiv \alpha$$





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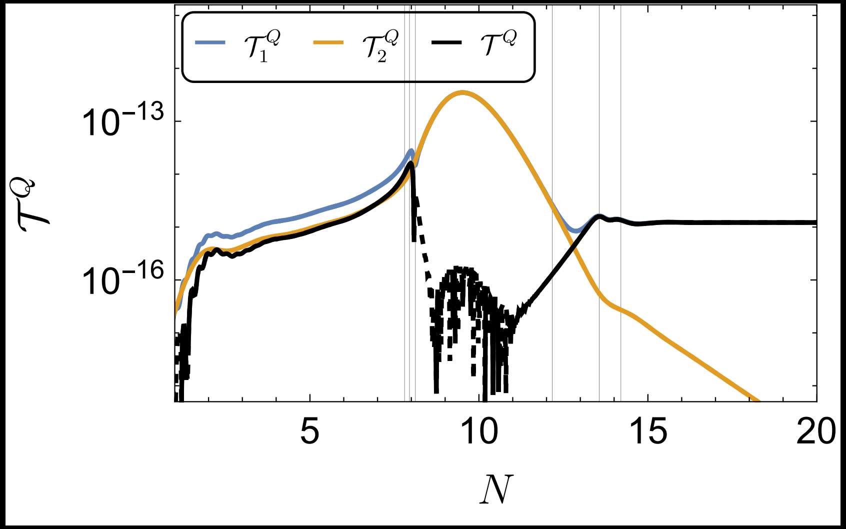

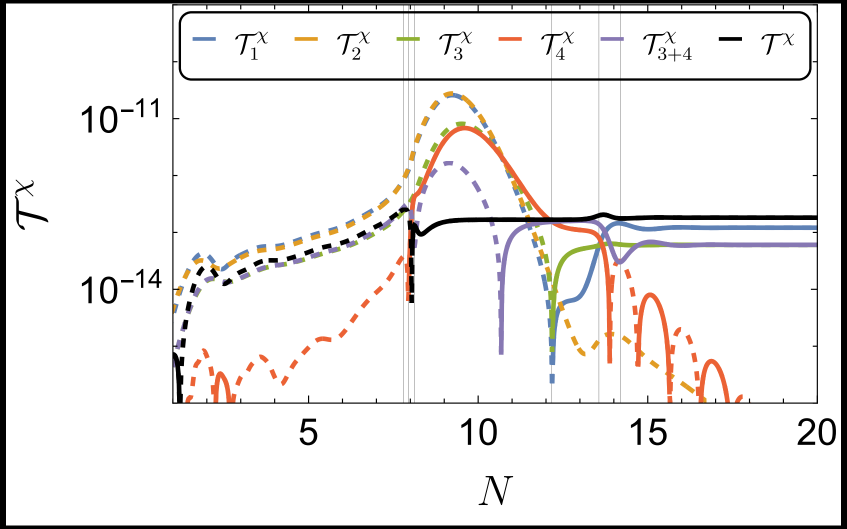

new solution:

$$\frac{\lambda}{af}\chi' \simeq -\frac{2H^2}{gQ}$$

$$U_\chi = -\frac{3g\lambda}{f}HQ^3 + \frac{1}{\alpha}\left(4H^2Q + 2g^2Q^3\right)$$

New dynamical attractor solution

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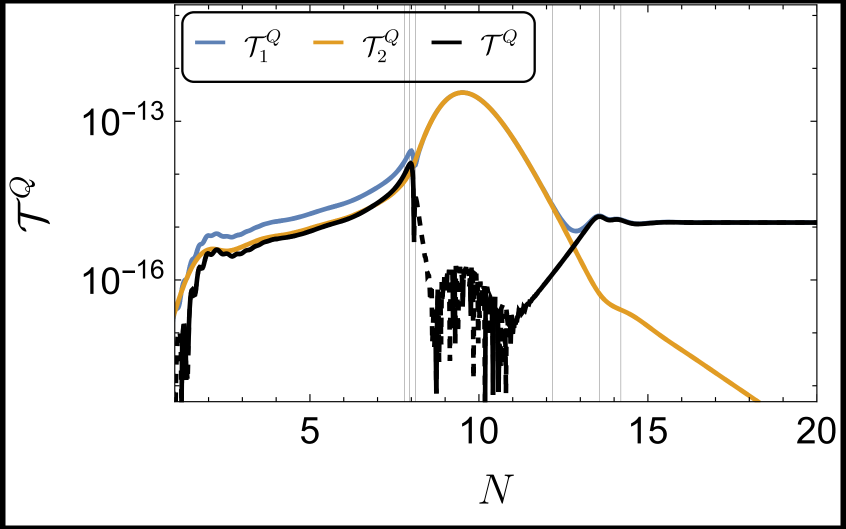

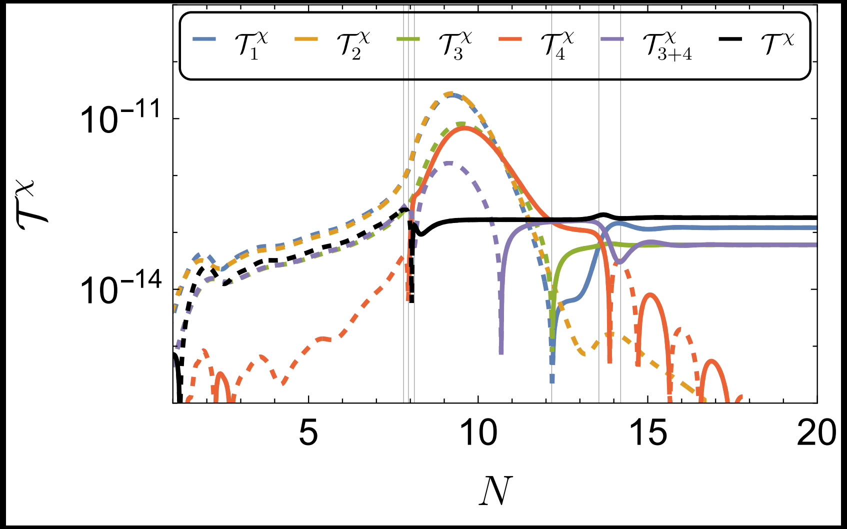

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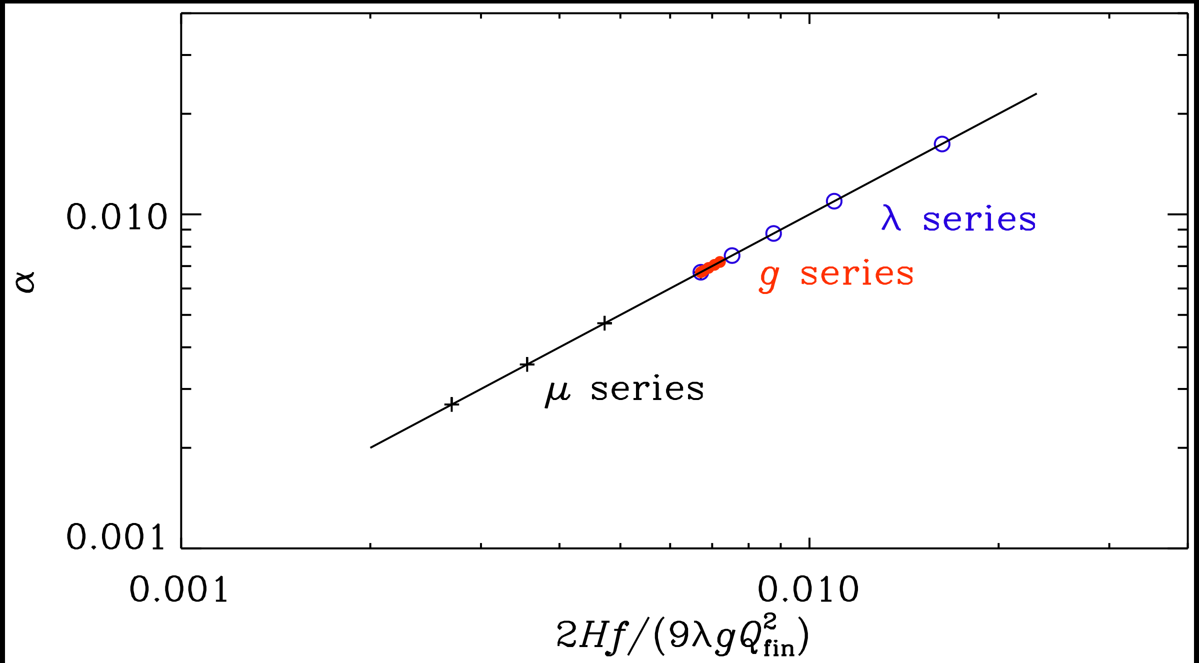
resembles CNI solution:

