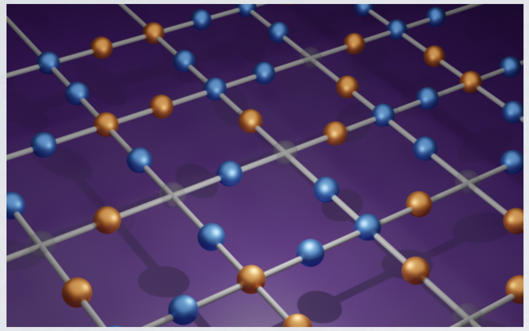

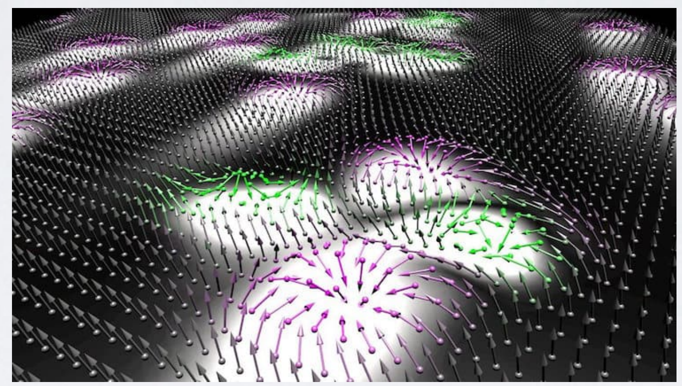

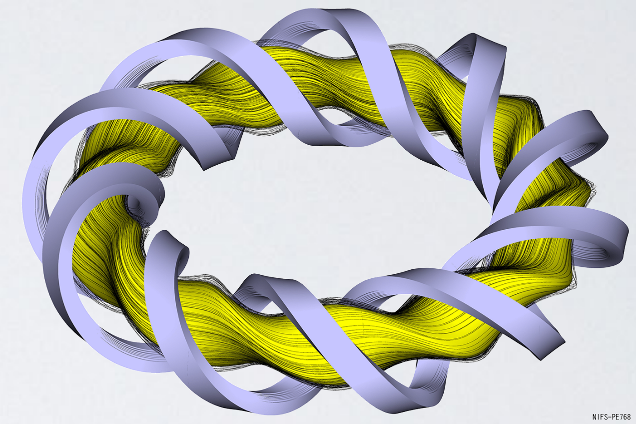

SIMULATING THE EARLY UNIVERSE

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UNITED KINGDOM · CHINA · MALAYSIA

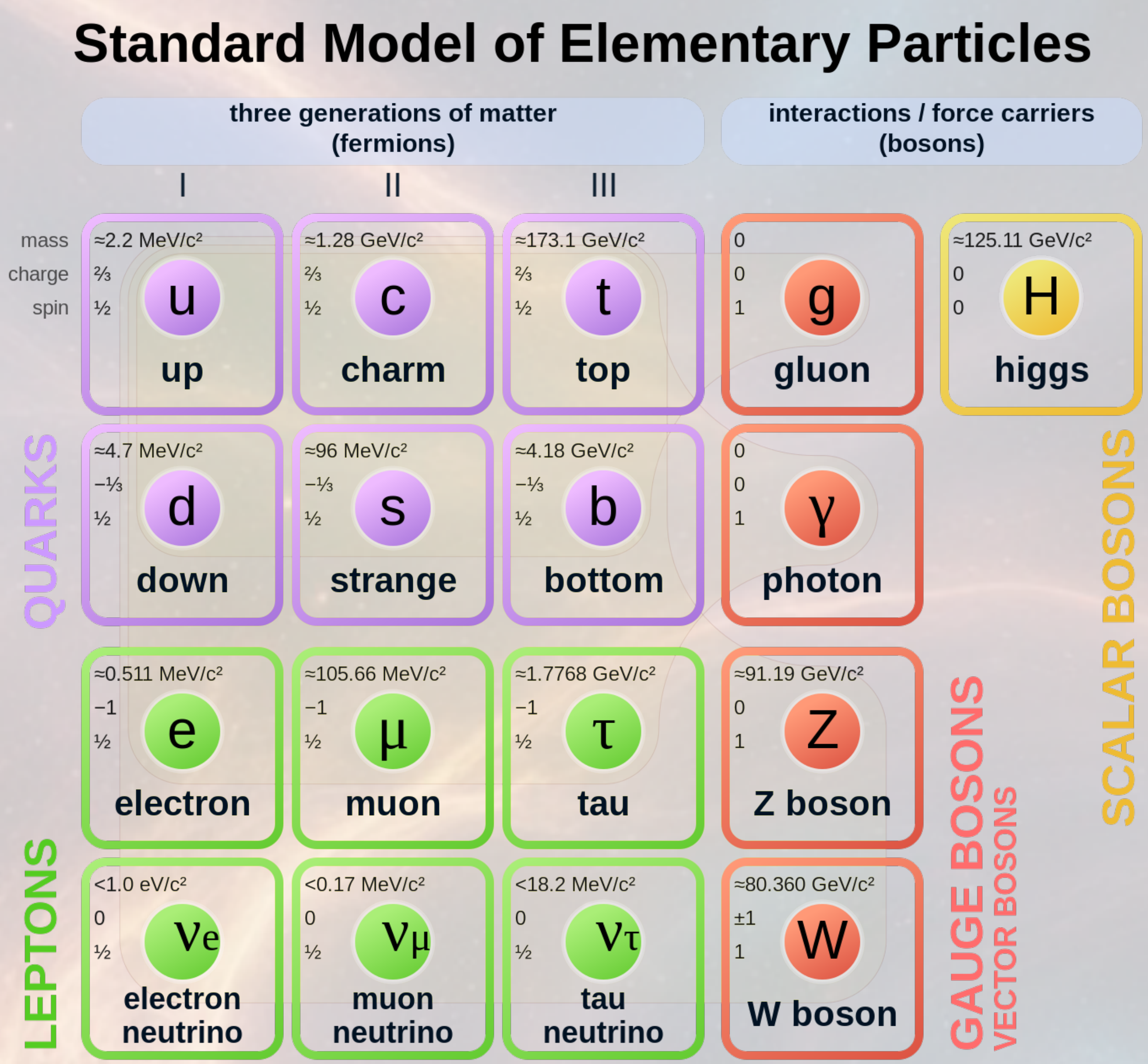
REAL-TIME EVOLUTION IN FIELD THEORY

- Magnetic fields
  - Berges, Felder, Garcia-Bellido, Garcia-Perez,Gelfand, Gonzalez-Arroyo, Khlebnikov, Kofman, Linde, Micha, Mou, Prokopec, Pruschke, Roos, Tkachev
- Preheating
  - Berges, Felder, Garcia-Bellido, Garcia-Perez,Gelfand, Gonzalez-Arroyo, Khlebnikov, Kofman, Linde, Micha, Mou, Prokopec, Pruschke, Roos, Tkachev
- Baryogenesis
  - Bodecker, Grigoriev, Hu, Kusenko, Moore, Mou, Muller, Rummukainen, PMS, Shaposhnikov,, Smit, Tranberg
- FRW evolution
  - Baacke, Covi, Heitmann, Kevlishvili, Patzold, Tranberg
- Soliton dynamics
  - Berges, Borsanyi, Hertzberg, Hindmarsh, Mou,PMS, Roth, TognarelliTranberg
- Thermalization
  - Aarts, Attems, Berges, Bonini, Gelfand, Kurkela, Philipsen, Pruschke, Schlicting, Sexty, Shafer, Wagenbach, Wetterich, Zafeiropoulos
- Phase transitions
  - Blaizot, Hatta, Rajantie, Sexty, Smit, Tranberg, Tsutsui
- Langevin
  - Aarts, Scherzer, Seiler, Sexty, Stamatescu

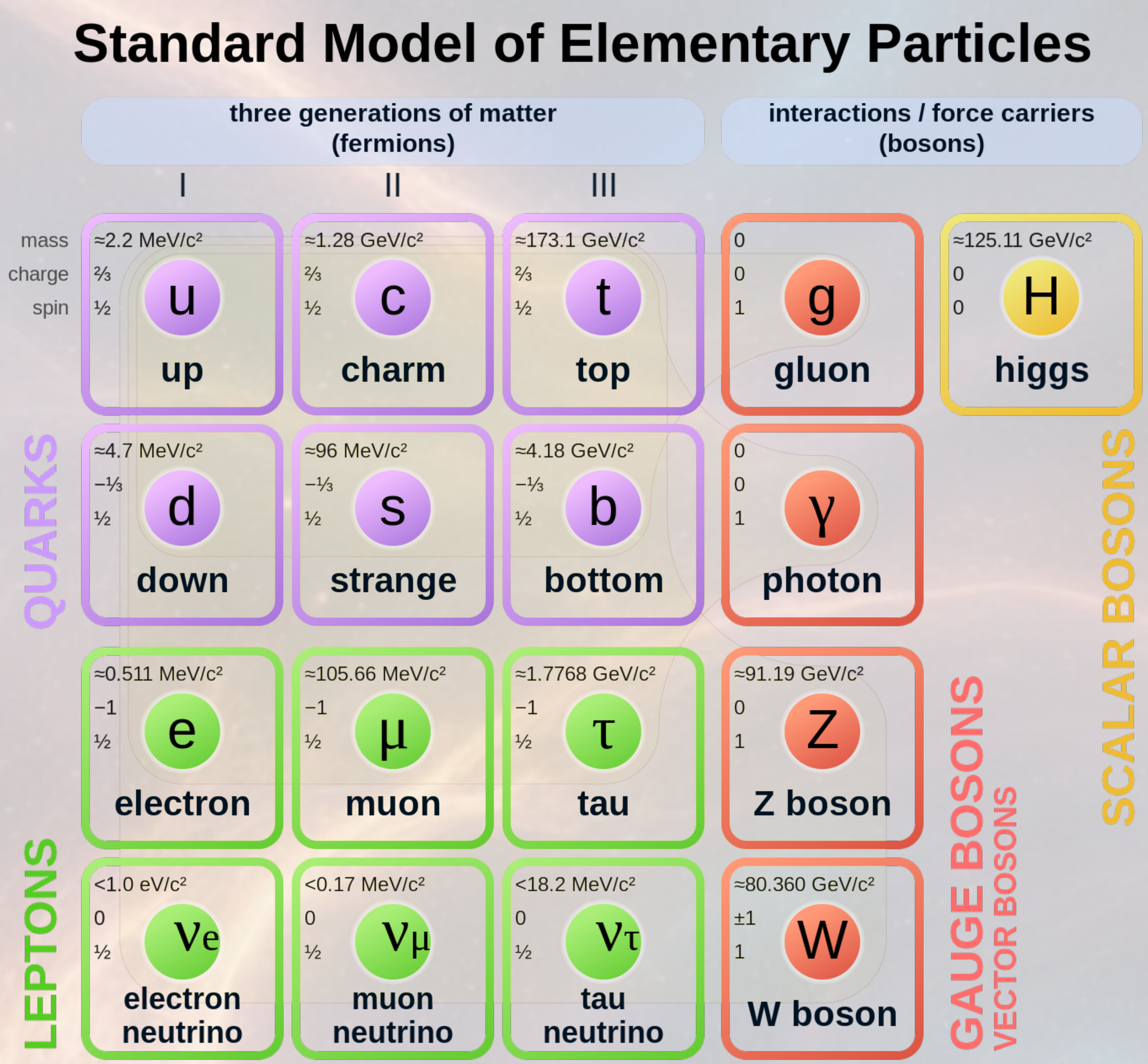
OVERVIEW OF TALK

- The (standard) model
  - gauge fields, scalars, fermions
- The methods
  - Lattice gauge theory
  - Helical fields, parity, baryogenesis
  - PDE approach
- Classical/quantum simulations
- Summary

What to simulate?



What to simulate?



$$
\begin{gathered}
\mathcal{L}_{SM} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4} g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
igs_w(\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w (A_\mu Z^0_\nu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
\beta_h \left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g} H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-) \right) + \tfrac{2M^4}{g^2}\alpha_h - \\
g\alpha_h M \left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
\tfrac{1}{8} g^2 \alpha_h \left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2\right) - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2} g \tfrac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \\
\tfrac{1}{2} ig \left(W^+_\mu (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W^-_\mu (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0) \right) + \\
\tfrac{1}{2} g \left(W^+_\mu (H\partial_\mu \phi^- - \phi^- \partial_\mu H) + W^-_\mu (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H) \right) + \tfrac{1}{2} g \tfrac{1}{c_w} (Z^0_\mu (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) + \\
M \left(\tfrac{1}{c_w} Z^0_\mu \partial_\mu \phi^0 + W^+_\mu \partial_\mu \phi^- + W^-_\mu \partial_\mu \phi^+ \right) - ig \tfrac{s_w^2}{c_w} M Z^0_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) + igs_w M A_\mu (W^+_\mu \phi^- - \\
W^-_\mu \phi^+) - ig \tfrac{1-2c_w^2}{2c_w} Z^0_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + igs_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \\
\tfrac{1}{4} g^2 W^+_\mu W^-_\mu \left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) - \tfrac{1}{8} g^2 \tfrac{1}{c_w^2} Z^0_\mu Z^0_\mu \left(H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+\phi^-\right) - \\
\tfrac{1}{2} g^2 \tfrac{s_w^2}{c_w} Z^0_\mu \phi^0 (W^+_\mu \phi^- + W^-_\mu \phi^+) - \tfrac{1}{2} ig^2 \tfrac{s_w^2}{c_w} Z^0_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) + \tfrac{1}{2} g^2 s_w A_\mu \phi^0 (W^+_\mu \phi^- + \\
W^-_\mu \phi^+) + \tfrac{1}{2} ig^2 s_w A_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) - g^2 \tfrac{s_w}{c_w} (2c_w^2 - 1) Z^0_\mu A_\mu \phi^+ \phi^- - \\
g^2 s_w^2 A_\mu A_\mu \phi^+ \phi^- + \tfrac{1}{2} ig_s \lambda^a_{ij} (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu - \bar{e}^\lambda (\gamma\partial + m^\lambda_e) e^\lambda - \bar{\nu}^\lambda (\gamma\partial + m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j (\gamma\partial + \\
m^\lambda_u) u^\lambda_j - \bar{d}^\lambda_j (\gamma\partial + m^\lambda_d) d^\lambda_j + igs_w A_\mu \left(-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j \gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j \gamma^\mu d^\lambda_j) \right) + \\
\tfrac{ig}{4c_w} Z^0_\mu \{ (\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{d}^\lambda_j \gamma^\mu (\tfrac{4}{3}s_w^2 - 1 - \gamma^5) d^\lambda_j) + \\
(\bar{u}^\lambda_j \gamma^\mu (1 - \tfrac{8}{3}s_w^2 + \gamma^5) u^\lambda_j) \} + \tfrac{ig}{2\sqrt{2}} W^+_\mu \left((\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) U^{lep}{}_{\lambda\kappa} e^\kappa) + (\bar{u}^\lambda_j \gamma^\mu (1+\gamma^5) C_{\lambda\kappa} d^\kappa_j) \right) + \\
\tfrac{ig}{2\sqrt{2}} W^-_\mu \left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda} \gamma^\mu (1+\gamma^5) \nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda} \gamma^\mu (1+\gamma^5) u^\lambda_j) \right) + \\
\tfrac{ig}{2M\sqrt{2}} \phi^+ \left(-m^\kappa_e (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1-\gamma^5) e^\kappa) + m^\lambda_\nu (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1+\gamma^5) e^\kappa \right) + \\
\tfrac{ig}{2M\sqrt{2}} \phi^- \left(m^\lambda_e (\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa} (1+\gamma^5) \nu^\kappa) - m^\kappa_\nu (\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa} (1-\gamma^5) \nu^\kappa \right) - \tfrac{g}{2} \tfrac{m^\lambda_\nu}{M} H (\bar{\nu}^\lambda \nu^\lambda) - \\
\tfrac{g}{2} \tfrac{m^\lambda_e}{M} H (\bar{e}^\lambda e^\lambda) + \tfrac{ig}{2} \tfrac{m^\lambda_\nu}{M} \phi^0 (\bar{\nu}^\lambda \gamma^5 \nu^\lambda) - \tfrac{ig}{2} \tfrac{m^\lambda_e}{M} \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda) - \tfrac{1}{4} \bar{\nu}_\lambda M^R_{\lambda\kappa} (1-\gamma_5) \hat{\nu}_\kappa - \\
\tfrac{1}{4} \bar{\nu}_\lambda M^R_{\lambda\kappa} (1-\gamma_5) \hat{\nu}_\kappa + \tfrac{ig}{2M\sqrt{2}} \phi^+ \left(-m^\kappa_d (\bar{u}^\lambda_j C_{\lambda\kappa} (1-\gamma^5) d^\kappa_j) + m^\lambda_u (\bar{u}^\lambda_j C_{\lambda\kappa} (1+\gamma^5) d^\kappa_j \right) + \\
\tfrac{ig}{2M\sqrt{2}} \phi^- \left(m^\lambda_d (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1+\gamma^5) u^\kappa_j) - m^\kappa_u (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1-\gamma^5) u^\kappa_j \right) - \tfrac{g}{2} \tfrac{m^\lambda_u}{M} H (\bar{u}^\lambda_j u^\lambda_j) - \\
\tfrac{g}{2} \tfrac{m^\lambda_d}{M} H (\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2} \tfrac{m^\lambda_u}{M} \phi^0 (\bar{u}^\lambda_j \gamma^5 u^\lambda_j) - \tfrac{ig}{2} \tfrac{m^\lambda_d}{M} \phi^0 (\bar{d}^\lambda_j \gamma^5 d^\lambda_j) + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g^c_\mu +
\end{gathered}
$$