$$\frac{\lambda}{af}\chi' = \cancel{2gQ} + \frac{2H^2}{gQ}$$

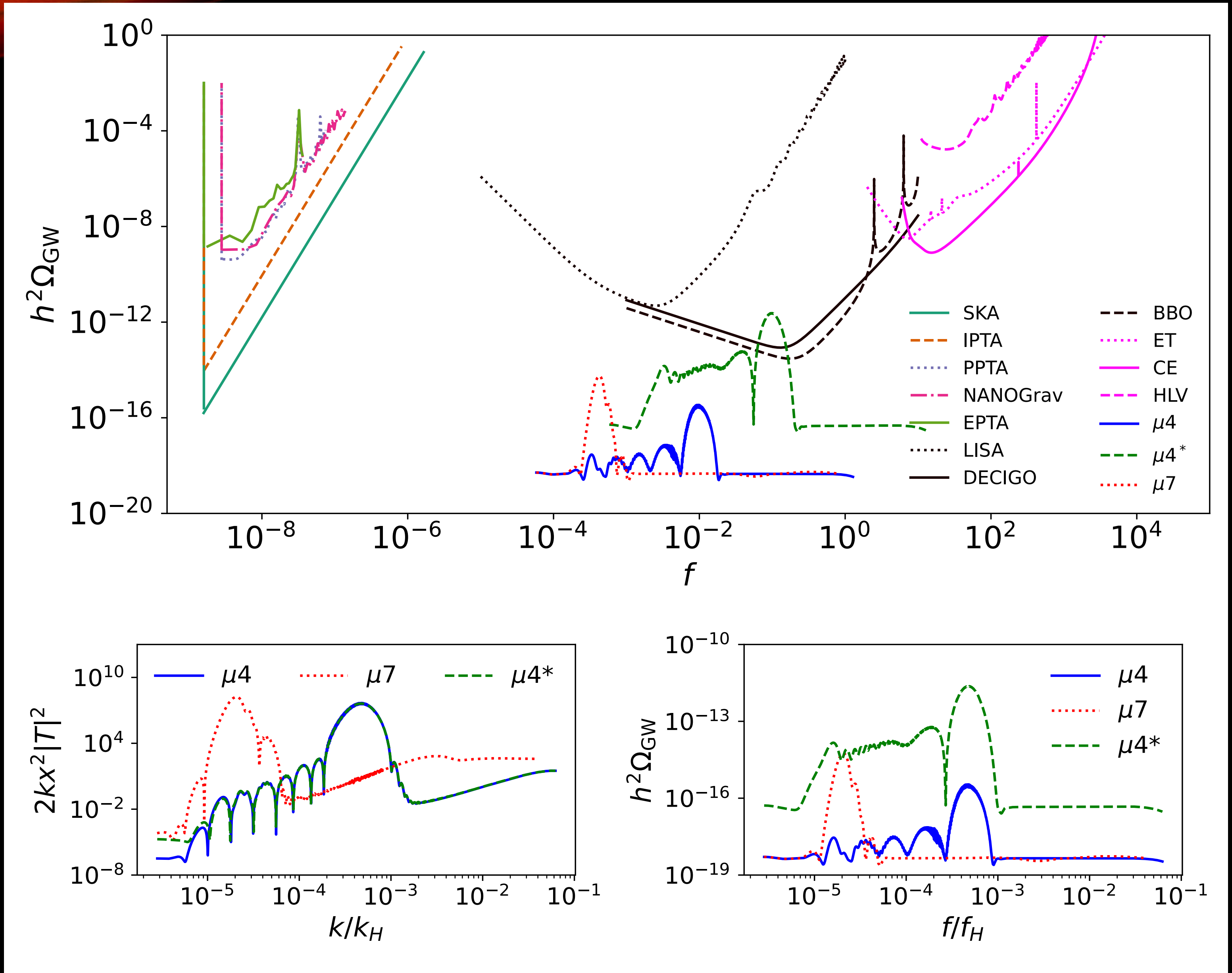
$$\cancel{\dot{Q}} = -HQ + \frac{f}{3g\lambda}\frac{U_\chi}{Q^2}$$

Dynamical attractor solution



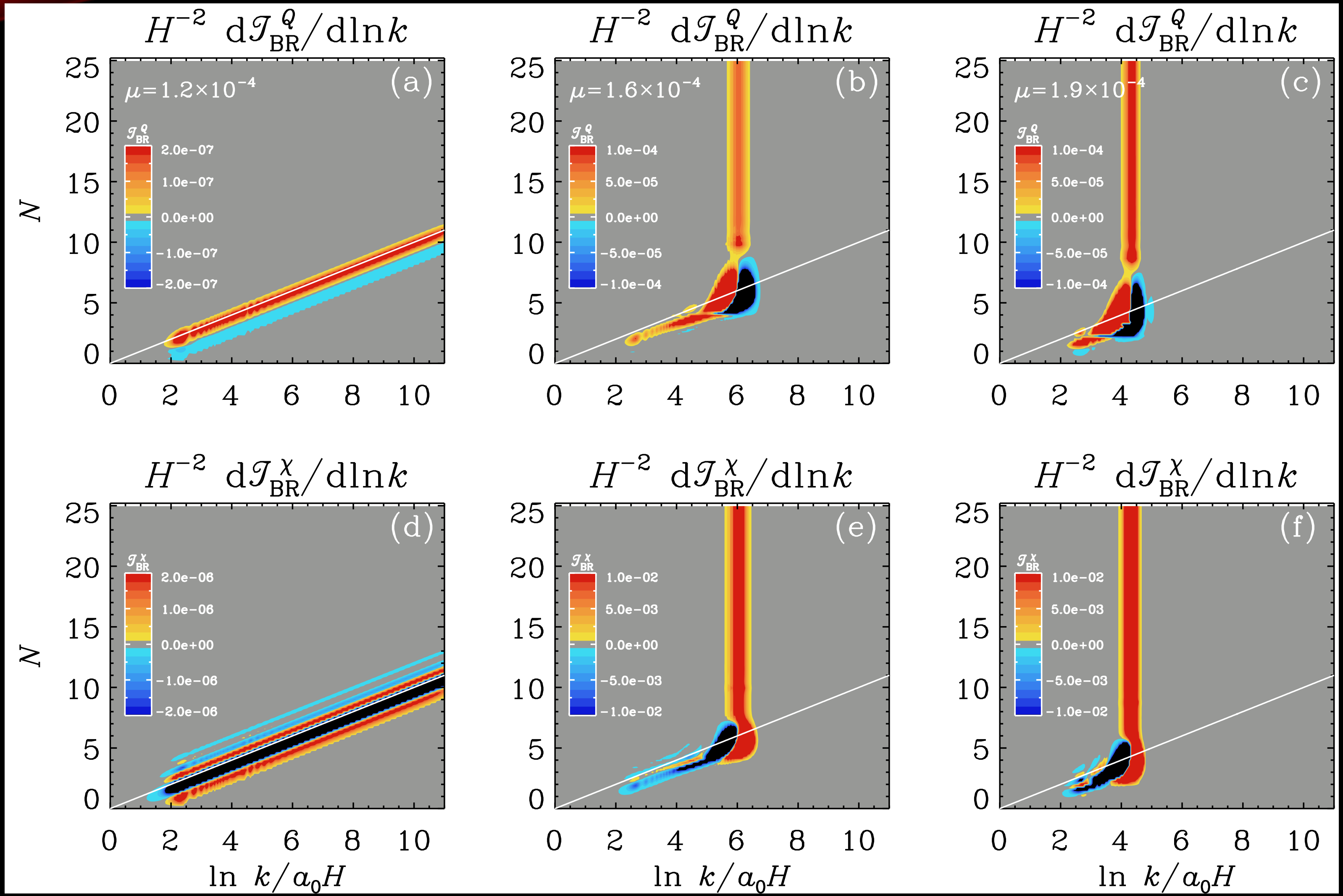
$$\frac{\mathcal{T}_{\rm BR}^{Q}}{\mathcal{T}_{\rm BR}^{\chi}} \equiv \alpha \quad \rightarrow \quad \alpha = \frac{2}{9}\frac{H f}{\lambda g\, Q_{\rm fin}^2}$$

Gravitational waves



$m_Q = 3.78, 5.15$

Backreaction integrands

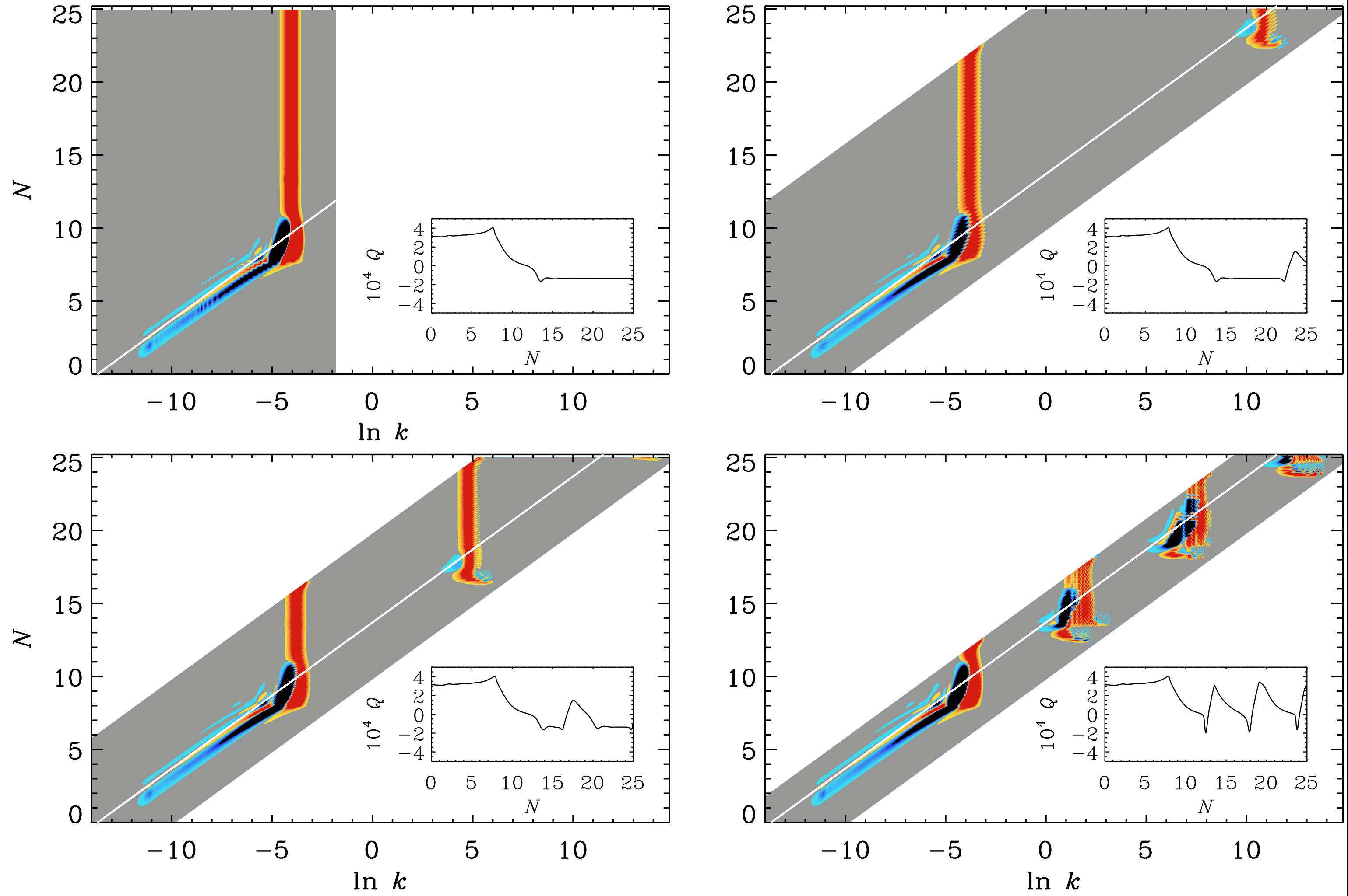


$m_Q = 2.44$ $m_Q = 3.58$ $m_Q = 4.19$

Most of the contribution comes from a fixed narrow range of wave numbers!

Artifacts from not resolving the superhorizon modes

Integration over a comoving strip: $n_{\min} \leq \ln[k/a(\eta)H] \leq n_{\max}$



Take-home message about backreaction:

- A new dynamical attractor solution for the axion field and the vacuum expectation value of the gauge field, where the latter has an opposite sign with respect to the chromo-natural inflation solution.

- Phenomenology: redefining parts of the viable parameter space.

- Characteristic oscillatory features in the primordial gravitational wave background that are potentially detectable with upcoming gravitational wave detectors.

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- A new dynamical attractor solution for the axion field and the vacuum expectation value of the gauge field, where the latter has an opposite sign with respect to the chromo-natural inflation solution.

- Phenomenology: redefining parts of the viable parameter space.