Marcoli

Lots of structure in the Standard Model, three principle difficulties:

- Gauge invariance
- Fermions
- $\hbar$

Gauge invariance

Wigner: massless spin-one particle have two polarisations

A_μ

Gauge invariance

Wigner: massless spin-one particle have two polarisations

$$A_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu : \quad \mathcal{L} \sim -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

Gauge invariance

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$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \Lambda$$

Gauge invariance

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$$A_0 : \quad \underline{\nabla}.\underline{E} = \rho$$

Gauge invariance

Abelian Higgs:

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$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \Lambda$$

$$A_0 : \quad \underline{\nabla}.\underline{E} = \rho$$

Charged scalar field:

$$\phi \to \phi' = e^{-iq\Lambda}\phi$$

$$\underline{p} \to \underline{p} - q\underline{A}$$

$$D_\mu \phi = \partial_\mu \phi - ieA_\mu \phi$$

$$D_\mu \phi \to (D_\mu \phi)' = e^{-iq\Lambda} D_\mu \phi$$

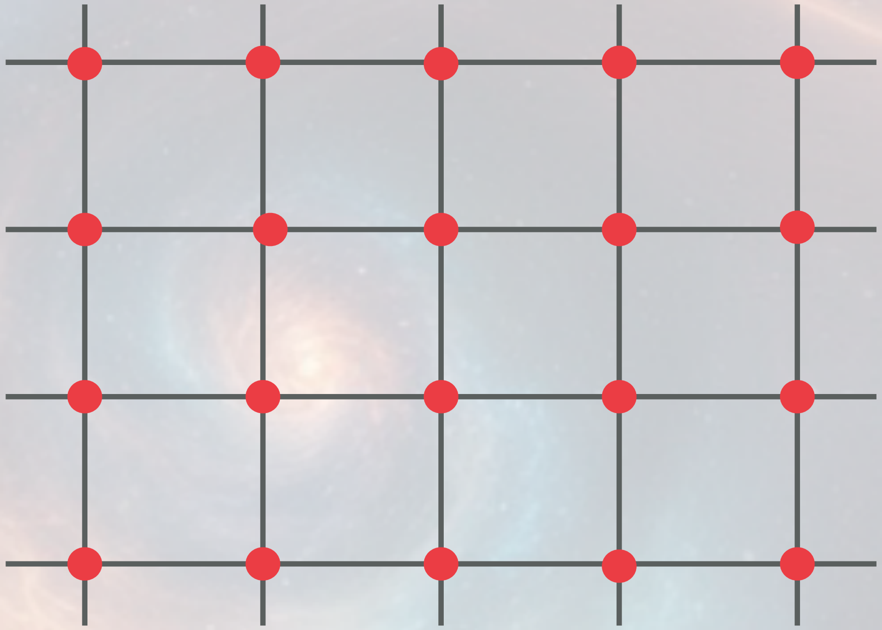
Lattice Gauge Theory approach

- Create a lattice

Wilson; Kogut and Susskind.

Lattice Gauge Theory approach

- Create a lattice
- Put ϕ on the lattice sites

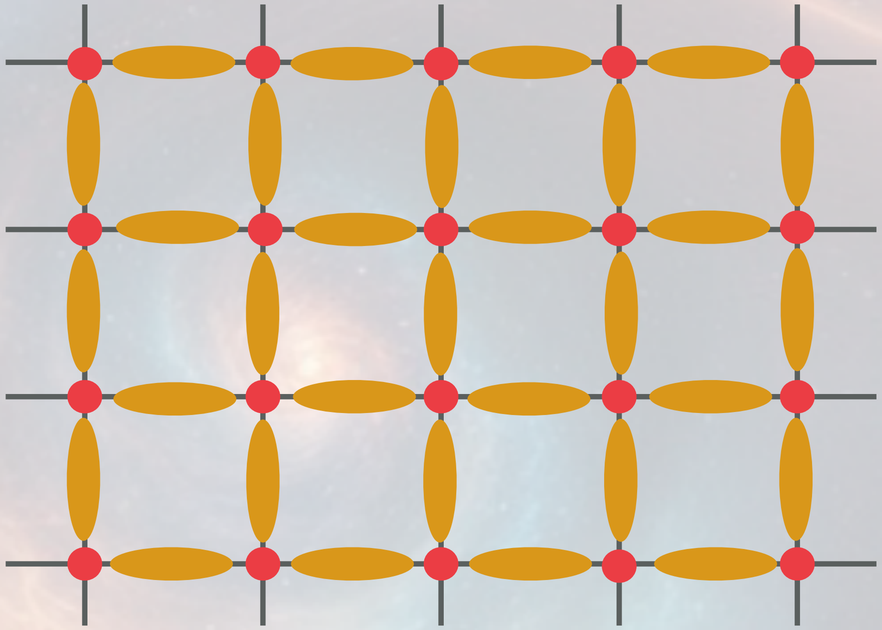


Wilson; Kogut and Susskind.

Lattice Gauge Theory approach

- Create a lattice
- Put ϕ on the lattice sites
- Gauge fields are replaced by holonomy that lives on links

$$\phi(x_f) = \mathcal{P}\exp\left[ie\int_{x_i}^{x_f} A\right]\phi(x_i),$$
$$= U(x_i,\, x_f)\phi(x_i)$$



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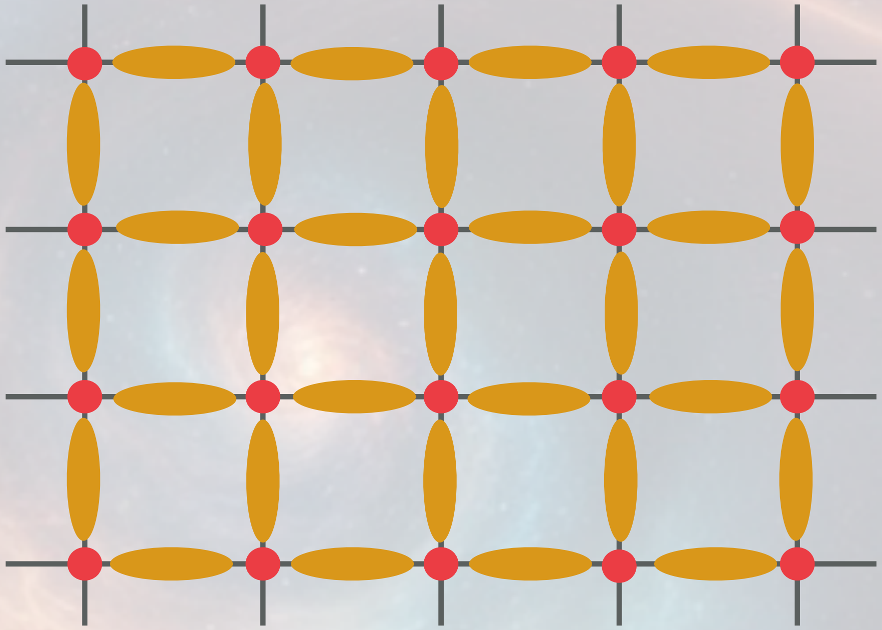

$$\phi'(x) = \Omega(x)\phi(x), \\ U'(x_1, x_2) = \Omega(x_1)U(x_1, x_2)\Omega^\dagger(x_2)$$

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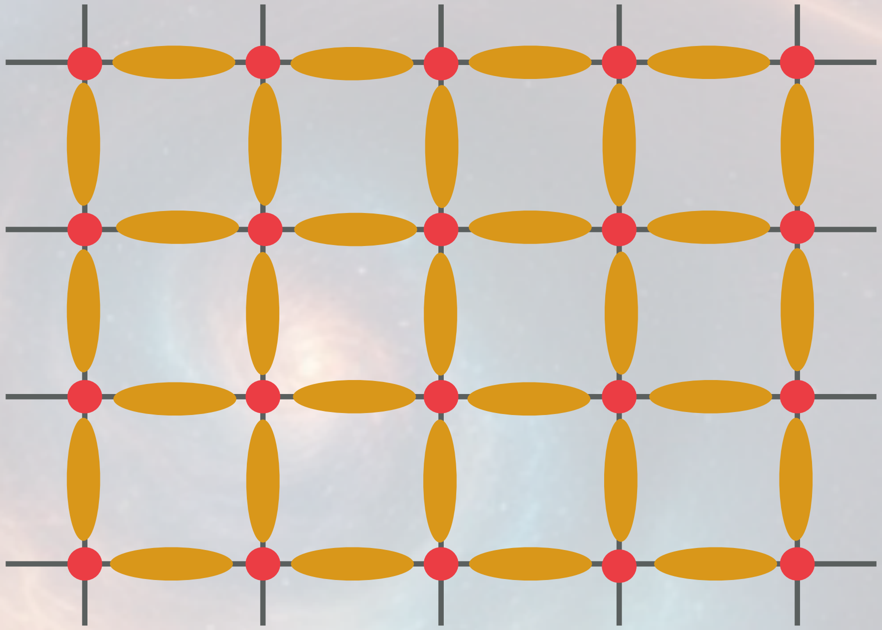

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Wilson; Kogut and Susskind.

Lattice Gauge Theory approach

$$D_\mu \phi = \frac{1}{\mathrm{d}x^\mu} \left[U_\mu(x) \phi(x + \hat{x}^\mu) - \phi(x) \right]$$

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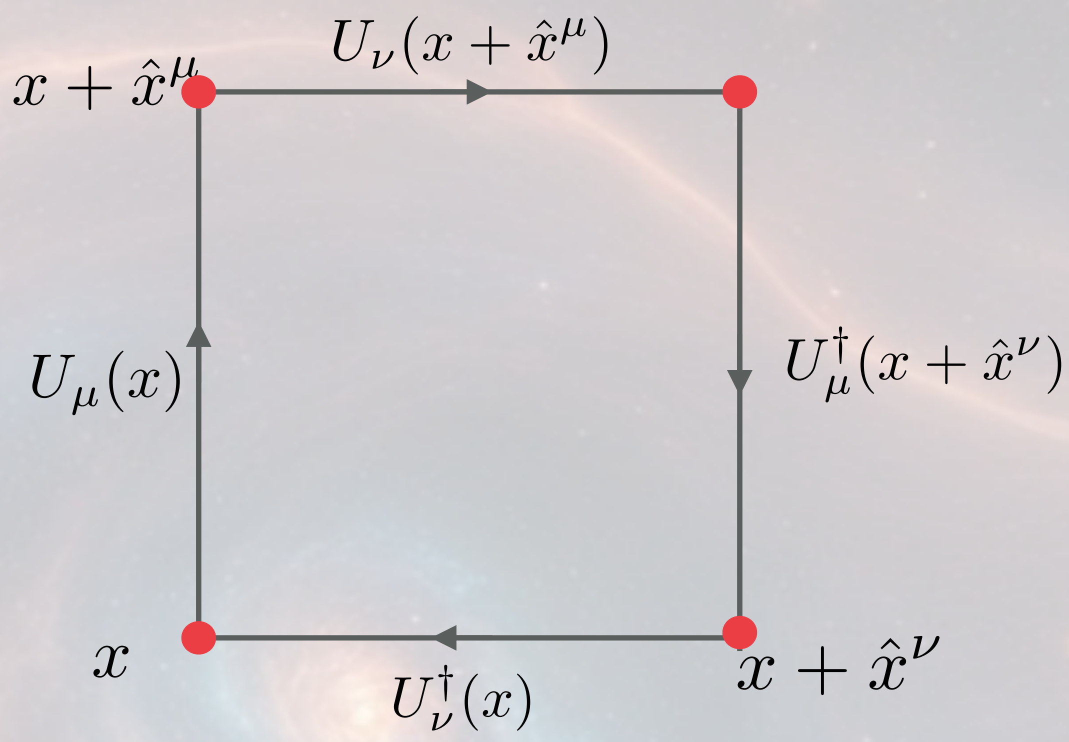

Lattice Gauge Theory approach

$$S = \sum \mathrm{d}t\mathrm{d}^3x \left[D_0\phi^\dagger D_0\phi - D_i\phi^\dagger D_i\phi - V(|\phi|) - \frac{2}{\mathrm{d}t^2\mathrm{d}x^2g^2}\sum_i \mathrm{Tr}(U_{0i}) + \frac{1}{\mathrm{d}x^4g^2\sum_{i,j}}\mathrm{Tr}(U_{ij}) \right]$$

$$D_\mu\phi = \frac{1}{\mathrm{d}x^\mu}\left[U_\mu(x)\phi(x+\hat{x}^\mu) - \phi(x)\right]$$

$$U_{\mu\nu} = U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x)$$

$$U_\mu(x) = \exp\left[-ig\ \mathrm{d}x^\mu A_\mu\right] \quad \text{no sum over } \mu$$

Lattice Gauge Theory approach

$$S = \sum \mathrm{d}t\mathrm{d}^3x \left[D_0\phi^\dagger D_0\phi - D_i\phi^\dagger D_i\phi - V(|\phi|) - \frac{2}{\mathrm{d}t^2\mathrm{d}x^2g^2}\sum_i \mathrm{Tr}(U_{0i}) + \frac{1}{\mathrm{d}x^4g^2\sum_{i,j}}\mathrm{Tr}(U_{ij}) \right]$$

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- **Baryogengesis**: Moore, Turok, Ambjörn, Krasnitz, Smit, Hu, Müller, Rajantie, PMS, Copeland, Rummukainen, Annala, Askgaard, Porter, Shaposhnikov…
- (**P)reheating**: Adshead, Giblin, Weiner, Amin, Lozano, Cuissa, Figueroa, Mou, PMS, Tranberg, Smit, Skullerud, Dufaux, Figueroa, Garcia-Bellido…
- **Cosmic Strings**: Moriarty, Myers, Rebbi, Hindmarsh, Vincent, Antunes, Shellard, Battye, Lizarraga, Urrestilla, Daverio, Kunz, Rummukainen, Tenkanen, Weir, PMS, Bevis, Portsidou, Copeland, Martin, Niz, Stuckey,…

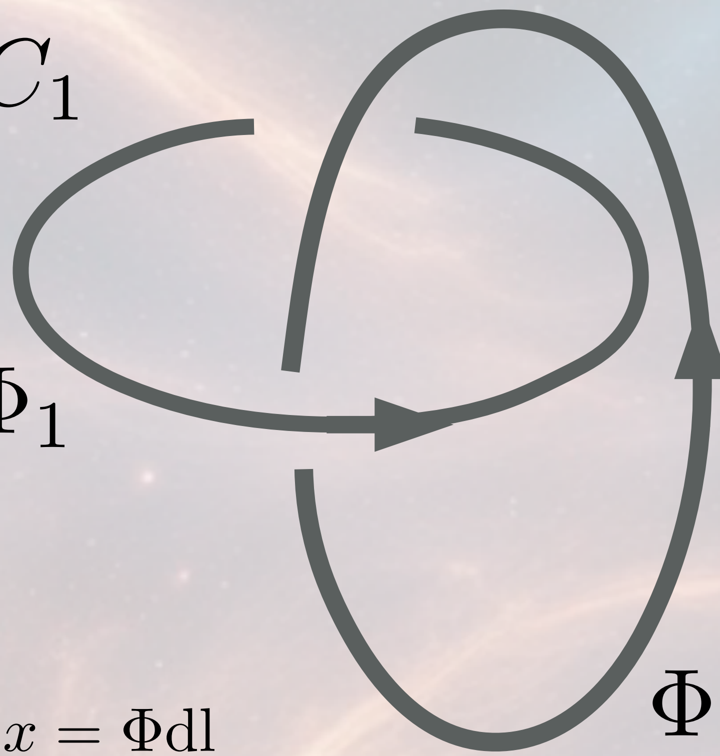
Helical B-fields

$$\mathcal{H} = \int \mathrm{d}^3x \; \underline{A}.\underline{B}$$

$$\mathcal{H} \to \mathcal{H}' = \int \mathrm{d}^3x \; (\underline{A} + \underline{\nabla}\Lambda).\underline{\nabla} \times \underline{A} = \mathcal{H} + \int_{\partial V} \Lambda \underline{B}.\underline{\mathrm{d}s}$$

Helical B-fields

$$\mathcal{H} = \int \mathrm{d}^3x \; \underline{A}.\underline{B}$$



$\underline{B}\mathrm{d}^3x = \Phi\underline{\mathrm{dl}}$

$$\mathcal{H} = \Phi_1 \int_{C_1=\partial S_1} \underline{A}.\underline{\mathrm{d}l} + \Phi_2 \int_{C_2=\partial S_2} \underline{A}.\underline{\mathrm{d}l}$$