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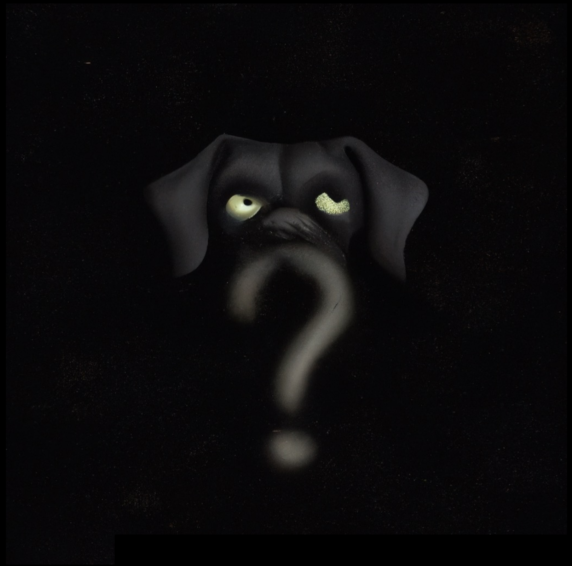
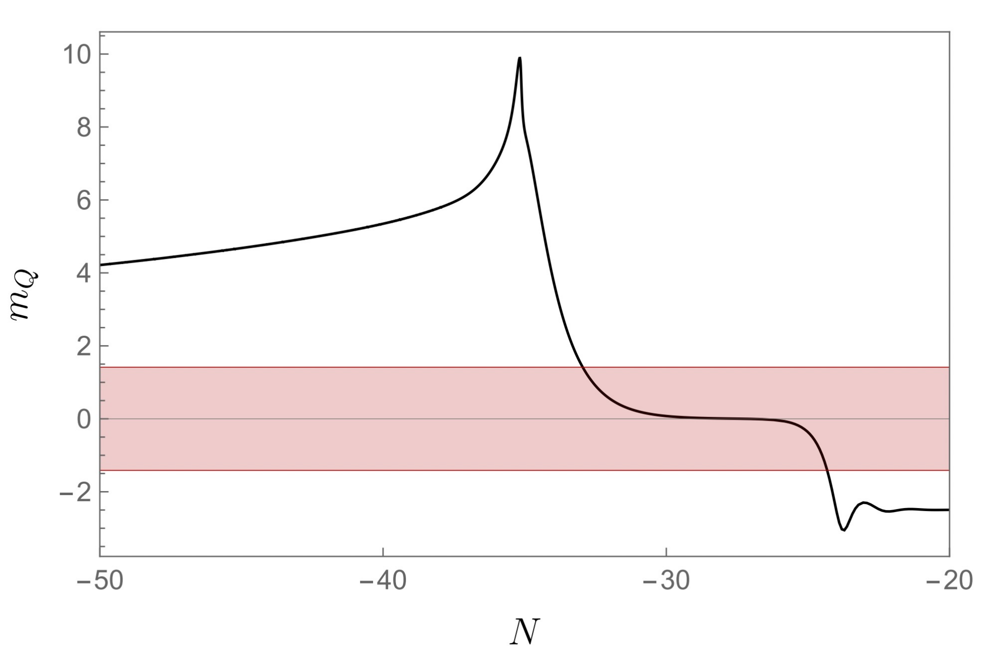
But

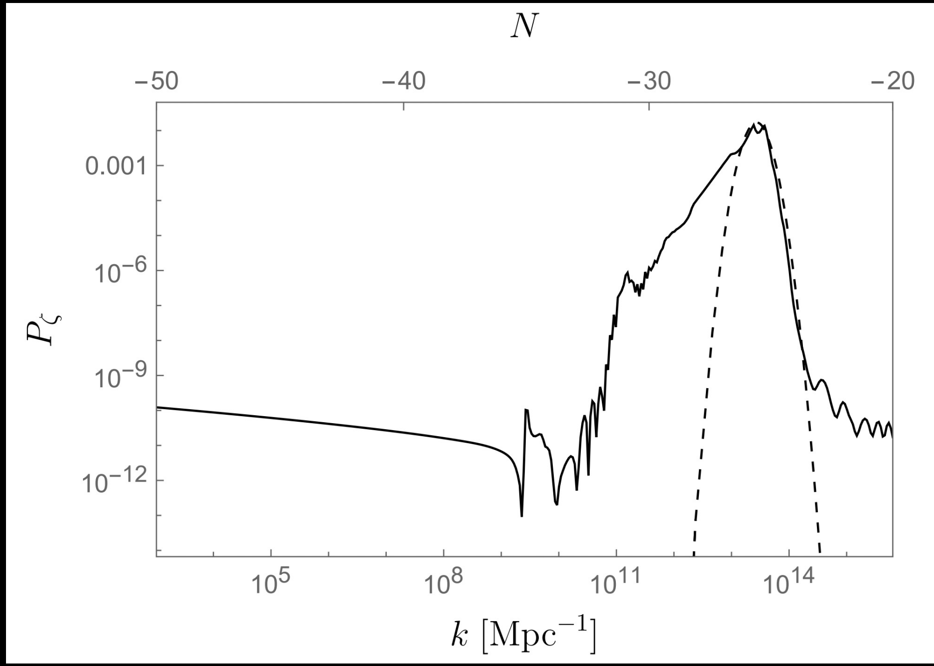
- Scalar perturbations?
- End of inflation $|\dot{H}/H^2| \sim 1$
- Inhomogenious backreaction



Scalar perturbations

$|m_Q| < \sqrt{2}$ instability band leads to primordial black holes production





[E. Dimastrogiovanni, M. Fasiello, A. Papageorgiou, 2024]

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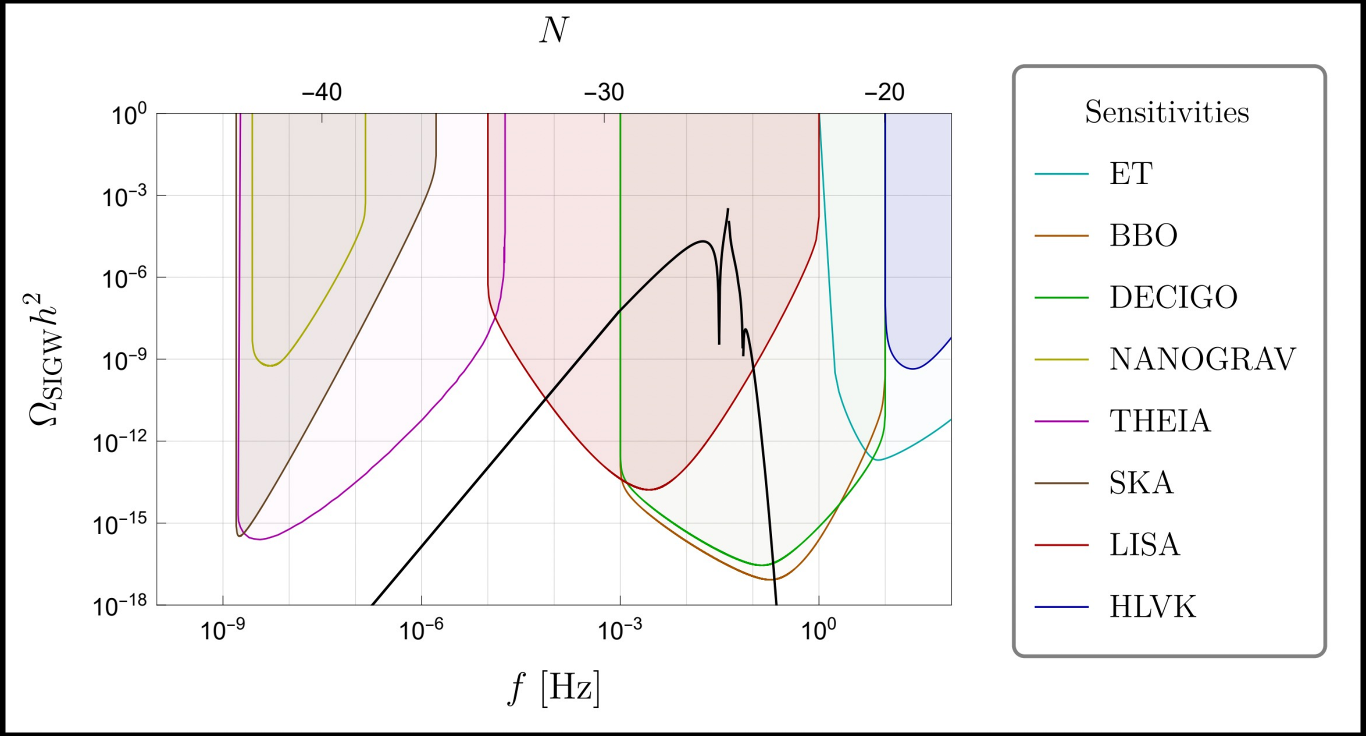
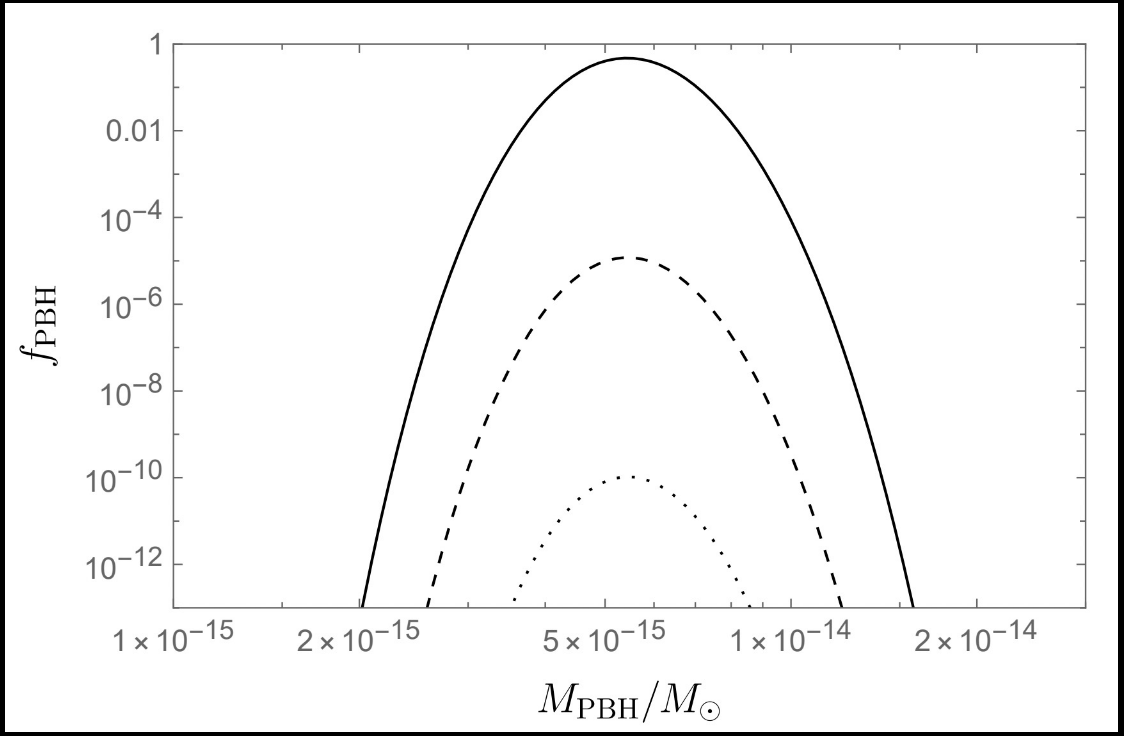
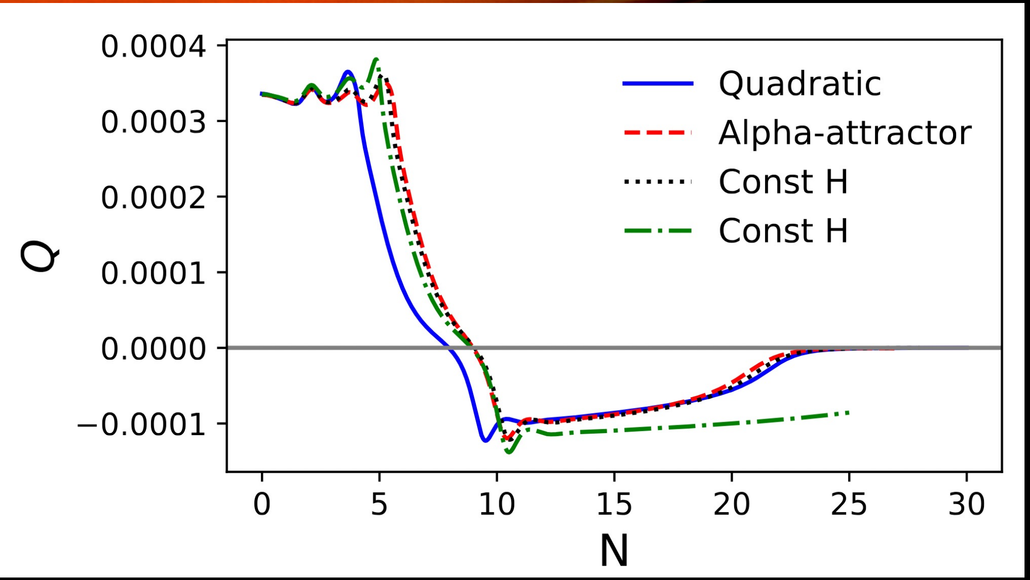
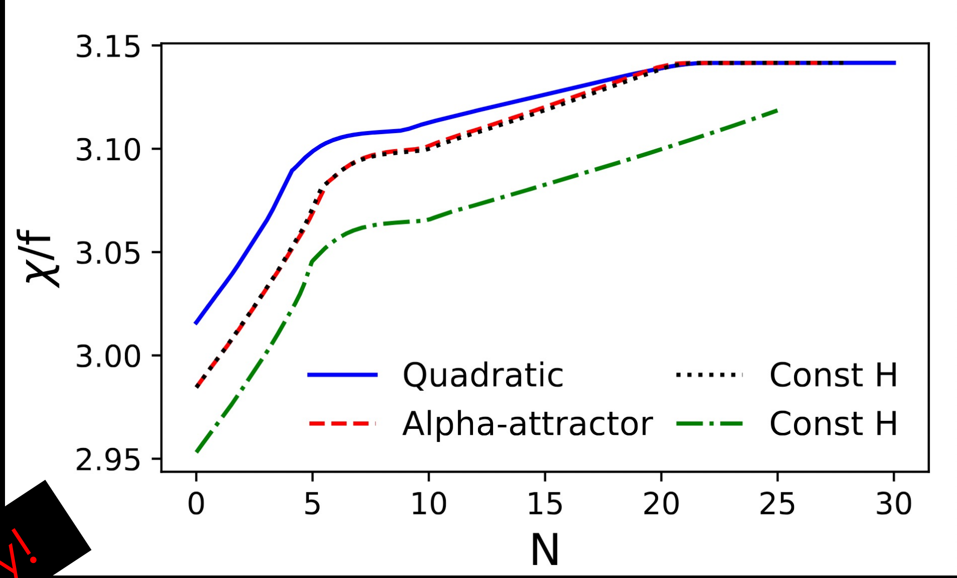