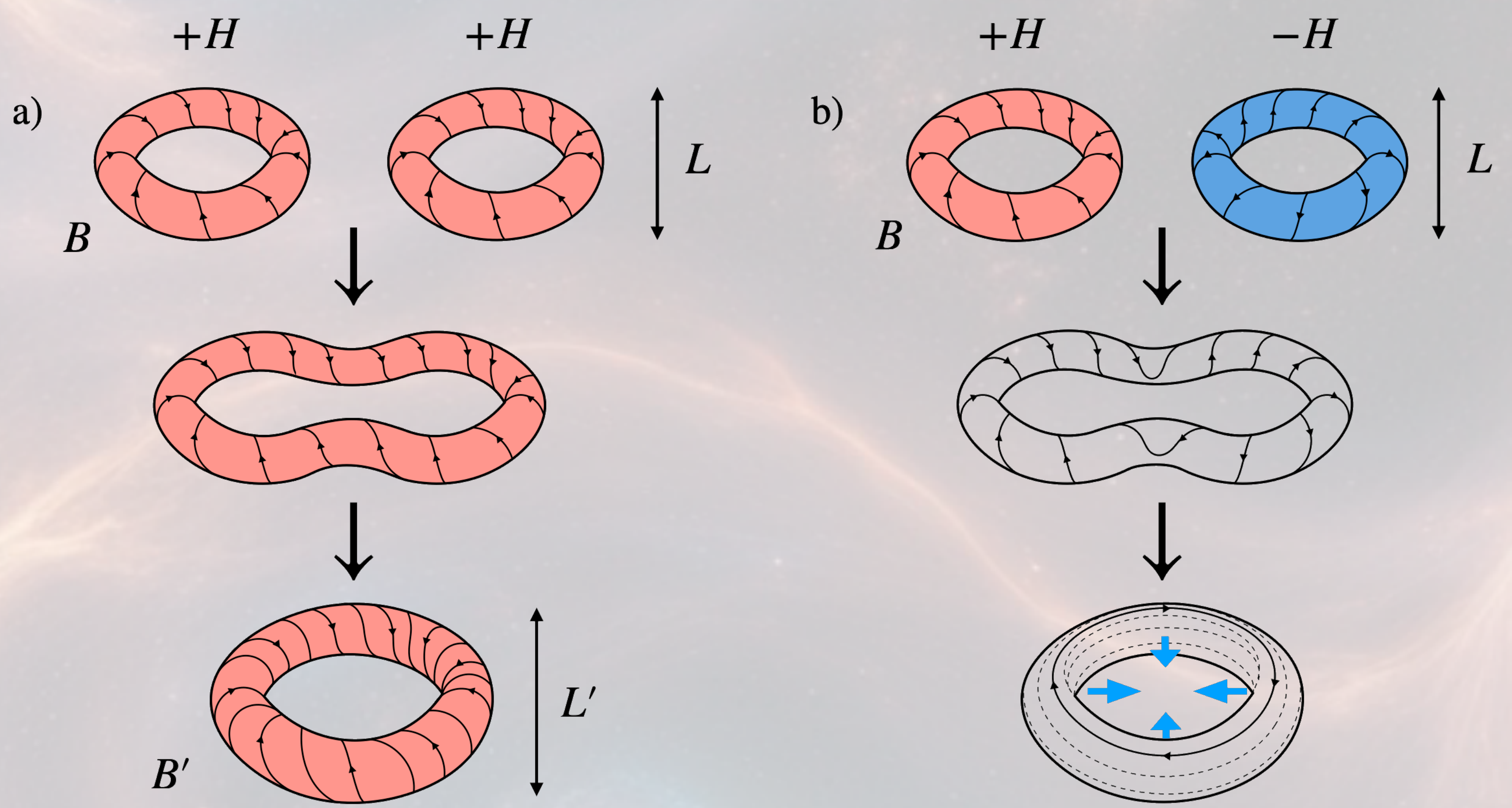
$$= \Phi_1 \underbrace{\int_{S_1} \underline{\nabla} \times \underline{A}.\mathrm{d}\underline{S}}_{\Phi_2} + \Phi_2 \underbrace{\int_{S_2} \underline{\nabla} \times \underline{A}.\mathrm{d}\underline{S}}_{\Phi_1}$$

$$= 2\Phi_1\Phi_2$$

$$\mathcal{H} \to \mathcal{H}' = \int \mathrm{d}^3x \; (\underline{A} + \underline{\nabla}\Lambda).\underline{\nabla} \times \underline{A} = \mathcal{H} + \int_{\partial V} \Lambda\underline{B}.\underline{\mathrm{d}s}$$

Helical B-fields

- **Inverse cascade**
- Baryogenesis



- Image from Hosking, Schekochihin

- Pouquet, Frisch, Leorat,; Meneguzzi, Frisch, Pouquet; Balsara, Pouquet; Müller, Biskamp; Christensson, Hindmarsh, Brandenburg; Cho; Müller, Tashihiro, Vachaspati, Vilenkin; Malapaka, Busse; Hirono, Kharzeev, Yin; Gorbar, Rudenok, Shovkovy, Vilchinskii; Hatori; Hosking, Schekochihin

Helical B-fields

- Inverse cascade
- **Baryogenesis**

$$N_{CS,1} = -\frac{g'^2}{32\pi^2} \int \mathrm{d}^3x \; \epsilon_{ijk} Y_i Y_{jk}$$

$$N_{CS,2} = -\frac{g^2}{32\pi^2} \int \mathrm{d}^3x \; \epsilon_{ijk} \left(W_i^a W_{jk}^a - \frac{g}{3} \epsilon^{abc} W_i^a W_j^b W_k^c \right)$$

Helical B-fields

- Inverse cascade
- **Baryogenesis**

$$N_{CS,1} = -\frac{g'^2}{32\pi^2}\int \mathrm{d}^3x\ \epsilon_{ijk} Y_i Y_{jk}$$

$$N_{CS,2} = -\frac{g^2}{32\pi^2}\int \mathrm{d}^3x\ \epsilon_{ijk}\left(W_i^a W_{jk}^a - \frac{g}{3}\epsilon^{abc} W_i^a W_j^b W_k^c\right)$$

$$\partial_\mu j_B^\mu = n_f\left[\frac{g^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma} W_{\mu\nu}^a W_{\rho\sigma}^a - \frac{g'^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma} Y_{\mu\nu} Y_{\rho\sigma}\right]$$

$$\Delta N_B(t) = 3\left[\Delta N_{CS,2}(t) - \Delta N_{CS,1}(t)\right]$$

Helical B-fields and baryogenesis

$$A_\mu = n^a \sin\theta_w W^a_\mu - \cos\theta_w Y_\mu$$

$$\sin\theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$n^a = -\frac{\Phi^\dagger \sigma^a \Phi}{\Phi^\dagger \Phi}$$

Helical B-fields and baryogenesis

$$A_\mu = n^a \sin\theta_w W^a_\mu - \cos\theta_w B_\mu$$

$$\sin\theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$n^a = -\frac{\Phi^\dagger \sigma^a \Phi}{\Phi^\dagger \Phi}$$

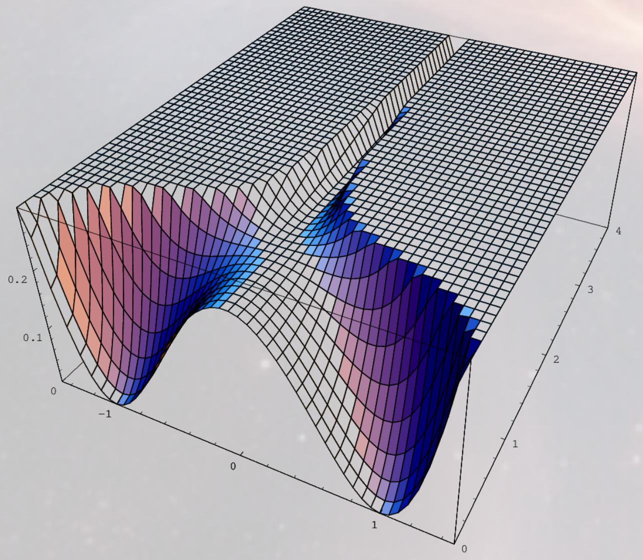
$$\mathcal{H}/V \sim n_b/\alpha$$

- Uncertainty in modelling $\mathcal{H}$ from sphaleron decay
- Uncertainty in n_b vs $N_b + \bar{N}_b$

Vachaspati; Cornwall; Copi, Ferrer, Vachaspati, Achucarro; Chu, Dent, Vachaspati; Kharzeev, Shuryak, Zahed;

Helical B-fields and baryogenesis

- Cold EW baryogengesis



- CP violation

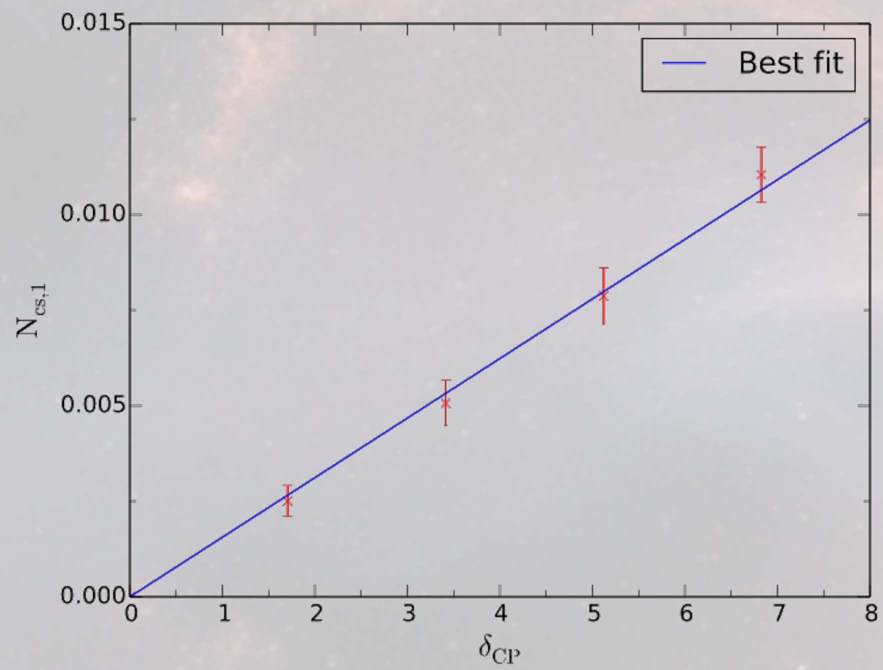
$$\mathcal{L}_{CP} = \mathcal{L} - \frac{3\delta_{CP}}{16\pi^2 m_W^2}\phi^\dagger\phi \mathrm{Tr}\left[W^{\mu\nu}\tilde{W}_{\mu\nu}\right]$$

- Classical-Statistical initial data

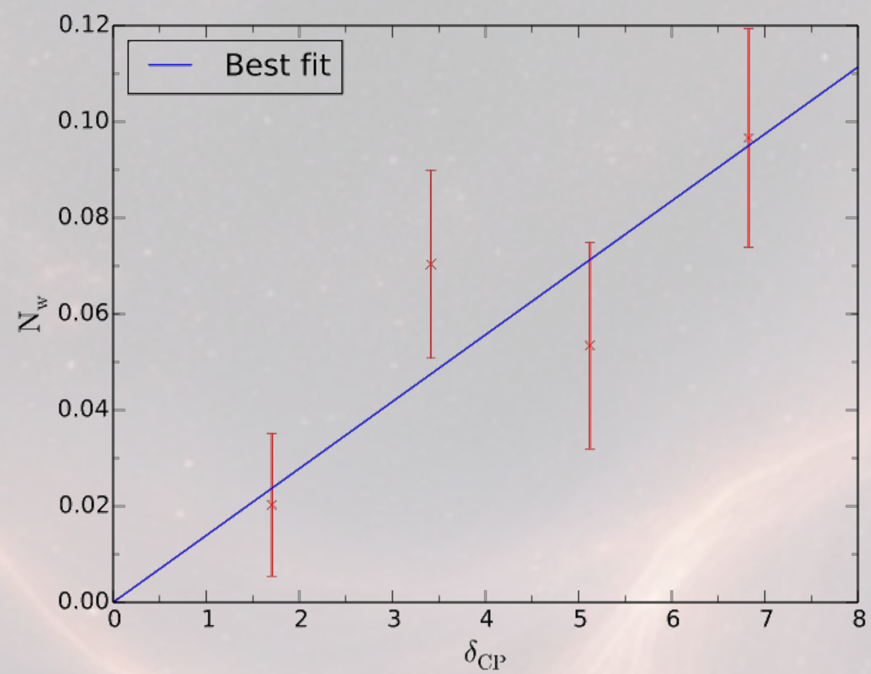
$$\hbar$$

Mou, PMS, Tranberg

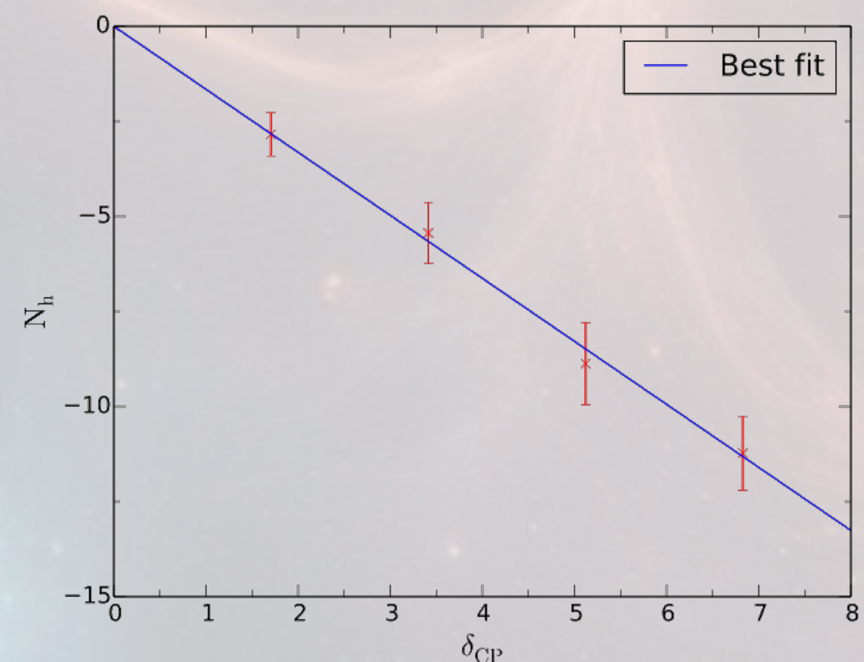
- Hypercharge Chern-Simons:



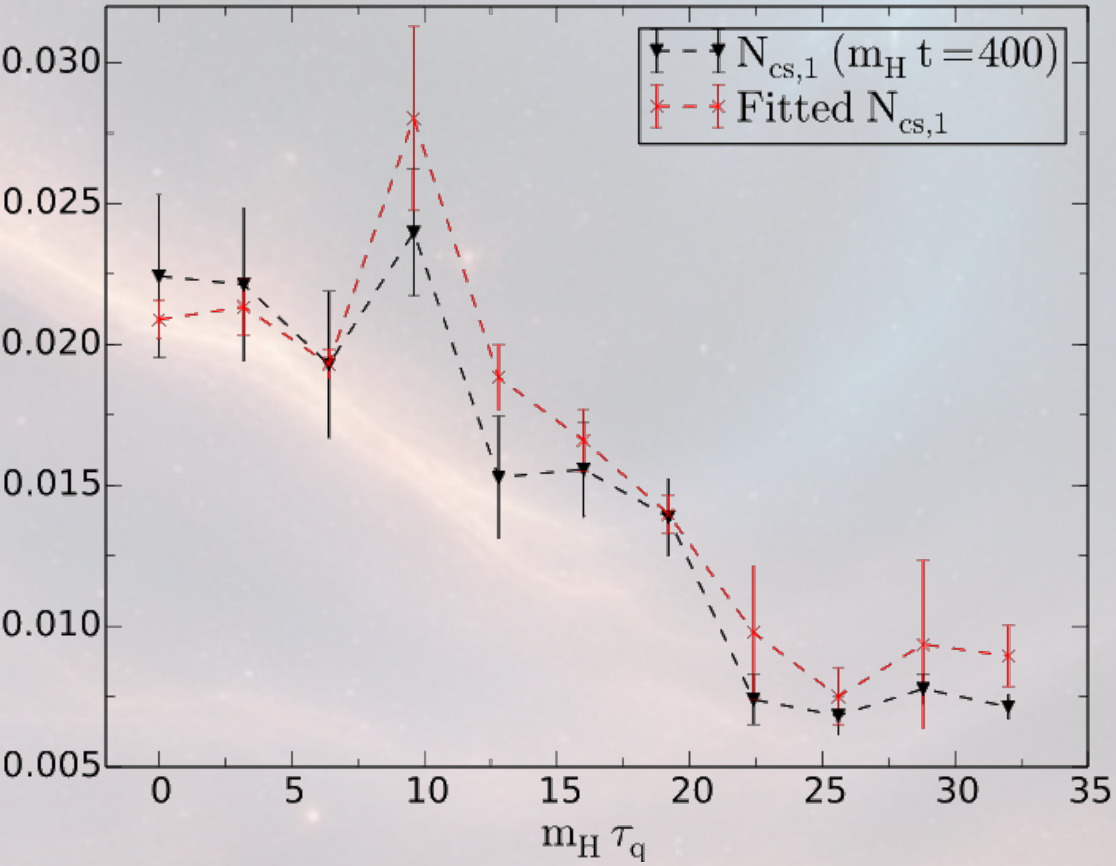
- non-Abelian Chern-Simons:

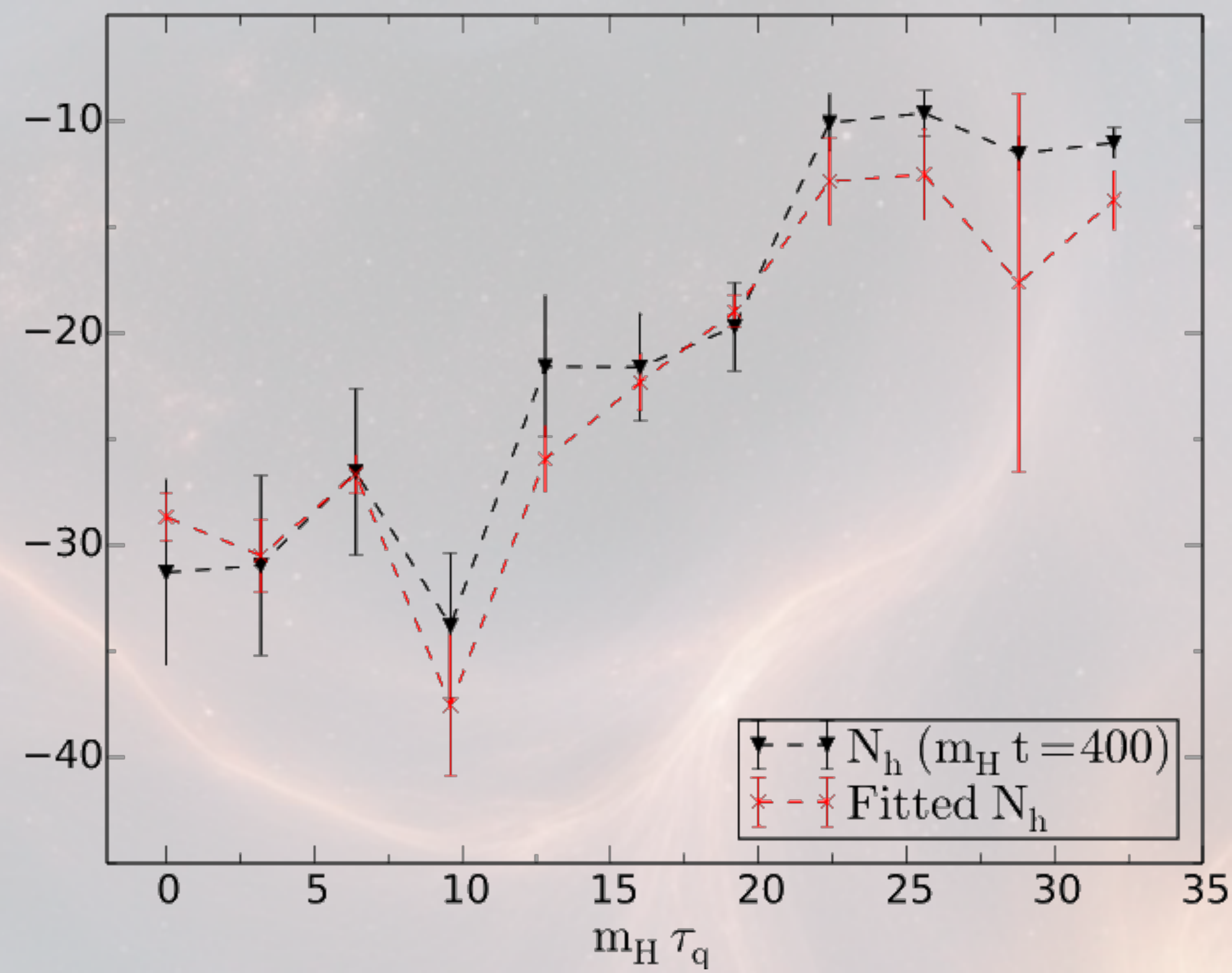


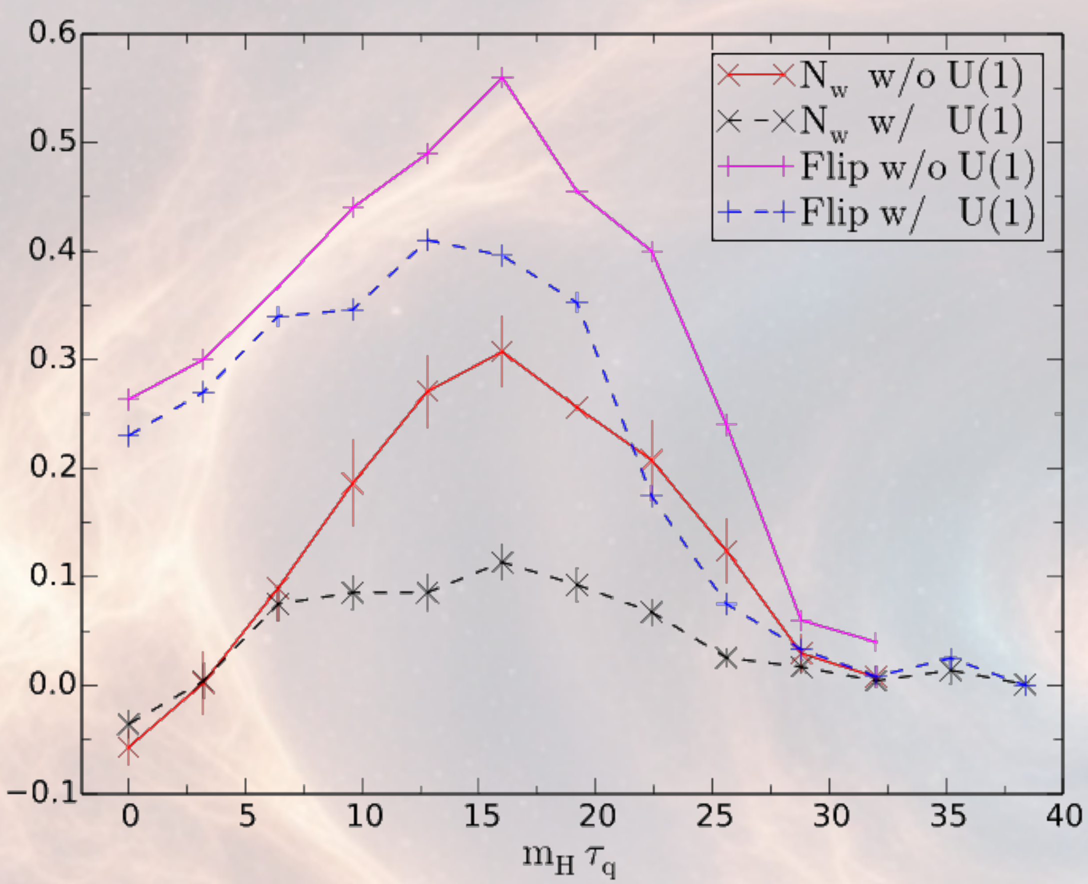
- Helicity:



$\mathcal{H} \propto n_b$







Lattice approach

- Second order* (space and time)
- Implicit equation of motion for P violation

Lattice approach

- Second order* (space and time)
- Implicit eom for CP violation

PDE approach

$$\partial_0 \phi = \pi,$$

$$\partial_0 Y_i = -E_i,$$

$$\partial_0 W_i^a = -E_i^a,$$

$$\partial_0 \pi = D_i D_i \phi - 2\lambda(\phi^\dagger \phi - \eta^2)\phi - \frac{1}{2}\beta_W \epsilon^{ijk} E_i^a W_{jk}^a \phi - \frac{1}{2}\beta_Y \epsilon^{ijk} E_i Y_{jk} \phi,$$

$$\partial_0 E^i = -\partial_k Y^{ki} + \frac{1}{2}\beta_Y (\pi^\dagger \phi + \phi^\dagger \pi)\epsilon^{ijk} Y_{jk} + \beta_Y \epsilon^{ijk} \partial_j (\phi^\dagger \phi) E_k$$
$$+ \frac{i}{2} g' (\phi^\dagger D^i \phi - D^i \phi^\dagger \phi)$$

$$\partial_0 E^{a,i} = -D_k W^{a,ki} + \frac{1}{2}\beta_W (\pi^\dagger \phi + \phi^\dagger \pi)\epsilon^{ijk} W_{jk}^a + \beta_W \epsilon^{ijk} \partial_j (\phi^\dagger \phi) E_k^a$$
$$+ \frac{i}{2} g (\phi^\dagger \sigma^a D^i \phi - D^i \phi^\dagger \sigma^a \phi)$$

- Abelian: Deskins, Giblin, Caldwell; Adshead, Giblin, Skully, Sfakianakis; Garcia-Bellido, Grigoriev, Kusenko, Shaposhnikov;
- non-Abelian: Lozano, Amin; Adshead, Giblin, Weiner,

PDE approach

- Easier to use bigger stencils
- Easier to use higher order time integrators
- Easier to include parity violation

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Gauss constraints not automatically satisfied

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- Easier to use higher order time integrators
- Easier to include parity violation

Gauss constraints not automatically satisfied

Numerical relativity also has gauge invariance

$$x^\mu \to x'^\mu(x)$$

And constraint equations

$$G_{00},\ G_{0i}$$

Numerically improved EM

$$\dot{A}_i = -E_i - \partial_i \phi$$

$$\dot{E}_i = -\nabla^2 A_i + \partial_i \partial_j A_j - J_i$$

$$\mathcal{C} = \partial_i E_i - \rho$$

- Knapp, Walker, Baumgarte

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$$\dot{E}_i = -\nabla^2 A_i + \partial_i \Gamma - J_i$$

$$\dot{\Gamma} = -\nabla^2 \phi - \rho + a^2 \mathcal{C}$$

$$= (a^2 - 1)\partial_i E_i - \nabla^2 \phi - a^2 \rho$$

- Knapp, Walker, Baumgarte

Numerically improved EM

$$\dot{A}_i = -E_i - \partial_i \phi$$

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$$\dot{\Gamma} = -\nabla^2 \phi - \rho + a^2 \mathcal{C}$$

$$= (a^2 - 1)\partial_i E_i - \nabla^2 \phi - a^2 \rho$$

$$-\ddot{\mathcal{C}} + a^2 \nabla^2 \mathcal{C} = 0$$

- Knapp, Walker, Baumgarte

Standard Model version

$$\partial_0 \pi = \nabla^2 \phi - \frac{i}{2}(g' Y_i + y\sigma^a W^{a_i})(\partial_i \phi + D_i \phi) - 2\lambda(\phi^\dagger \phi - \eta^2)\phi$$
$$- \frac{1}{2}\beta_Y \epsilon_{ijk} E_i Y_{jk} \phi - \frac{1}{2}\beta_W \epsilon_{ijk} E_i^a W_{jk}^a \phi - \frac{i}{2} g' \Gamma \phi - \frac{i}{2} g \sigma^a \Gamma^a \phi,$$
$$\partial_0 E_i = -\nabla^2 Y_i + \partial_i \Gamma + \frac{i}{2} g' (\phi^\dagger D_i \phi - D_i \phi^\dagger \phi)$$
$$+ \frac{1}{2}\beta_Y (\pi^\dagger \phi + \phi^\dagger \pi)\epsilon_{ijk} Y_{jk} + \beta_Y \epsilon_{ijk} \partial_j (\phi^\dagger \phi) E_k,$$
$$\partial_0 E_i^a = -\nabla^2 W_i^a - g\epsilon^{abc} W_j^b (\partial_j W_i^c + W_{ji}^c) + D_i \Gamma^a + \frac{i}{2} g (\phi^\dagger \sigma^a D_i \phi - D_i \phi^\dagger \sigma^a \phi)$$
$$+ \frac{1}{2}\beta_W (\pi^\dagger \phi + \phi^\dagger \pi)\epsilon_{ijk} W_{jk}^a + \beta_W \epsilon_{ijk} \partial_j (\phi^\dagger \phi) E_k^a$$

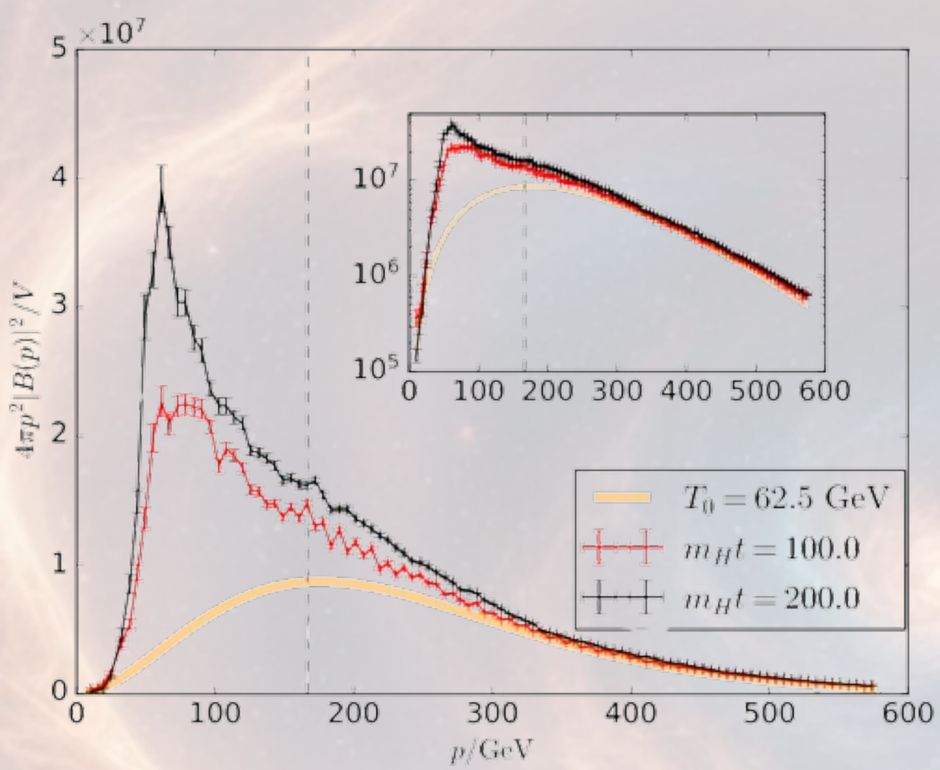
$$\partial_0 \Gamma = -\partial_i E_i + g_p^2 G,$$
$$\partial_0 \Gamma^a = -\partial_i E_i^a + g_p^2 G^a$$

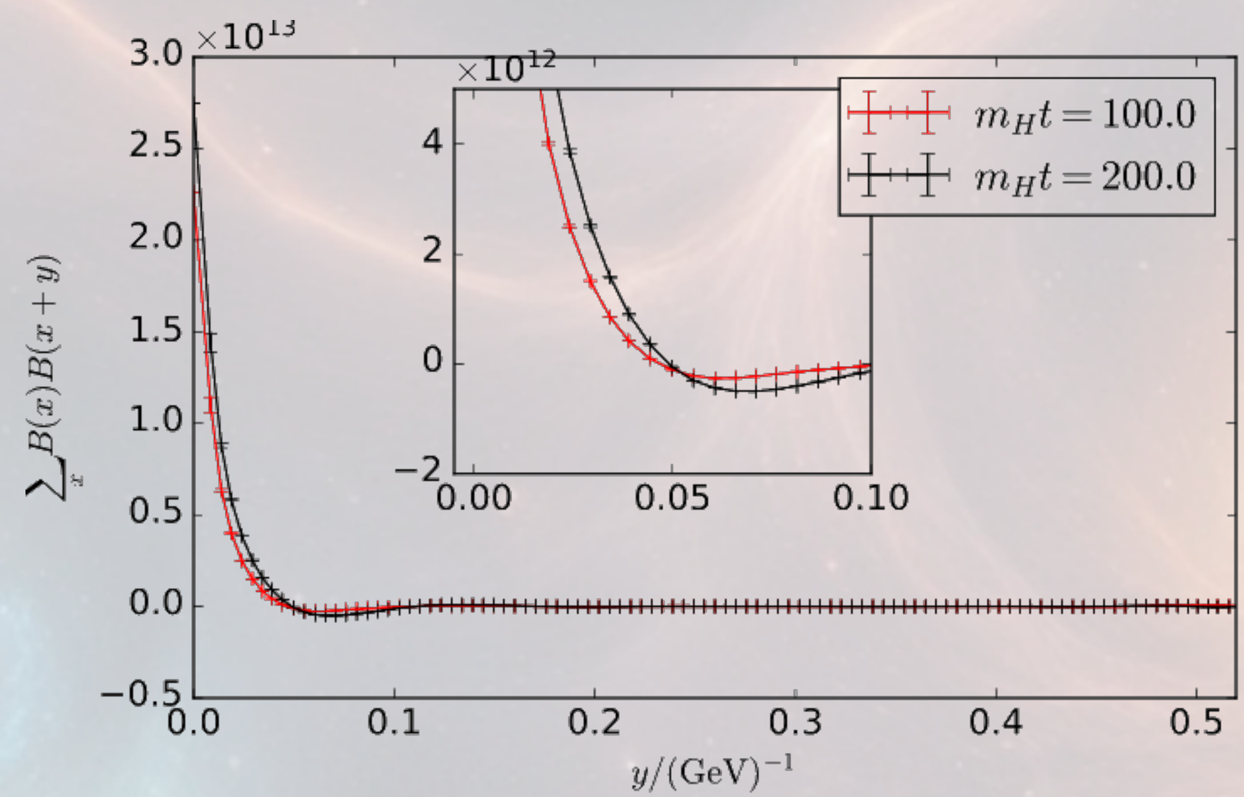
$$\Gamma = \partial_i Y_i,$$
$$\Gamma^a = \partial_i W_i^a$$