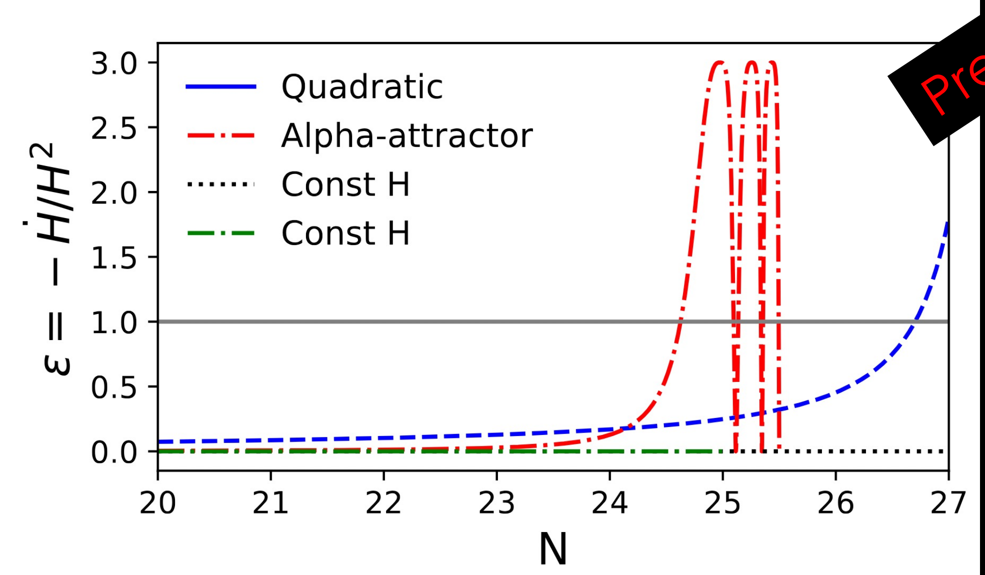


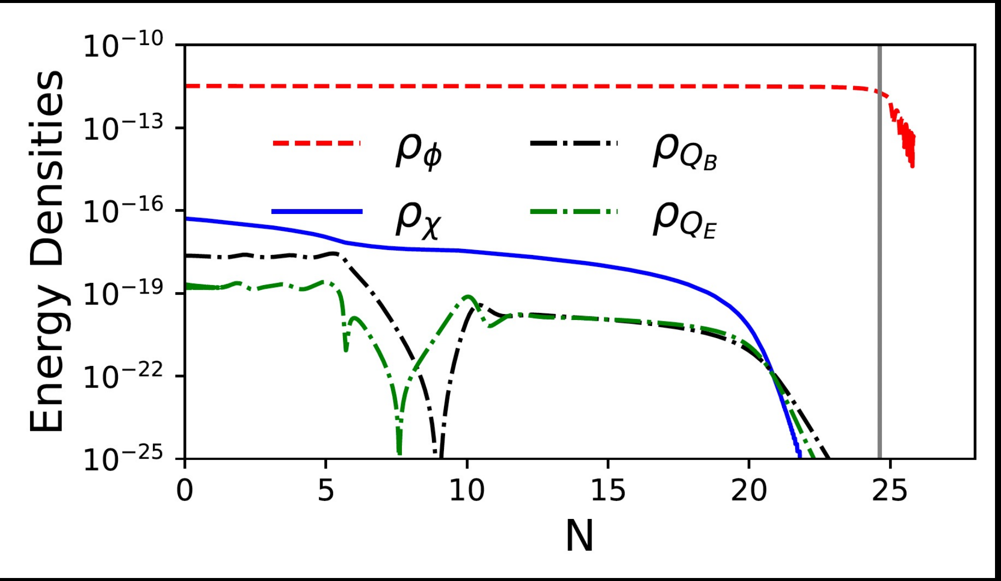
[E. Dimastrogiovanni, M. Fasiello, A. Papageorgiou, 2024]