- Vachaspati

Standard Model version

$$
\begin{aligned}
\partial_0\pi &= \nabla^2\phi - \frac{i}{2}(g'Y_i + y\sigma^a W^{a_i})(\partial_i\phi + D_i\phi) - 2\lambda(\phi^\dagger\phi - \eta^2)\phi \\
&\quad - \frac{1}{2}\beta_Y\epsilon_{ijk}E_iY_{jk}\phi - \frac{1}{2}\beta_W\epsilon_{ijk}E_i^aW_{jk}^a\phi - \frac{i}{2}g'\Gamma\phi - \frac{i}{2}g\sigma^a\Gamma^a\phi, \\
\partial_0 E_i &= -\nabla^2 Y_i + \partial_i\Gamma + \frac{i}{2}g'(\phi^\dagger D_i\phi - D_i\phi^\dagger\phi) \\
&\quad + \frac{1}{2}\beta_Y(\pi^\dagger\phi + \phi^\dagger\pi)\epsilon_{ijk}Y_{jk} + \beta_Y\epsilon_{ijk}\partial_j(\phi^\dagger\phi)E_k, \\
\partial_0 E_i^a &= -\nabla^2 W_i^a - g\epsilon^{abc}W_j^b(\partial_j W_i^c + W_{ji}^c) + D_i\Gamma^a + \frac{i}{2}g(\phi^\dagger\sigma^a D_i\phi - D_i\phi^\dagger\sigma^a\phi) \\
&\quad + \frac{1}{2}\beta_W(\pi^\dagger\phi + \phi^\dagger\pi)\epsilon_{ijk}W_{jk}^a + \beta_W\epsilon_{ijk}\partial_j(\phi^\dagger\phi)E_k^a
\end{aligned}
$$





- Mou, Patel, PMS, Vachaspati

Fermions

easy:

- Linear in fermion field

$$\mathcal{L} = ... + \bar{\psi}_{L,R}\gamma^{\mu}D_{\mu}\psi_{L,R} + g_Y\bar{\psi}_L\phi\psi_R + ...$$

Fermions

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- Linear in fermion field

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hard:

- Intrinsically quantum: Grassmann
- First order derivatives: doublers

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hard:

- Intrinsically quantum: Grassmann
- First order derivatives: doublers

$$\frac{df}{dx} = f \quad \rightarrow \quad \frac{f_{n+1} - f_{n-1}}{2dx} = f_n \quad \Rightarrow f_{n+1} = f_{n-1} + 2dx\ f_n$$

Fermions

$$\hat{\psi}(t,\underline{x}) = \sum_s \int \frac{d^3p}{(2\pi)^3}\frac{1}{2\omega_p}\left[u^s(x,\underline{p})\hat{b}_s(\underline{p}) + v^s(x,\underline{p})\hat{d}_s^\dagger(\underline{p})\right]$$

Fermions

$$\hat{\psi}(t,\underline{x}) = \sum_s \int \frac{d^3p}{(2\pi)^3}\frac{1}{2\omega_p}\left[u^s(x,\underline{p})\hat{b}_s(\underline{p}) + v^s(x,\underline{p})\hat{d}_s^\dagger(\underline{p})\right]$$

$$u^s(t=0,\underline{x},\underline{p}) = U_s(\underline{p})e^{-i\underline{p}.\underline{x}},$$

$$v^s(t=0,\underline{x},\underline{p}) = V_s(\underline{p})e^{-i\underline{p}.\underline{x}}$$

Fermions

$$\hat{\psi}(t,\underline{x}) = \sum_s \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_p} \left[u^s(x,\underline{p}) \hat{b}_s(\underline{p}) + v^s(x,\underline{p}) \hat{d}_s^\dagger(\underline{p}) \right]$$

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One quantum field, $\hat{\psi}(t,\underline{x})$, ~ N³ classical fields $u^s(x,\underline{p}),\ v^s(x,\underline{p})$

$$\gamma^\mu \partial_\mu \hat{\psi} + m\hat{\psi} = 0 \rightarrow \begin{cases} \gamma^\mu \partial_\mu u^s(x,\underline{p}) + m u^s(x,\underline{p}) = 0, & \forall \underline{p} \\ \gamma^\mu \partial_\mu v^s(x,\underline{p}) + m v^s(x,\underline{p}) = 0, & \forall \underline{p} \end{cases}$$

Fermions

$$\hat{\psi}(t,\underline{x}) = \sum_s \int \frac{d^3p}{(2\pi)^3}\frac{1}{2\omega_p}\left[u^s(x,\underline{p})\hat{b}_s(\underline{p}) + v^s(x,\underline{p})\hat{d}_s^\dagger(\underline{p})\right]$$

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One quantum field, $\hat{\psi}(t,\underline{x})$, ~ N[3] classical fields $u^s(x,\underline{p}),\ v^s(x,\underline{p})$

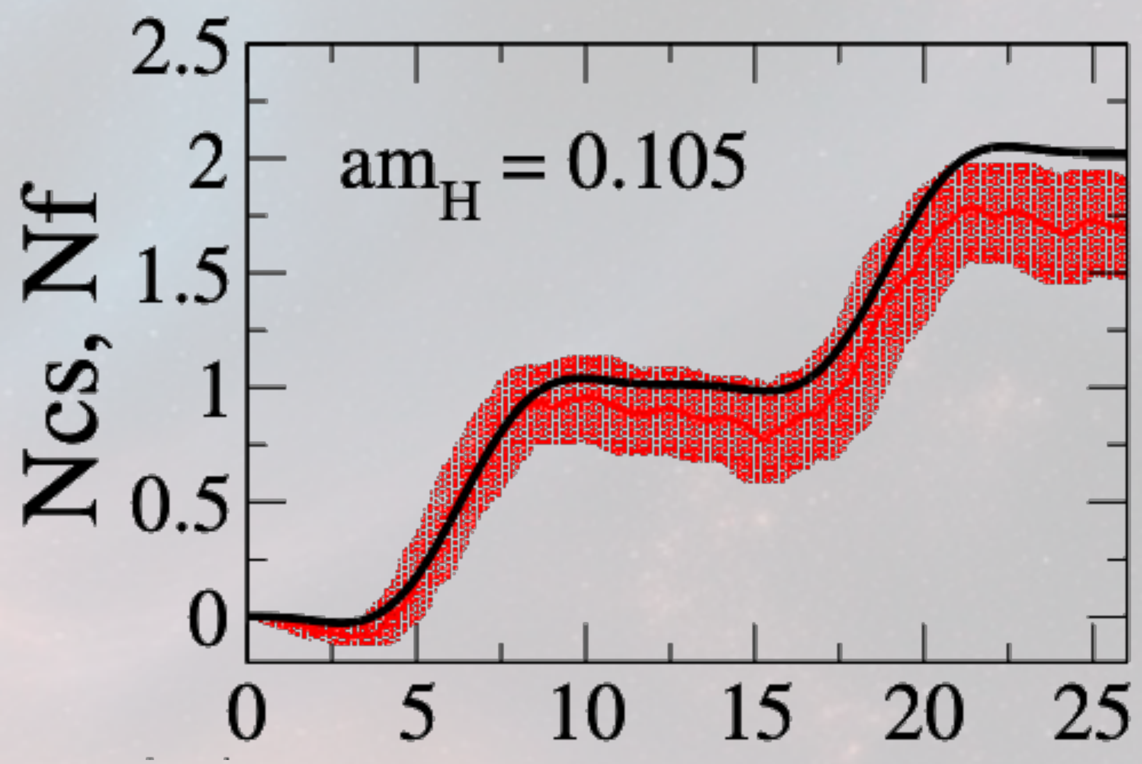
$$\gamma^\mu\partial_\mu\hat{\psi} + m\hat{\psi} = 0 \rightarrow \begin{cases} \gamma^\mu\partial_\mu u^s(x,\underline{p}) + mu^s(x,\underline{p}) = 0, & \forall\underline{p} \\ \gamma^\mu\partial_\mu v^s(x,\underline{p}) + mv^s(x,\underline{p}) = 0, & \forall\underline{p} \end{cases}$$

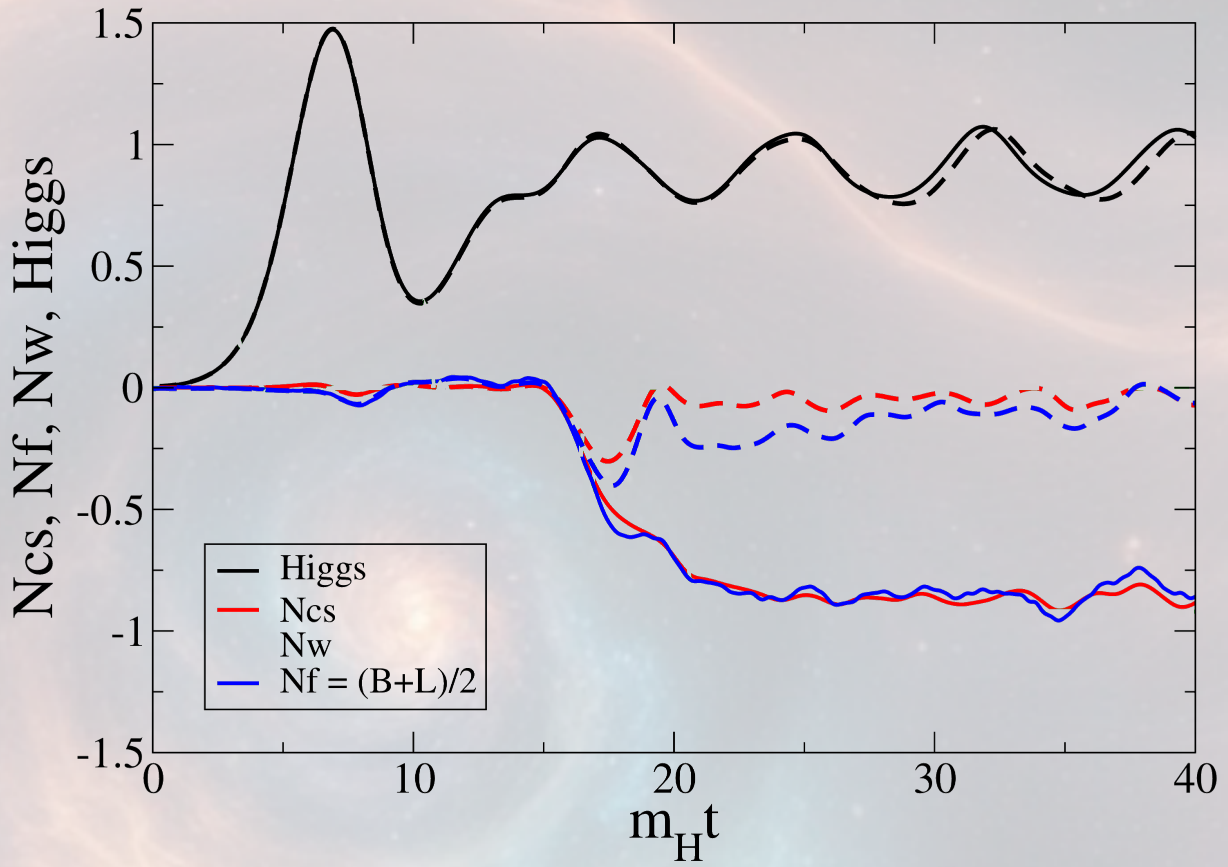
$$\text{“}\Box\phi... = g_Y\langle\bar{\psi}(x)\psi(x)\rangle\text{”} \qquad \hat{b}_s(\underline{p})|0\rangle = 0,\ \hat{d}_s(\underline{p})|0\rangle = 0$$

$$\text{“}\Box\phi... = g_Y\int\frac{d^3p}{(2\pi)^3}\left[\bar{u}^s(x,\underline{p})u^s(x,\underline{p}) - \bar{v}^s(x,\underline{p})v^s(x,\underline{p})\right]\text{”}$$

- Borsanyi , Hindmarsh
- Berges, Gelfand, Pruschke; PMS, Tranberg; Mou, PMS, Tranberg; Hebenstreit, Berges, Gelfand; Tanji, Mueller, Berges; Mueller, Schlichting, Sharma.

Simulating fermions





- PMS, Tranberg

Classical/quantum

$$\left\langle \hat{\mathcal{O}}\left(\hat{\Phi},\hat{\Pi}\right)\right\rangle = \mathrm{Tr}\left[\hat{\mathcal{O}}\left(\hat{\Phi},\hat{\Pi}\right)\hat{\rho}\left(\hat{\Phi}(t_0),\hat{\Pi}(t_0)\right)\right]$$

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$$\left\langle \hat{\mathcal{O}}\left(\hat{\Phi},\hat{\Pi}\right)\right\rangle = \mathrm{Tr}\left[\hat{\mathcal{O}}\left(\hat{\Phi},\hat{\Pi}\right)\hat{\rho}\left(\hat{\Phi}(t_0),\hat{\Pi}(t_0)\right)\right]$$

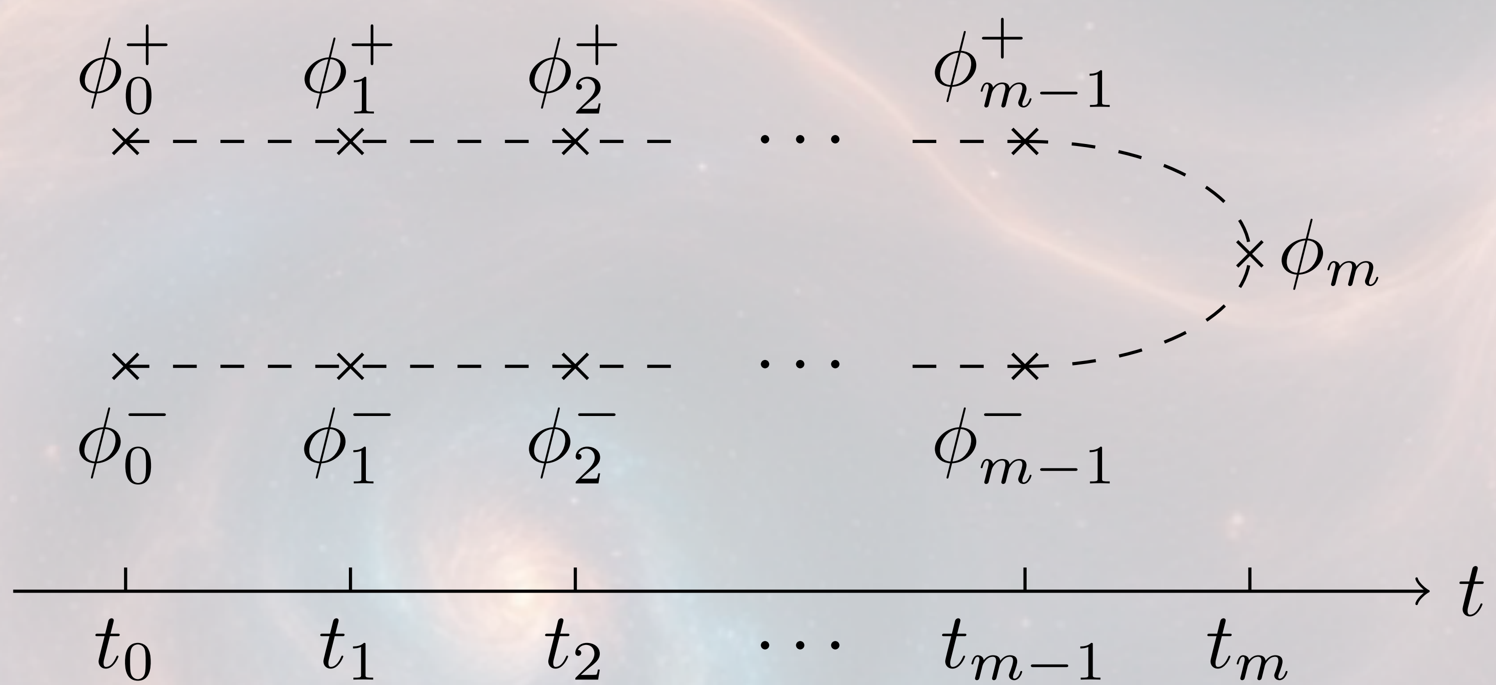
$$= \int \mathcal{D}\phi_0^- \langle \phi_0^-;t_0|\hat{\mathcal{O}}(t_m)\hat{\rho}(t_0)|\phi_0^-;t_0\rangle$$

Classical/quantum

$$\left\langle \hat{\mathcal{O}}\left(\hat{\Phi},\hat{\Pi}\right)\right\rangle = \mathrm{Tr}\left[\hat{\mathcal{O}}\left(\hat{\Phi},\hat{\Pi}\right)\hat{\rho}\left(\hat{\Phi}(t_0),\hat{\Pi}(t_0)\right)\right]$$

$$= \int \mathcal{D}\phi_0^- \langle \phi_0^-;t_0|\hat{\mathcal{O}}(t_m)\hat{\rho}(t_0)|\phi_0^-;t_0\rangle$$

$$= \int \mathcal{D}\phi \langle \phi_0^-;t_0 \underbrace{|\phi_1^-;t_1\rangle\langle\phi_1^-;t_1|}_{\mathbb{I}} \ldots\hat{\mathcal{O}}(t_m)\ldots \underbrace{|\phi_1^+;t_1\rangle\langle\phi_1^+;t_1|}_{\mathbb{I}}\underbrace{\phi_0^+;t_0\rangle\langle\phi_0^+;t_0|}_{\mathbb{I}}\hat{\rho}(t_0)|\phi_0^-;t_0\rangle$$



- Schwinger; Keldysh

Classical/quantum

$$\left\langle \hat{\mathcal{O}}\left(\hat{\Phi},\hat{\Pi}\right)\right\rangle = \mathrm{Tr}\left[\hat{\mathcal{O}}\left(\hat{\Phi},\hat{\Pi}\right)\hat{\rho}\left(\hat{\Phi}(t_0),\hat{\Pi}(t_0)\right)\right]$$

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$$= \int \mathcal{D}\phi \left\langle \phi_0^+; t_0\right| \hat{\rho}\left[\hat{\Phi}(t_0),\hat{\Pi}(t_0)\right]\left|\phi_0^-; t_0\right\rangle \mathcal{O}(t_m) \exp\left\{\frac{i}{\hbar}S(\phi^-,\phi^+)\right\}$$

- Schwinger; Keldysh

Classical/quantum

$$\left\langle \hat{\mathcal{O}}\left(\hat{\Phi},\hat{\Pi}\right)\right\rangle = \mathrm{Tr}\left[\hat{\mathcal{O}}\left(\hat{\Phi},\hat{\Pi}\right)\hat{\rho}\left(\hat{\Phi}(t_0),\hat{\Pi}(t_0)\right)\right]$$

$$= \int \mathcal{D}\phi_0^- \langle\phi_0^-;t_0|\hat{\mathcal{O}}(t_m)\hat{\rho}(t_0)|\phi_0^-;t_0\rangle$$

$$= \int \mathcal{D}\phi\langle\phi_0^-;t_0 \underbrace{|\phi_1^-;t_1\rangle\langle\phi_1^-;t_1|}_{\mathbb{I}} ...\hat{\mathcal{O}}(t_m)... \underbrace{|\phi_1^+;t_1\rangle\langle\phi_1^+;t_1|}_{\mathbb{I}} \underbrace{\phi_0^+;t_0\rangle\langle\phi_0^+;t_0|}_{\mathbb{I}} \hat{\rho}(t_0)|\phi_0^-;t_0\rangle$$

$$= \int \mathcal{D}\phi \left\langle\phi_0^+;t_0\right| \hat{\rho}\left[\hat{\Phi}(t_0),\hat{\Pi}(t_0)\right]\left|\phi_0^-;t_0\right\rangle \mathcal{O}(t_m)\exp\left\{\frac{i}{\hbar}S(\phi^-,\phi^+)\right\}$$

$$\phi_i^+(x) = \phi_i^{cl}(x) + \frac{1}{2}\phi_i^q(x), \qquad \phi_i^-(x) = \phi_i^{cl}(x) - \frac{1}{2}\phi_i^q(x)$$

- Keldysh; Aarts, Smit Greiner, Müller; Kameen, Levchenko

Classical/quantum

$$\langle \phi_0^{cl}(p)\phi_0^{cl\dagger}(p')\rangle = \frac{\hbar}{\omega}\left(n_p + \frac{1}{2}\right)(2\pi)^3\delta(\underline{p} - \underline{p}')$$

$$\langle \dot{\phi}_0^{cl}(p)\dot{\phi}_0^{cl\dagger}(p')\rangle = \hbar\omega\left(n_p + \frac{1}{2}\right)(2\pi)^3\delta(\underline{p} - \underline{p}')$$

Classical/quantum

$$\langle \phi_0^{cl}(p) \phi_0^{cl\dagger}(p') \rangle = \frac{\hbar}{\omega} \left(n_p + \frac{1}{2} \right) (2\pi)^3 \delta(\underline{p} - \underline{p}')$$

$$\langle \dot{\phi}_0^{cl}(p) \dot{\phi}_0^{cl\dagger}(p') \rangle = \hbar\omega \left(n_p + \frac{1}{2} \right) (2\pi)^3 \delta(\underline{p} - \underline{p}')$$

Unique saddle:

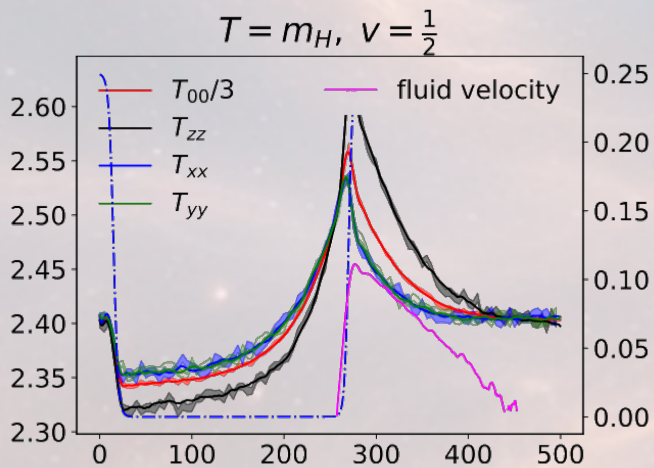
$$\phi^{cl}|_{saddle} = \varphi, \qquad \phi^{q}|_{saddle} = 0$$

$$\Box\varphi - V'(\varphi) = 0$$

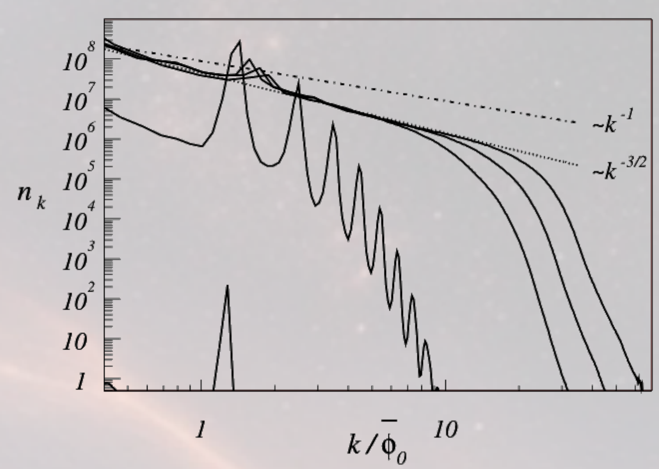
- Mou, PMS, Tranberg, Woodward

Classical-statistical approximation

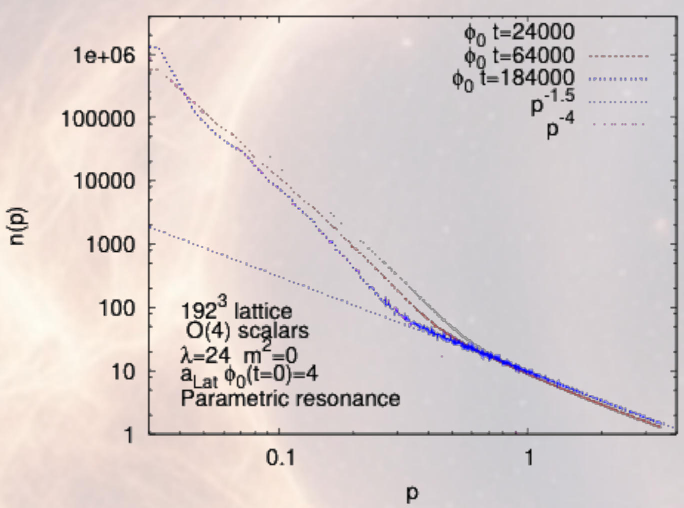
- Gaussian initial data - quantum distribution
- Classical evolution



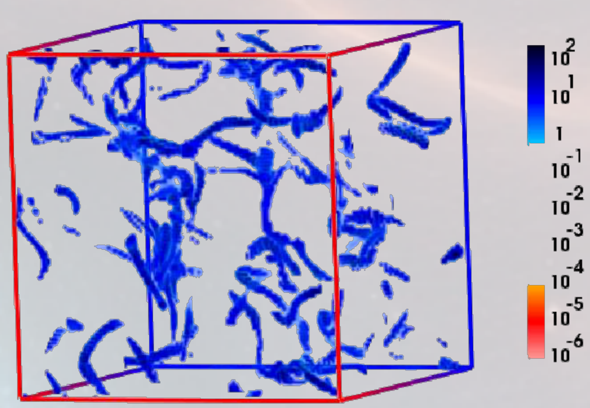
- Mou, PMS, Tranberg



- Micha, Tkachev



- Berges, Sexty



- Diaz-Gil, Garcia-Bellido, Perez, González-Arroyo

- Aarts, Smit; Aarts, Berges; Buchmüller, Jakovác;; Cooper, Share, Rose; Blagoev, Cooper, Dawson, Mihail; Berges, Boguslavski, Schlichting, Venugopalan; Mou, PMS, Tranberg; Micha, Tkachev; Diaz-Gil, Garcia-Bellido, Perez, González-Arroyo…

Thimbles

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}\Phi \; \mathcal{O} e^{iS/\hbar}}{\int \mathcal{D}\Phi \; e^{iS/\hbar}}$$

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- It's big

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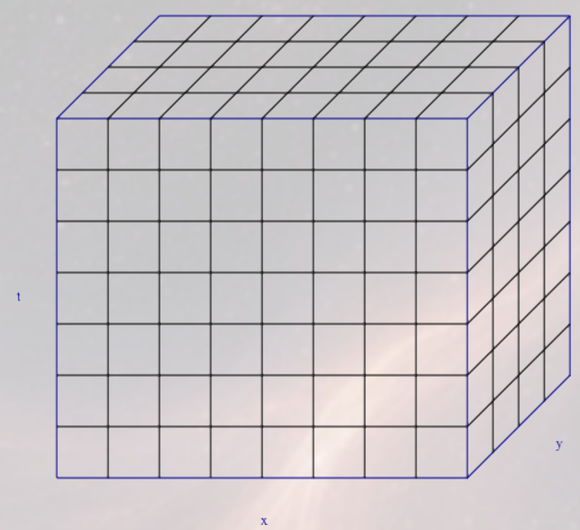

Thimbles

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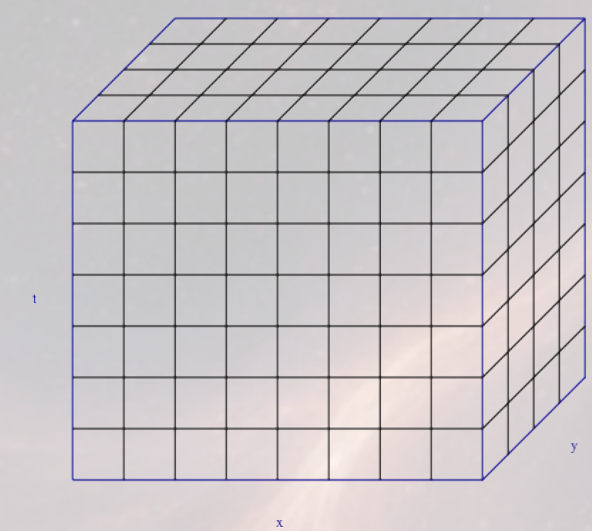
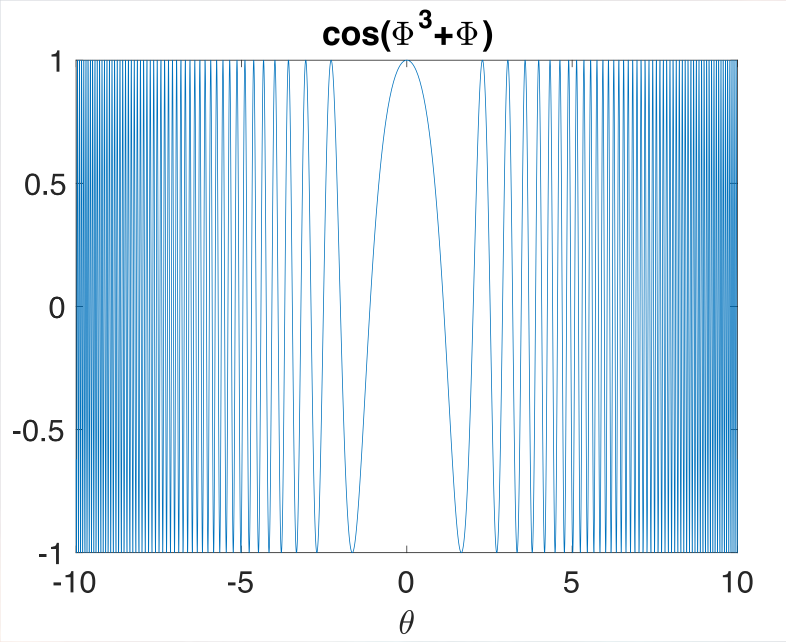
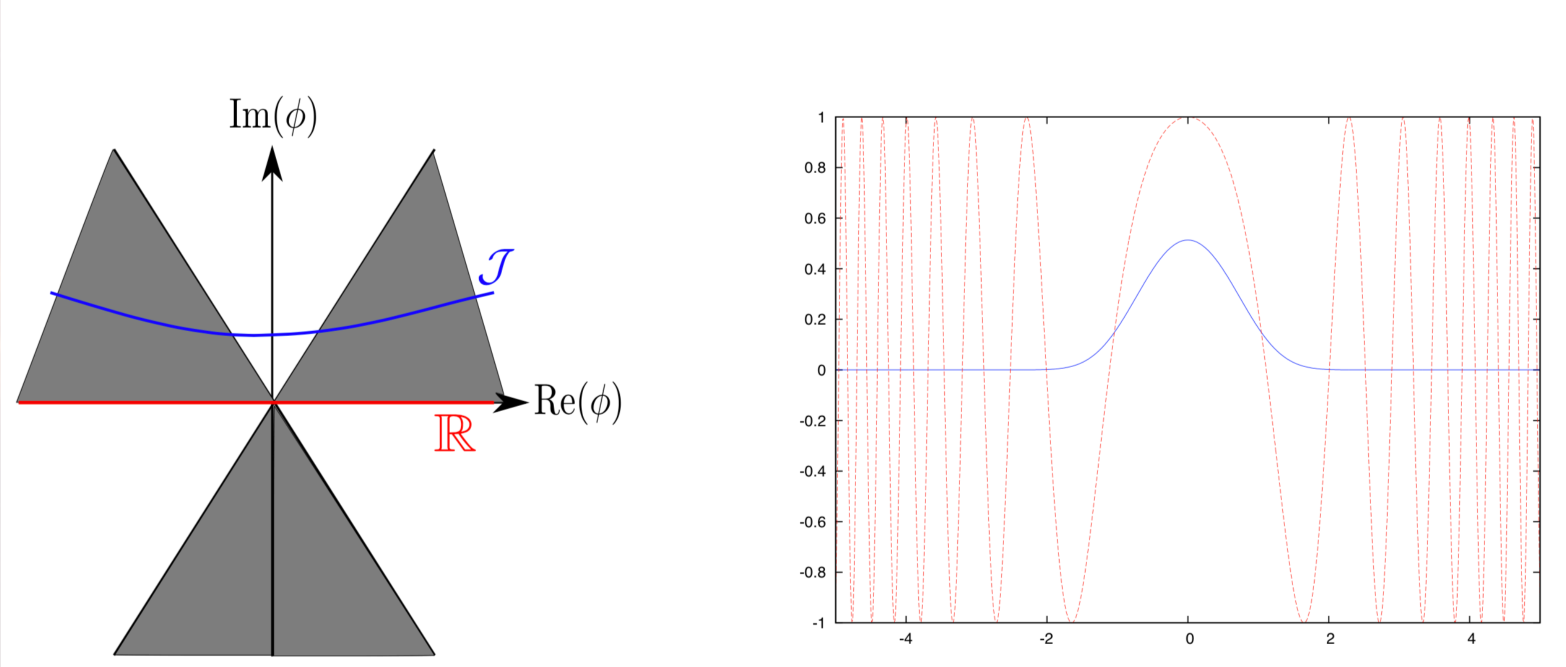