End of inflation



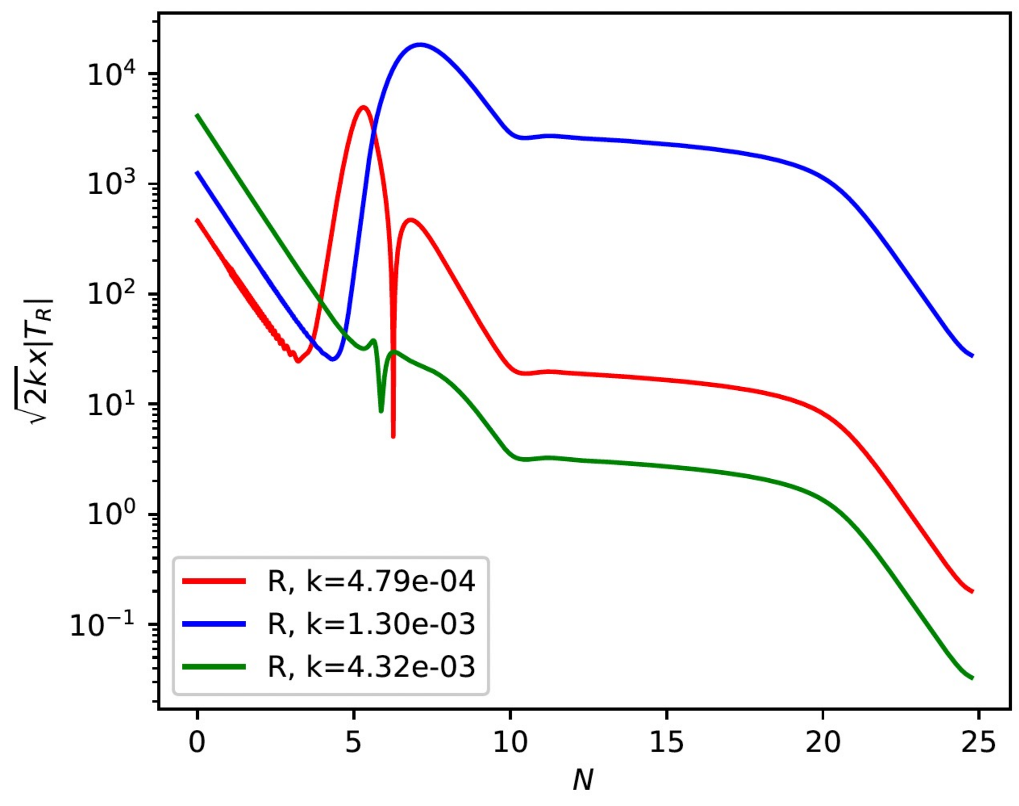


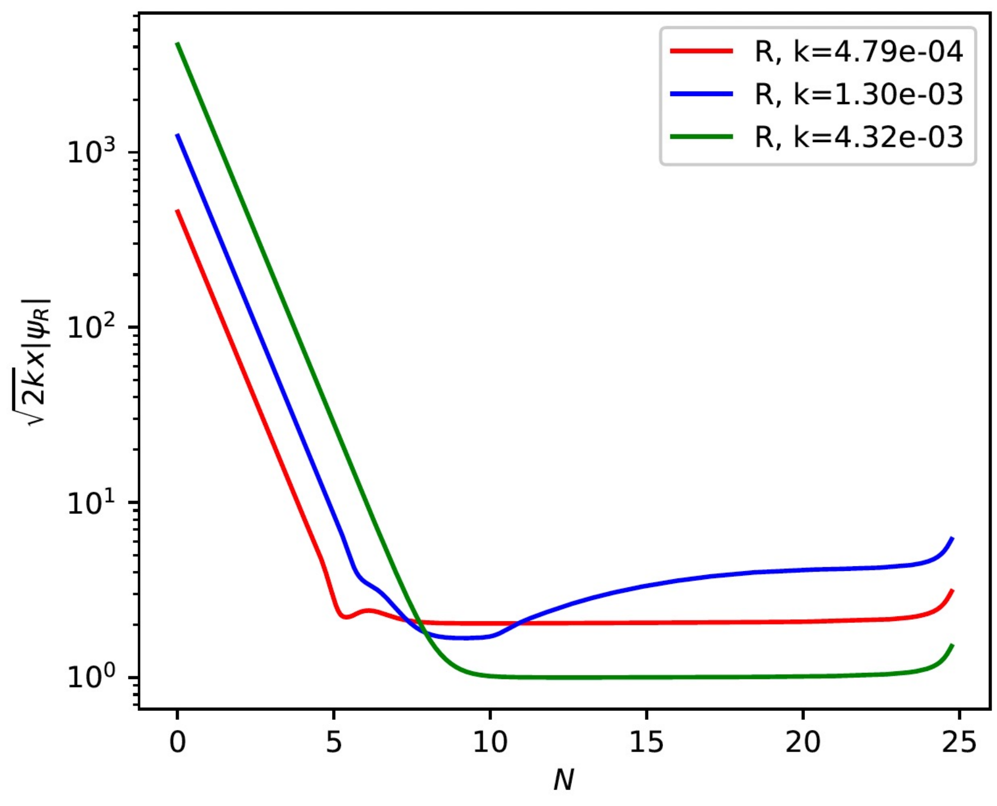




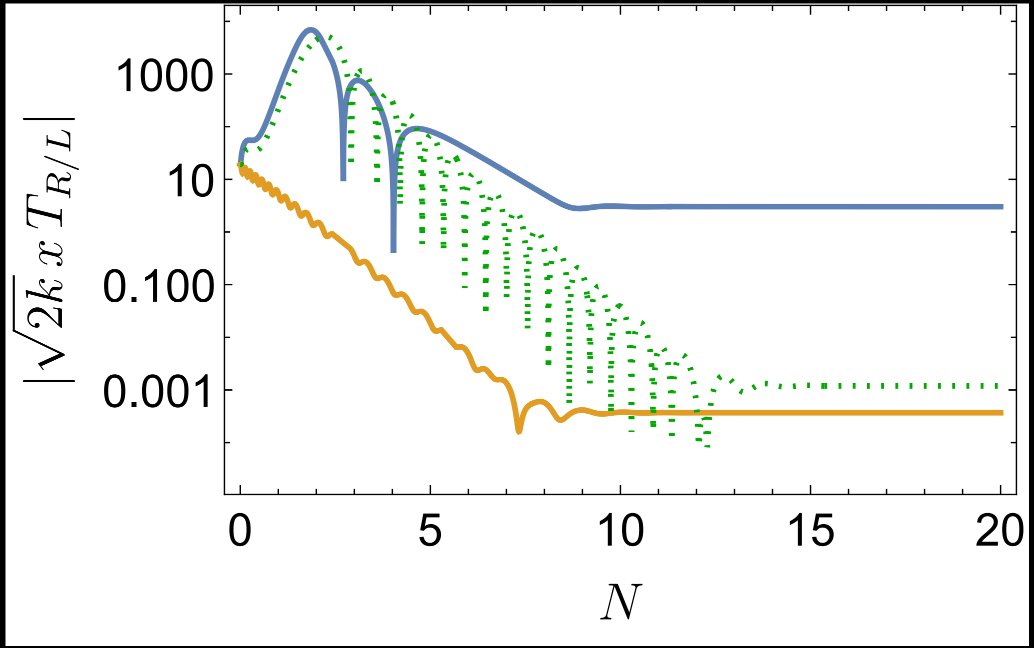
Preliminary!

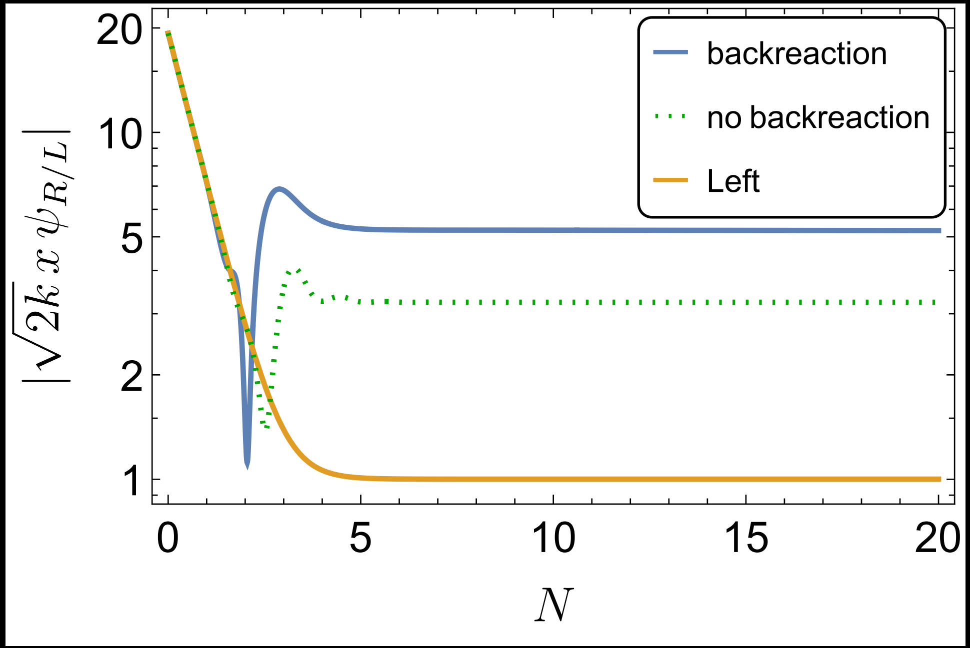
Tensor pertutbations





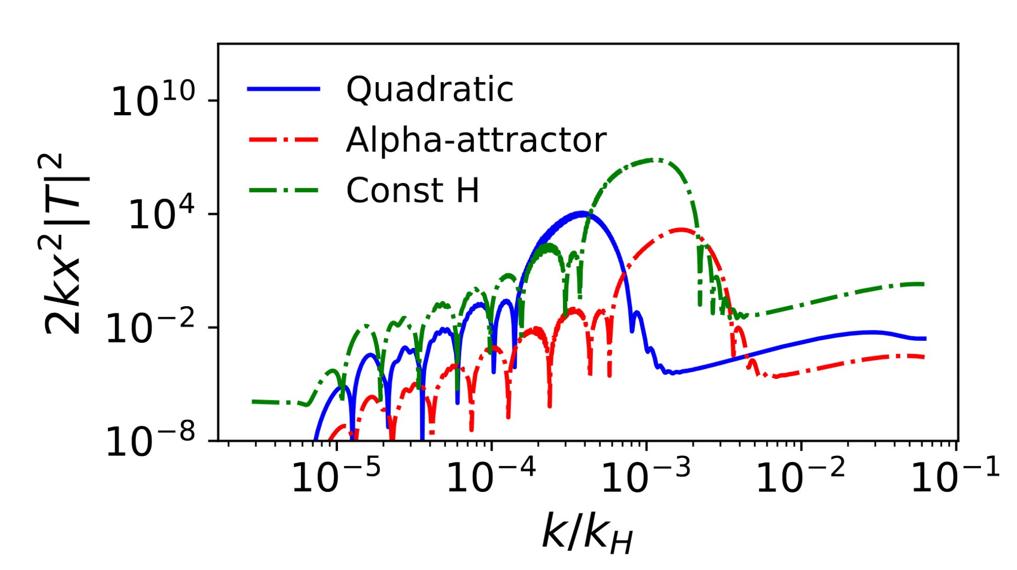
Difference with no backreaction regime





Matching solutions with the radiation-dominated era

$$\sqrt{2k}xT_L(x) = c_2(k)$$

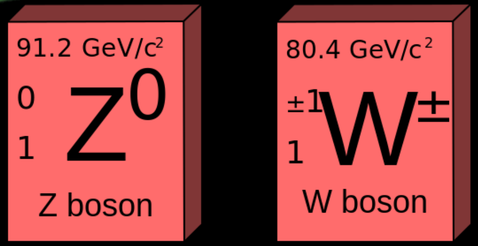


Assuming instanteneous reheating:

$$\partial_\tau^2 T_L + k^2 T_L = 0$$

$$T_L \approx \frac{c_2(k)}{\sqrt{2k}} \frac{a_e^2 H_f^2}{k^2} \sin k\tau$$

SU(2) $\longleftrightarrow$

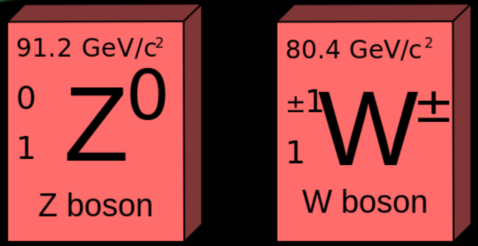


transverse vector perturbations

$$W_0^a = a(Y_a + \partial_a Y)\,,$$

$$W_i^a = a\left[(Q + \delta Q)\,\delta_{ai} + \partial_i\,(M_a + \partial_a M) + \epsilon_{iac}\,(U_c + \partial_c U) + t_{ia}\right]$$

SU(2) $\longleftrightarrow$



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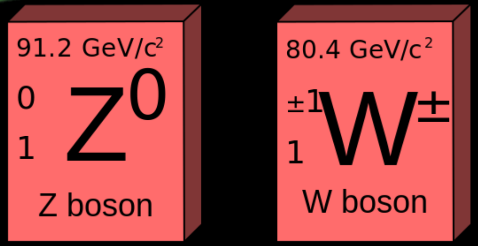
traceless and transverse tensor

$$\delta W_3^0 = 0,$$

$$\delta W_i^3 = a t_{i3}$$

$$|\tilde{t}_{i3}|^2 = \frac{1}{2}\left(|\tilde{t}^L|^2 + |\tilde{t}^R|^2\right)$$