- It's a phase

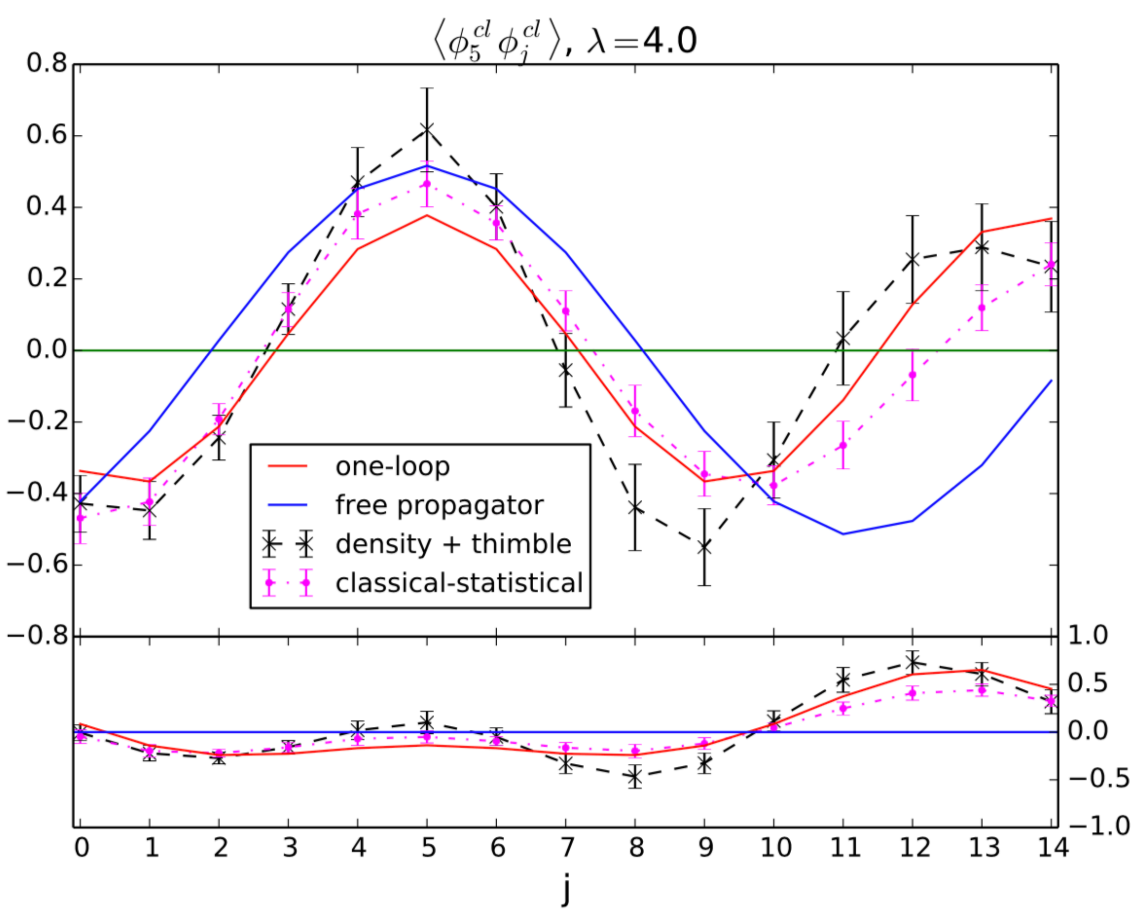


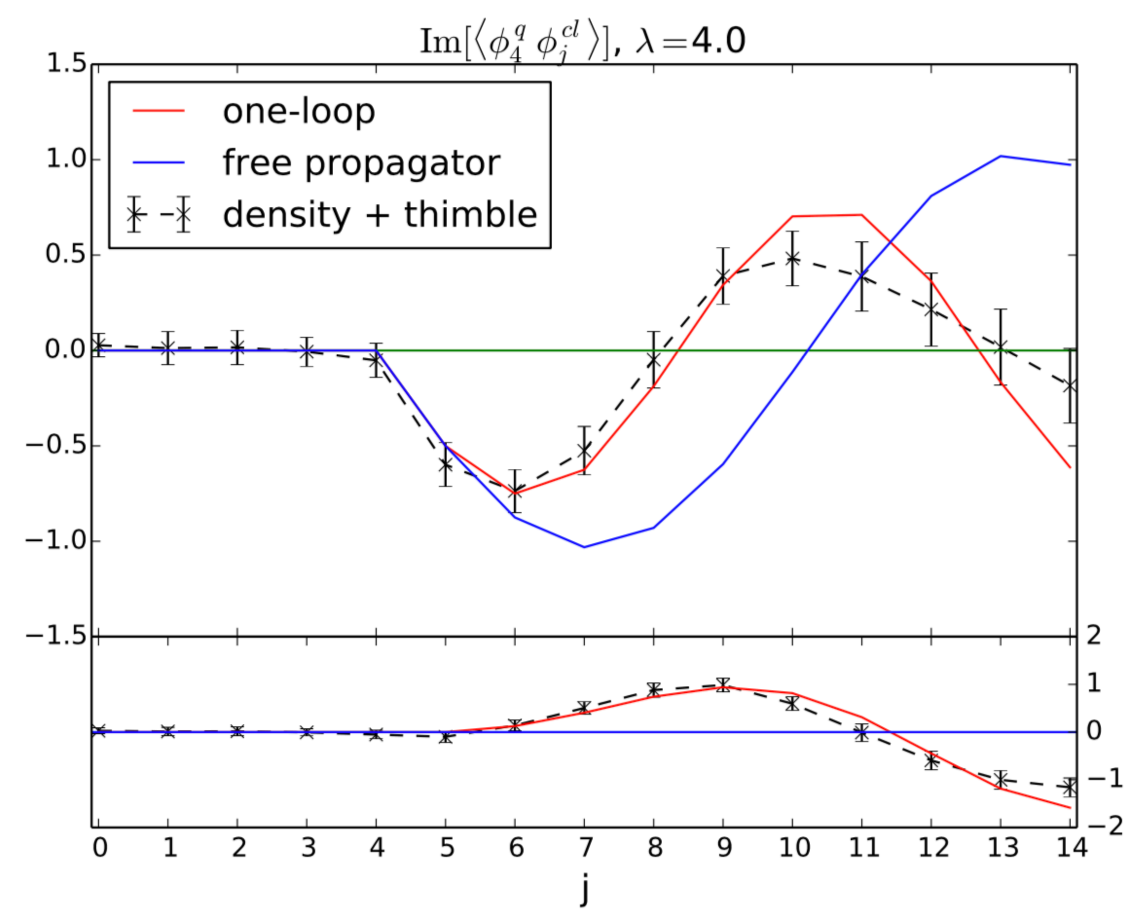
- It’s big:
    - use Monte Carlo sampling to evaluate the integrals

- It's big:
  - use Monte Carlo sampling to evaluate the integrals
- It's a phase:
  - use Picard-Lefschetz (Cauchy's theorem) to improve convergence

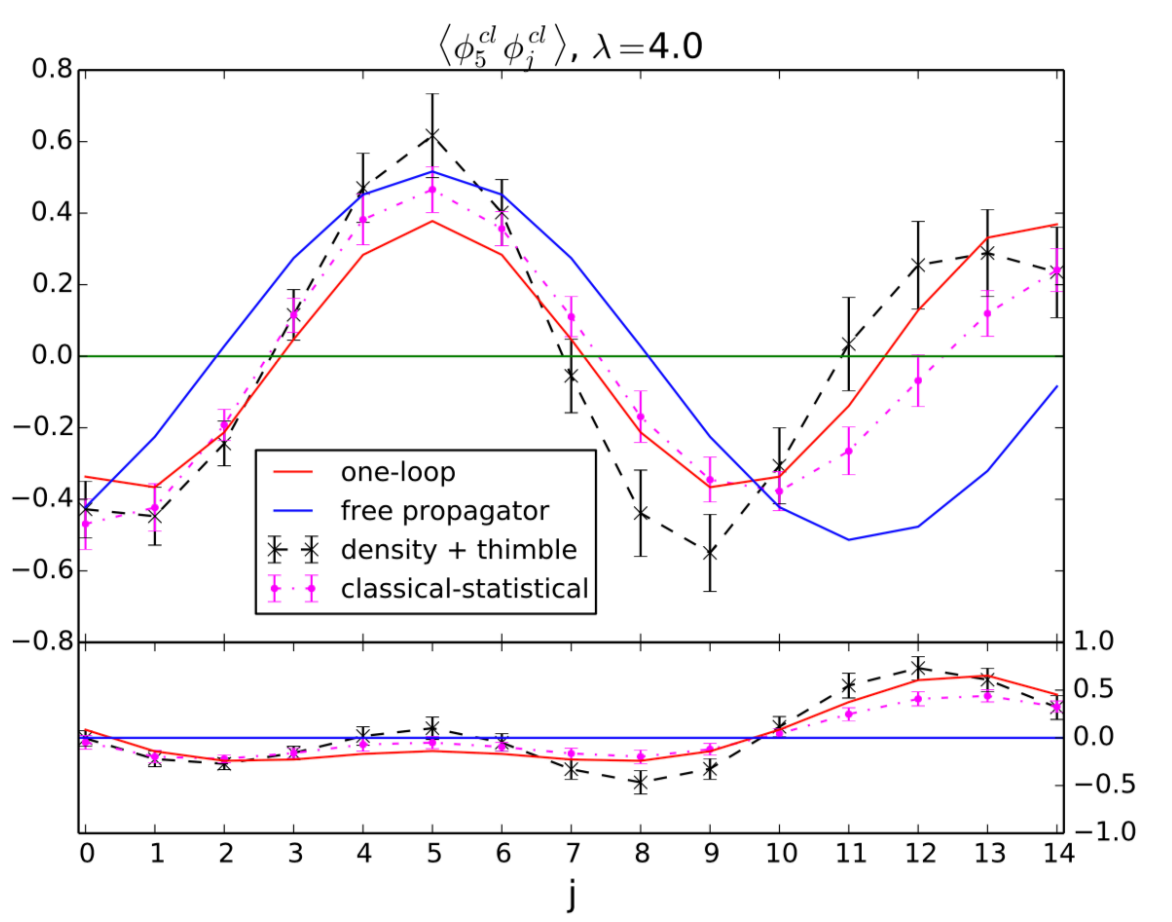


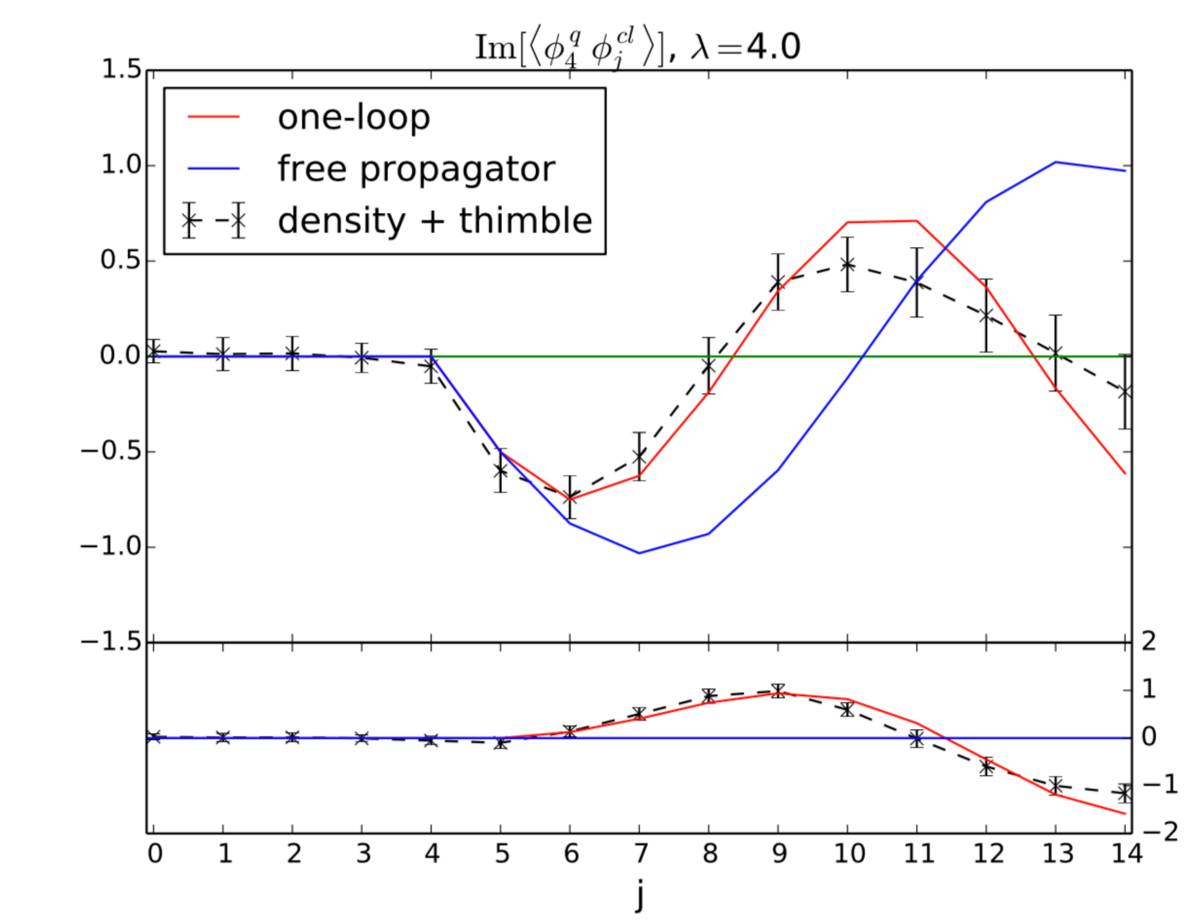
- Aarts, Alexandru, Basar, Bedaque, Christoforetti, , Feldbrugge, Fujii, Honda, Kato, Kikukawa, Komatsu, Lehners, Mukherjee, Di Renz Ridgeway, Sano, Scorzato, Seiler, Sexty, Turok, Warrington, Witten,…

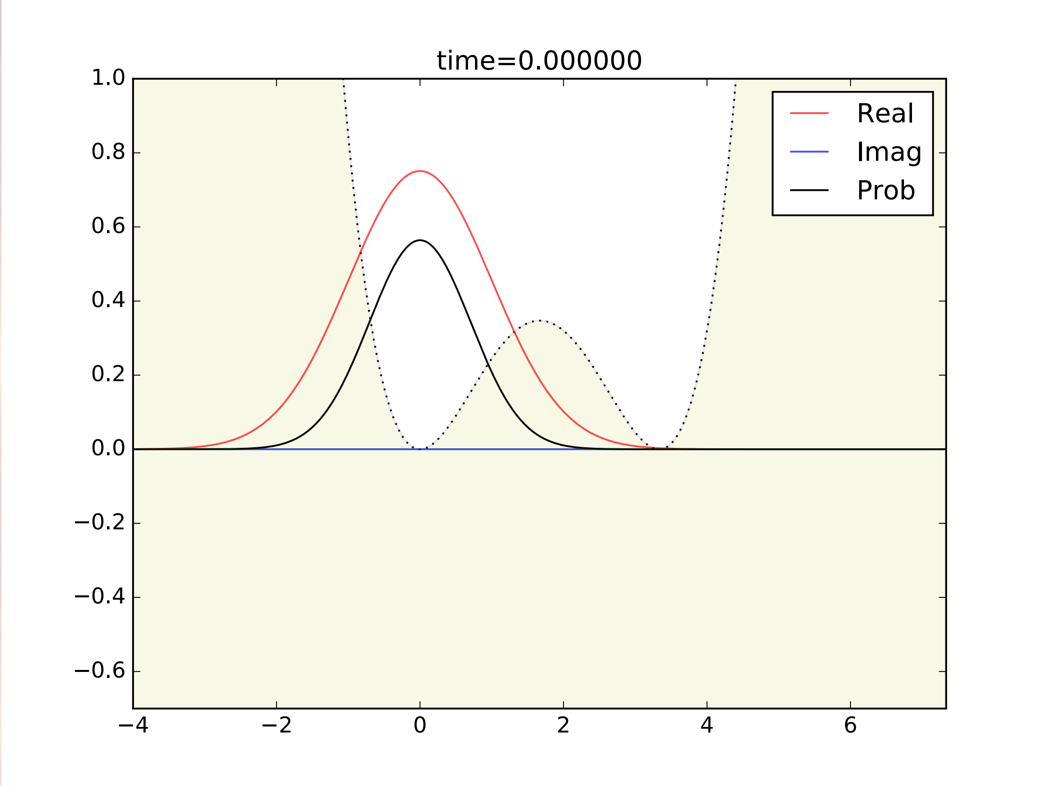


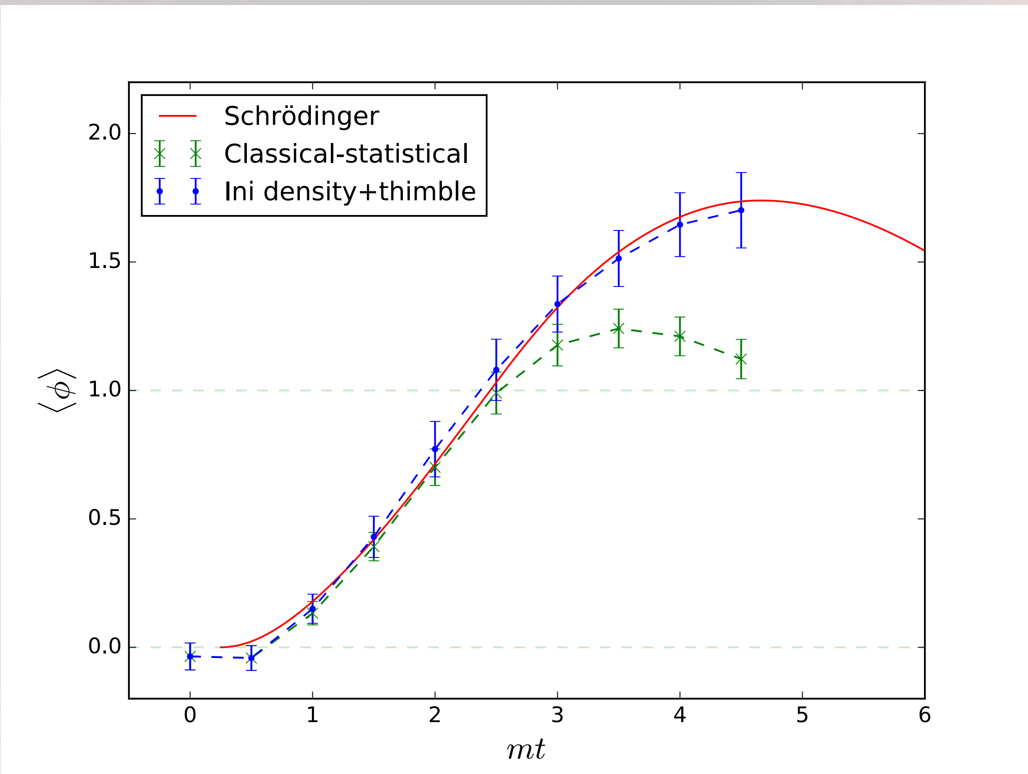


- Mou, PMS, Tranberg, Woodward









- Mou, PMS, Tranberg, Woodward
- Mou, PMS, Tranberg

Summary

- How to simulate?
  - Lattice gauge methods
  - PDE methods

$$
\begin{gathered}
\mathcal{L}_{SM} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w(A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
\beta_h\left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \tfrac{2M^4}{g^2}\alpha_h - \\
g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
\tfrac{1}{8}g^2\alpha_h\left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \\
\tfrac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\tfrac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\left(\tfrac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w MA_\mu(W^+_\mu\phi^- - \\
W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \\
\tfrac{1}{4}g^2 W^+_\mu W^-_\mu\left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) - \tfrac{1}{8}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-\right) - \\
\tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \tfrac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^2 s_w^2 A_\mu A_\mu\phi^+\phi^- + \tfrac{1}{2}ig_s\,\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial + m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + \\
m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_w A_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\tfrac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - 1 - \gamma^5)d^\lambda_j) + \\
(\bar{u}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 + \gamma^5)u^\lambda_j)\} + \tfrac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \\
\tfrac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \tfrac{g}{2}\tfrac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \\
\tfrac{g}{2}\tfrac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) + \tfrac{ig}{2}\tfrac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \tfrac{ig}{2}\tfrac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda) - \tfrac{1}{4}\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa - \\
\tfrac{1}{4}\overline{\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa} + \tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \tfrac{g}{2}\tfrac{m^\lambda_u}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \\
\tfrac{g}{2}\tfrac{m^\lambda_d}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \tfrac{ig}{2}\tfrac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu\bar{G}^a G^b g^c_\mu +
\end{gathered}
$$

Summary

- How to simulate?
  - Lattice gauge methods
  - PDE methods

$$
\begin{gathered}
\mathcal{L}_{SM} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4} g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
igs_w(\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w (A_\mu Z^0_\nu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
\beta_h \left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g} H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-) \right) + \tfrac{2M^4}{g^2}\alpha_h - \\
g\alpha_h M\,(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-) - \\
\tfrac{1}{8} g^2 \alpha_h\,(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2) - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2} g \tfrac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \\
\tfrac{1}{2} ig \left(W^+_\mu (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W^-_\mu (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0) \right) + \\
\tfrac{1}{2} g \left(W^+_\mu (H\partial_\mu \phi^- - \phi^- \partial_\mu H) + W^-_\mu (H\partial_\mu \phi^+ - \phi^+ \partial_\mu H) \right) + \tfrac{1}{2} g \tfrac{1}{c_w} (Z^0_\mu (H\partial_\mu \phi^0 - \phi^0 \partial_\mu H) + \\
M\,(\tfrac{1}{c_w} Z^0_\mu \partial_\mu \phi^0 + W^+_\mu \partial_\mu \phi^- + W^-_\mu \partial_\mu \phi^+) - ig \tfrac{s_w^2}{c_w} M Z^0_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) + igs_w M A_\mu (W^+_\mu \phi^- - \\
W^-_\mu \phi^+) - ig \tfrac{1-2c_w^2}{2c_w} Z^0_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + igs_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \\
\tfrac{1}{4} g^2 W^+_\mu W^-_\mu\,(H^2 + (\phi^0)^2 + 2\phi^+\phi^-) - \tfrac{1}{8} g^2 \tfrac{1}{c_w^2} Z^0_\mu Z^0_\mu\,(H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+\phi^-) - \\
\tfrac{1}{2} g^2 \tfrac{s_w^2}{c_w} Z^0_\mu \phi^0 (W^+_\mu \phi^- + W^-_\mu \phi^+) - \tfrac{1}{2} ig^2 \tfrac{s_w^2}{c_w} Z^0_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) + \tfrac{1}{2} g^2 s_w A_\mu \phi^0 (W^+_\mu \phi^- + \\
W^-_\mu \phi^+) + \tfrac{1}{2} ig^2 s_w A_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) - g^2 \tfrac{s_w}{c_w} (2c_w^2 - 1) Z^0_\mu A_\mu \phi^+ \phi^- - \\
g^2 s_w^2 A_\mu A_\mu \phi^+ \phi^- + \tfrac{1}{2} ig_s\,\lambda^a_{ij} (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu - \bar{e}^\lambda (\gamma\partial + m^\lambda_e) e^\lambda - \bar{\nu}^\lambda (\gamma\partial + m^\lambda_\nu) \nu^\lambda - \bar{u}^\lambda_j (\gamma\partial + \\
m^\lambda_u) u^\lambda_j - \bar{d}^\lambda_j (\gamma\partial + m^\lambda_d) d^\lambda_j + igs_w A_\mu \left(-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \tfrac{2}{3} (\bar{u}^\lambda_j \gamma^\mu u^\lambda_j) - \tfrac{1}{3} (\bar{d}^\lambda_j \gamma^\mu d^\lambda_j) \right) + \\
\tfrac{ig}{4c_w} Z^0_\mu \{ (\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{d}^\lambda_j \gamma^\mu (\tfrac{4}{3} s_w^2 - 1 - \gamma^5) d^\lambda_j) + \\
(\bar{u}^\lambda_j \gamma^\mu (1 - \tfrac{8}{3} s_w^2 + \gamma^5) u^\lambda_j) \} + \tfrac{ig}{2\sqrt{2}} W^+_\mu \left((\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) U^{lep}{}_{\lambda\kappa} e^\kappa) + (\bar{u}^\lambda_j \gamma^\mu (1+\gamma^5) C_{\lambda\kappa} d^\kappa_j) \right) + \\
\tfrac{ig}{2\sqrt{2}} W^-_\mu \left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda} \gamma^\mu (1+\gamma^5) \nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda} \gamma^\mu (1+\gamma^5) u^\lambda_j) \right) + \\
\tfrac{ig}{2M\sqrt{2}} \phi^+ \left(-m^\kappa_e (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1-\gamma^5) e^\kappa) + m^\lambda_\nu (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1+\gamma^5) e^\kappa \right) + \\
\tfrac{ig}{2M\sqrt{2}} \phi^- \left(m^\lambda_e (\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa} (1+\gamma^5) \nu^\kappa) - m^\kappa_\nu (\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa} (1-\gamma^5) \nu^\kappa \right) - \tfrac{g}{2} \tfrac{m^\lambda_\nu}{M} H (\bar{\nu}^\lambda \nu^\lambda) - \\
\tfrac{g}{2} \tfrac{m^\lambda_e}{M} H (\bar{e}^\lambda e^\lambda) + \tfrac{ig}{2} \tfrac{m^\lambda_\nu}{M} \phi^0 (\bar{\nu}^\lambda \gamma^5 \nu^\lambda) - \tfrac{ig}{2} \tfrac{m^\lambda_e}{M} \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda) - \tfrac{1}{4}\,\bar{\nu}_\lambda\, M^R_{\lambda\kappa}\,(1-\gamma_5) \hat{\nu}_\kappa - \\
\tfrac{1}{4}\,\bar{\nu}_\lambda\, M^R_{\lambda\kappa}\,(1-\gamma_5) \hat{\nu}_\kappa + \tfrac{ig}{2M\sqrt{2}} \phi^+ \left(-m^\kappa_d (\bar{u}^\lambda_j C_{\lambda\kappa} (1-\gamma^5) d^\kappa_j) + m^\lambda_u (\bar{u}^\lambda_j C_{\lambda\kappa} (1+\gamma^5) d^\kappa_j \right) + \\
\tfrac{ig}{2M\sqrt{2}} \phi^- \left(m^\lambda_d (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1+\gamma^5) u^\kappa_j) - m^\kappa_u (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1-\gamma^5) u^\kappa_j \right) - \tfrac{g}{2} \tfrac{m^\lambda_u}{M} H (\bar{u}^\lambda_j u^\lambda_j) - \\
\tfrac{g}{2} \tfrac{m^\lambda_d}{M} H (\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2} \tfrac{m^\lambda_u}{M} \phi^0 (\bar{u}^\lambda_j \gamma^5 u^\lambda_j) - \tfrac{ig}{2} \tfrac{m^\lambda_d}{M} \phi^0 (\bar{d}^\lambda_j \gamma^5 d^\lambda_j) + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g^c_\mu +
\end{gathered}
$$

- Early Universe dynamics
  - Inflation
  - Preheating
  - EW epoch/BSM physics

Summary

- How to simulate?
  - Lattice gauge methods
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$$
\begin{aligned}
\mathcal{L}_{SM} = & -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4} g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
& M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - \\
& W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
& igs_w(\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - \\
& W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
& Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w (A_\mu Z^0_\nu (W^+_\mu W^-_\nu - \\
& W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
& \beta_h \left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g} H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \tfrac{2M^4}{g^2}\alpha_h - \\
& g\alpha_h M (H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-) - \\
& \tfrac{1}{8} g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2) - \\
& gMW^+_\mu W^-_\mu H - \tfrac{1}{2} g \tfrac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \\
& \tfrac{1}{2} ig \left(W^+_\mu (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W^-_\mu (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)\right) + \\
& \tfrac{1}{2} g \left(W^+_\mu (H\partial_\mu \phi^- - \phi^- \partial_\mu H) + W^-_\mu (H\partial_\mu \phi^+ - \phi^+ \partial_\mu H)\right) + \tfrac{1}{2} g \tfrac{1}{c_w} (Z^0_\mu (H\partial_\mu \phi^0 - \phi^0 \partial_\mu H) + \\
& M (\tfrac{1}{c_w} Z^0_\mu \partial_\mu \phi^0 + W^+_\mu \partial_\mu \phi^- + W^-_\mu \partial_\mu \phi^+) - ig \tfrac{s_w^2}{c_w} M Z^0_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) + igs_w M A_\mu (W^+_\mu \phi^- - \\
& W^-_\mu \phi^+) - ig \tfrac{1-2c_w^2}{2c_w} Z^0_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + igs_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \\
& \tfrac{1}{4} g^2 W^+_\mu W^-_\mu (H^2 + (\phi^0)^2 + 2\phi^+\phi^-) - \tfrac{1}{8} g^2 \tfrac{1}{c_w^2} Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+\phi^-) - \\
& \tfrac{1}{2} g^2 \tfrac{s_w^2}{c_w} Z^0_\mu \phi^0 (W^+_\mu \phi^- + W^-_\mu \phi^+) - \tfrac{1}{2} ig^2 \tfrac{s_w^2}{c_w} Z^0_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) + \tfrac{1}{2} g^2 s_w A_\mu \phi^0 (W^+_\mu \phi^- + \\
& W^-_\mu \phi^+) + \tfrac{1}{2} ig^2 s_w A_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) - g^2 \tfrac{s_w}{c_w} (2c_w^2 - 1) Z^0_\mu A_\mu \phi^+\phi^- - \\
& g^2 s_w^2 A_\mu A_\mu \phi^+\phi^- + \tfrac{1}{2} ig_s \lambda^a_{ij} (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu - \bar{e}^\lambda (\gamma\partial + m^\lambda_e) e^\lambda - \bar{\nu}^\lambda (\gamma\partial + m^\lambda_\nu) \nu^\lambda - \bar{u}^\lambda_j (\gamma\partial + \\
& m^\lambda_u) u^\lambda_j - \bar{d}^\lambda_j (\gamma\partial + m^\lambda_d) d^\lambda_j + igs_w A_\mu \left(-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j \gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j \gamma^\mu d^\lambda_j)\right) + \\
& \tfrac{ig}{4c_w} Z^0_\mu \{(\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{d}^\lambda_j \gamma^\mu (\tfrac{4}{3} s_w^2 - 1 - \gamma^5) d^\lambda_j) + \\
& (\bar{u}^\lambda_j \gamma^\mu (1 - \tfrac{8}{3} s_w^2 + \gamma^5) u^\lambda_j)\} + \tfrac{ig}{2\sqrt{2}} W^+_\mu \left((\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) U^{lep}{}_{\lambda\kappa} e^\kappa) + (\bar{u}^\lambda_j \gamma^\mu (1+\gamma^5) C_{\lambda\kappa} d^\kappa_j)\right) + \\
& \tfrac{ig}{2\sqrt{2}} W^-_\mu \left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda} \gamma^\mu (1+\gamma^5) \nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda} \gamma^\mu (1+\gamma^5) u^\lambda_j)\right) + \\
& \tfrac{ig}{2M\sqrt{2}} \phi^+ \left(-m^\kappa_e (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1-\gamma^5) e^\kappa) + m^\lambda_\nu (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1+\gamma^5) e^\kappa\right) + \\
& \tfrac{ig}{2M\sqrt{2}} \phi^- \left(m^\lambda_e (\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa} (1+\gamma^5) \nu^\kappa) - m^\kappa_\nu (\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa} (1-\gamma^5) \nu^\kappa\right) - \tfrac{g}{2} \tfrac{m^\lambda_\nu}{M} H (\bar{\nu}^\lambda \nu^\lambda) - \\
& \tfrac{g}{2} \tfrac{m^\lambda_e}{M} H (\bar{e}^\lambda e^\lambda) + \tfrac{ig}{2} \tfrac{m^\lambda_\nu}{M} \phi^0 (\bar{\nu}^\lambda \gamma^5 \nu^\lambda) - \tfrac{ig}{2} \tfrac{m^\lambda_e}{M} \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda) - \tfrac{1}{4} \bar{\nu}_\lambda M^R_{\lambda\kappa} (1-\gamma_5) \hat{\nu}_\kappa - \\
& \tfrac{1}{4} \bar{\nu}_\lambda M^R_{\lambda\kappa} (1-\gamma_5) \hat{\nu}_\kappa + \tfrac{ig}{2M\sqrt{2}} \phi^+ \left(-m^\kappa_d (\bar{u}^\lambda_j C_{\lambda\kappa} (1-\gamma^5) d^\kappa_j) + m^\lambda_u (\bar{u}^\lambda_j C_{\lambda\kappa} (1+\gamma^5) d^\kappa_j\right) + \\
& \tfrac{ig}{2M\sqrt{2}} \phi^- \left(m^\lambda_d (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1+\gamma^5) u^\kappa_j) - m^\kappa_u (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1-\gamma^5) u^\kappa_j\right) - \tfrac{g}{2} \tfrac{m^\lambda_u}{M} H (\bar{u}^\lambda_j u^\lambda_j) - \\
& \tfrac{g}{2} \tfrac{m^\lambda_d}{M} H (\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2} \tfrac{m^\lambda_u}{M} \phi^0 (\bar{u}^\lambda_j \gamma^5 u^\lambda_j) - \tfrac{ig}{2} \tfrac{m^\lambda_d}{M} \phi^0 (\bar{d}^\lambda_j \gamma^5 d^\lambda_j) + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g^c_\mu +
\end{aligned}
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- Early Universe dynamics
  - Inflation
  - Preheating
  - EW epoch/BSM physics

- classical/quantum
  - Classical-statistical
  - 2PI
  - thimble