SU(2) $\longleftrightarrow$



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traceless and transverse tensor

$$A_\mu = W_\mu^3 \sin\theta_W + B_\mu \cos\theta_W$$

$$Z_\mu = W_\mu^3 \cos\theta_W - B_\mu \sin\theta_W$$

$$|\tilde{A}_i|^2 = \frac{1}{8}\left(|\tilde{T}^L|^2 + |\tilde{T}^R|^2\right)$$

Magnetic field energy spectrum

$$\rho_B \equiv \langle \frac{B^2}{2} \rangle = \int d\ln k \Delta_{\rm B}(k)$$

$$\Delta_{\rm B}(k) = \frac{1}{(2\pi)^2}\frac{k^5}{a^4}|\tilde{A}_i|^2$$

$$\Delta_{\rm B}(k) = \frac{1}{(2\pi)^2}\frac{k^5}{a^4}\frac{1}{8}\left(|\tilde{T}^L|^2 + |\tilde{T}^R|^2\right) \sim \frac{1}{(2\pi)^2}\frac{a_e^4 H_f^4}{a^4}\frac{1}{16}|c_2|^2 \sin^2 k\tau$$

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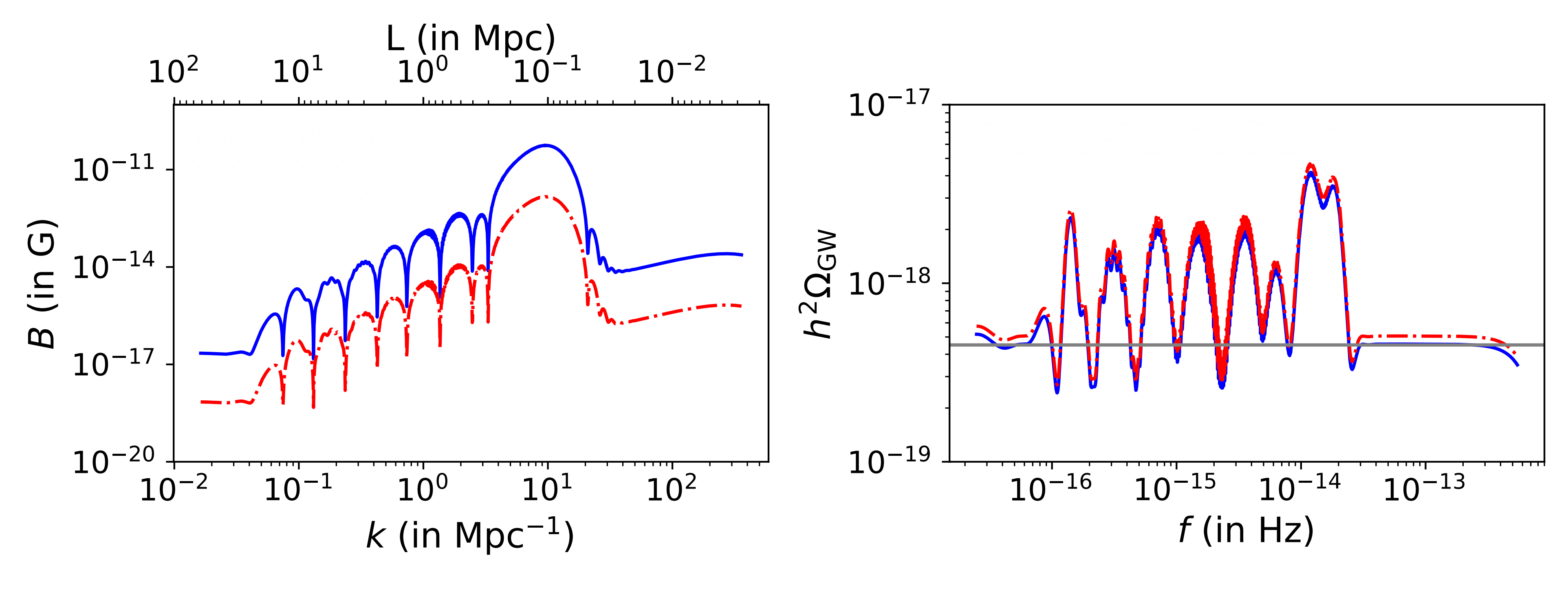
$$\Delta_{\rm B}(k) = \frac{1}{(2\pi)^2} \frac{k^5}{a^4} \frac{1}{8} \left(|\tilde{T}^L|^2 + |\tilde{T}^R|^2 \right) \sim \frac{1}{(2\pi)^2} \frac{a_e^4 H_f^4}{a^4} \frac{1}{16} |c_2|^2 \sin^2 k\tau$$

$$\frac{\Delta_{\rm B}(k)}{\rho_r} \simeq \frac{1}{96(2\pi)^2} \left(\frac{H_f}{M_{pl}} \right)^2 |c_2|^2$$

$$\sqrt{\Delta_{\rm B}(k)\big|_0} = 3.7 \times 10^{-7} \frac{H_f}{10^{-5} M_{pl}} |c_2| \ \mu{\rm G}$$

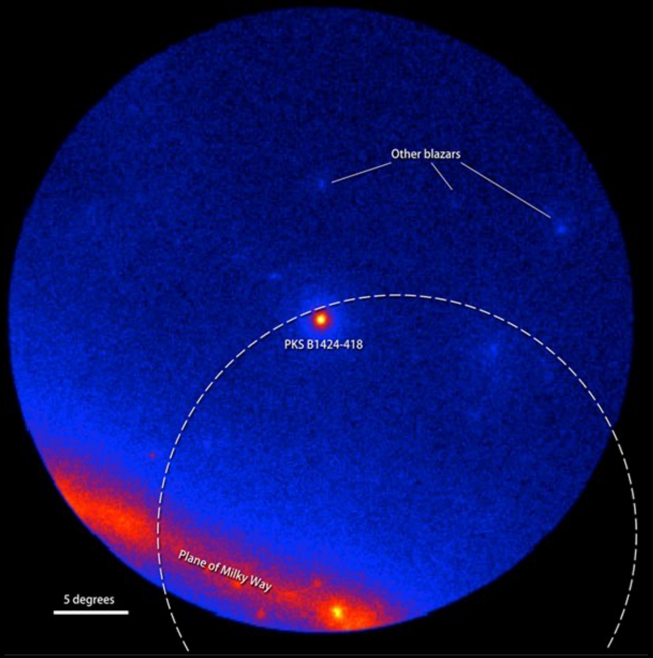
Estimates for present-day magnetic field and GWs

$$\sqrt{\Delta_{\rm B}(k)\big|_0} = 3.7 \times 10^{-7} \frac{H_f}{10^{-5} M_{pl}} |c_2| \ \mu{\rm G}$$



Conclusions

- Backreaction ‘helps’ with sustainging the primordial magnetic field.
- Axion-SU(2) inflation can potentially produce magentic fields compatible with blazar observations.
- Rich phenomenology in primordial gravitational waves and primordial black hole production .



Nordita program

‘Axions in Stockholm 2025’
23 June - 11 July, 2025

- Week 1: “Axions and gauge fields in the early and late universe”.
- Week 2: conference.
- Week 3: “Direct and indirect axion searches”.

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Alexander Millar
Hiranya Peiris
Evangelos Sfakianakis

Conclusions

- Backreaction ‘helps’ with sustainging the primordial magnetic field.
- Axion-SU(2) inflation can potentially produce magentic fields compatible with blazar observations.
- Rich phenomenology in primordial gravitational waves and primordial black hole production .

Thank you!